Department of Fluid Dynamics, Tel Aviv University, Israel

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Single and Multi-Grid Solution of Incompressible Navier- Stokes Equations on Massively Parallel Supercomputers

Yuri Feldman and Alexander Yu. Gelfgat

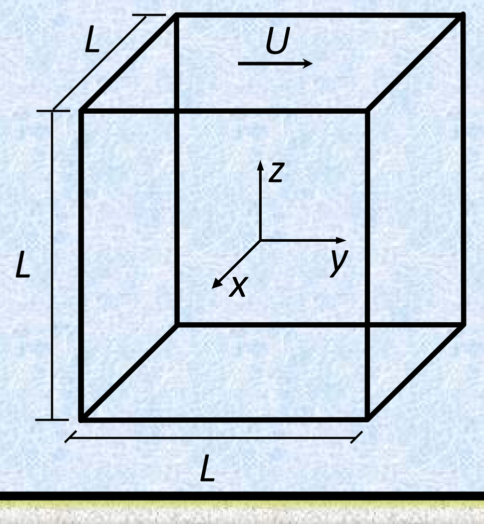
School of Mechanical Engineering, Faculty of Engineering, Tel-Aviv University, Ramat Aviv ,Israel

INTRODUCTION

Reliable and robust numerical solution of unsteady three-dimensional fluid dynamics problems at large Reynolds numbers remains a challenging task of modern computational fluid dynamics. In this context a parallel implementation of fully pressure-velocity coupled multigrid solver based on analytical solution accelerated (ASA-CLGS) approach has been developed and successfully parallelized for running on massively parallel platforms The parallelized algorithm is characterized by enhanced scalability taking advantage of an existence of analytical solution for the entire row (column) of control volumes. The parallel performances and speedups are presented for up to 2048 processors for both single- and multigrid approaches. The developed parallelized algorithm is applied for analysis of a time-dependent three-dimensional incompressible lid-driven cavity flow. As an example of fully 3D flow the lid-driven cubic cavity with the lid moving in parallel and at 45º relatively to its lateral boundaries is considered

GEOMETRY AND NUMERICAL MODEL

Geometry



Governing equations

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u}$$

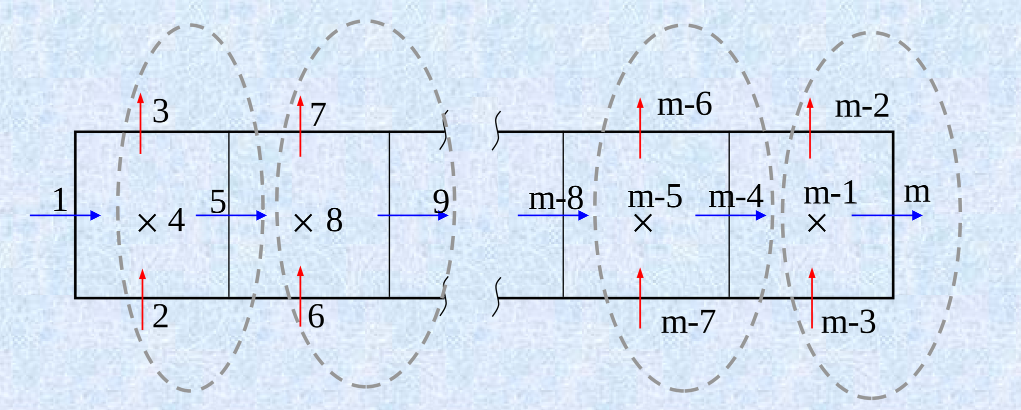
Boundary conditions

Non-slip velocities on all boundaries

No boundary conditions for pressure

MULTIGRID FORMULATION

Feldman and Gelfgat (2008) – ASA-CLGS:Horizontal (vertical) sweep *without* horizontally (vertically) adjacent pressure linkage



$$\ldots$$

$$A^{(x)}_{i+1/2,j} u'_{i+1/2,j} + B^{(x)}_{i+1/2,j} p'_{i,j} = R^{(x)}_{i+1/2,j}$$

$$A^{(x)}_{i-1/2,j} u'_{i-1/2,j} - B^{(x)}_{i-1/2,j} p'_{i,j} = R^{(x)}_{i-1/2,j}$$

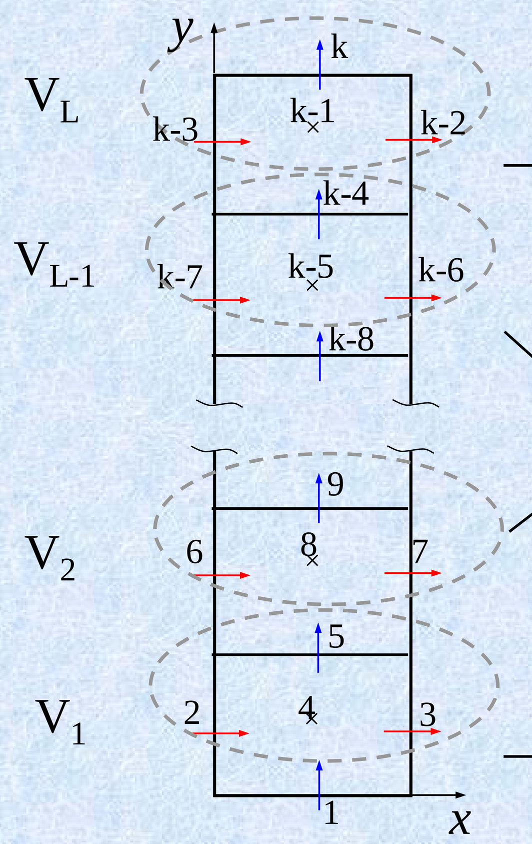
$$A^{(y)}_{i,j+1/2} v'_{ij+1/2} + B^{(y)}_{i,j+1/2} p'_{i,j} = \tilde{R}^{(y)}_{i,j+1/2}$$

$$A^{(x)}_{i,j}\left(u'_{i+1/2,j} - u'_{i-1/2,j}\right) + A^{(y)}_{i,j}\left(v'_{i,j+1/2} - v'_{i,j-1/2}\right) = 0$$

$$\ldots$$

where $\tilde{R}^{(y)}_{i,j+1/2} = R^{(y)}_{i,j+1/2} + B^{(y)}_{i,j+1/2} p'_{i,j+1}$

A schematic description of ASA-CLGS smoother

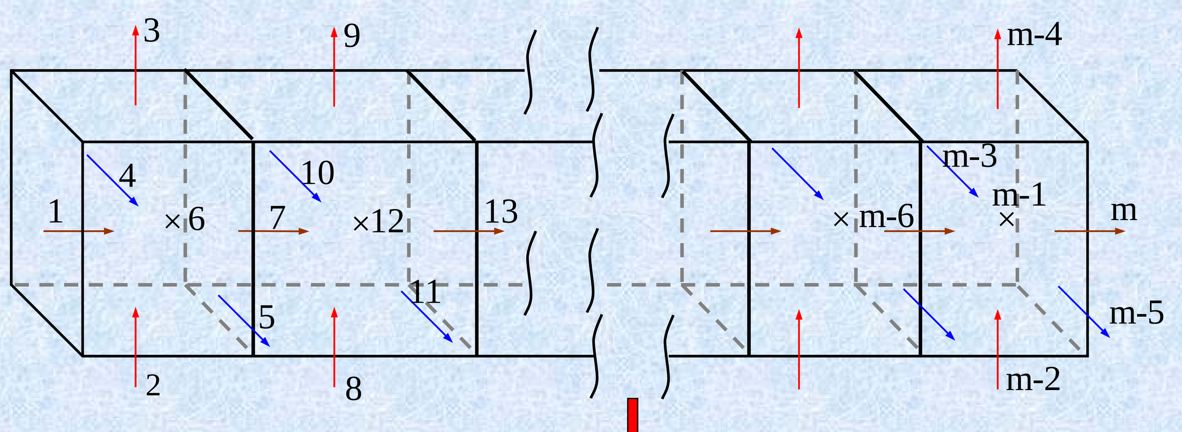


$$\begin{bmatrix} A_1^L & 0 & 0 & 0 & A_2^L \\ 0 & A_3^L & 0 & 0 & A_4^L \\ 0 & 0 & A_5^L & 0 & A_6^L \\ 0 & 0 & 0 & 1 & 0 \\ A_7^L - A_7^L & A_8^L & -A_8^L & 0 \end{bmatrix} \times \begin{bmatrix} u'_{k-2} \\ u'_{k-3} \\ v'_k \\ v'_{k-4} \\ p'_{k-1} \end{bmatrix} = \begin{bmatrix} R_{k-2} \\ R_{k-3} \\ R_k \\ R_{k-4} \\ R_{k-1} \end{bmatrix} \Rightarrow \begin{array}{l} u'_{k-2} = \alpha^L_{k-2} p'_{k-1} + \beta^L_{k-2} \\ u'_{k-3} = \alpha^L_{k-3} p'_{k-1} + \beta^L_{k-3} \\ v'_k = \textit{from b.c.} \\ v'_{k-4} \textit{ from volume } L-1 \\ p'_{k-1} = \alpha^L_{k-1} v'_{k-4} + \beta^L_{k-1} \end{array}$$

$$\begin{bmatrix} A_1^I & 0 & 0 & 0 & A_2^I \\ 0 & A_3^I & 0 & 0 & A_4^I \\ 0 & 0 & A_5^I & 0 & A_6^I \\ 0 & 0 & 0 & 1 & 0 \\ A_7^I - A_7^I & A_8^I & -A_8^I & 0 \end{bmatrix} \times \begin{bmatrix} u'_{m-2} \\ u'_{m-3} \\ v'_m \\ v'_{m-4} \\ p'_{m-1} \end{bmatrix} = \begin{bmatrix} R_{m-2} \\ R_{m-3} \\ R_m \\ R_{m-4} \\ R_{m-1} \end{bmatrix} \Rightarrow \begin{array}{l} u'_{m-2} = \alpha^L_{m-2} p'_{m-1} + \beta^L_{m-2} \\ u'_{m-3} = \alpha^L_{m-3} p'_{m-1} + \beta^L_{m-3} \\ v'_m = \alpha^L_m p'_{m-1} + \beta^L_{m-4} \\ v'_{m-4} \textit{ from volume } M-1 \\ p'_{m-1} = \alpha^L_{m-1} v'_{m-4} + \beta^L_{m-1} \end{array}$$

$$\begin{bmatrix} A_1^1 & 0 & 0 & 0 & A_2^1 \\ 0 & A_3^1 & 0 & 0 & A_4^1 \\ 0 & 0 & A_5^1 & 0 & A_6^1 \\ 0 & 0 & 0 & 1 & 0 \\ A_7^1 - A_7^1 & A_8^1 & -A_8^1 & 0 \end{bmatrix} \times \begin{bmatrix} u'_3 \\ u'_2 \\ v'_5 \\ v'_1 \\ p'_4 \end{bmatrix} = \begin{bmatrix} R_3 \\ R_2 \\ R_5 \\ R_1 \\ R_4 \end{bmatrix} \Rightarrow \begin{array}{l} u'_3 = \alpha^L_3 p'_4 + \beta^L_3 \\ u'_2 = \alpha^L_2 p'_4 + \beta^L_2 \\ v'_5 = \alpha^L_5 p'_4 + \beta^L_5 \\ v'_1 \textit{ from b.c.} \\ p'_4 = \alpha^L_4 v'_1 + \beta^L_4 \end{array}$$

ASA-CLGS –Efficiency Estimation for 3D



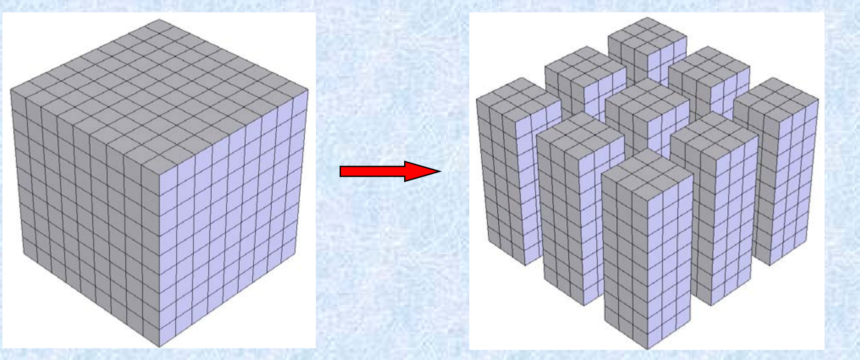
6 corrections for a single volume result in 17 multiplications and divisions and 11 summations $\approx O(5M)$

$$p'_p = \left(c_1^I w'_d + R_p^I + c_2^I R_e^I + c_3^I R_w^I + c_4^I R_n^I + c_5^I R_s^I + c_6^I R_d^I\right)/c_7^I$$

$$\begin{bmatrix} w'_d \\ v'_s \\ v'_n \\ u'_w \\ u'_e \end{bmatrix} = \begin{bmatrix} c_8^I \\ c_9^I \\ c_{10}^I \\ c_{11}^I \\ c_{12}^I \end{bmatrix} \times p'_p + \begin{bmatrix} c_{13}^I R_d^I \\ c_{14}^I R_s^I \\ c_{15}^I R_n^I \\ c_{16}^I R_w^I \\ c_{17}^I R_e^I \end{bmatrix}$$

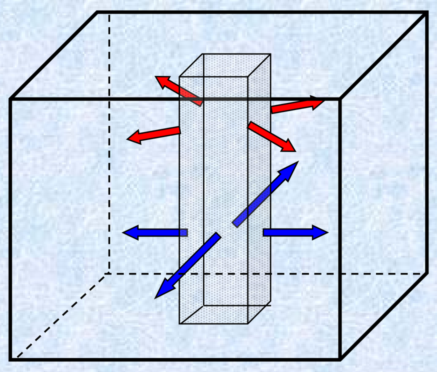
PARALLELIZATION

Domain partition



Existence of **analytical solution** for the whole column allows for 2D virtual topology of 3D configuration.

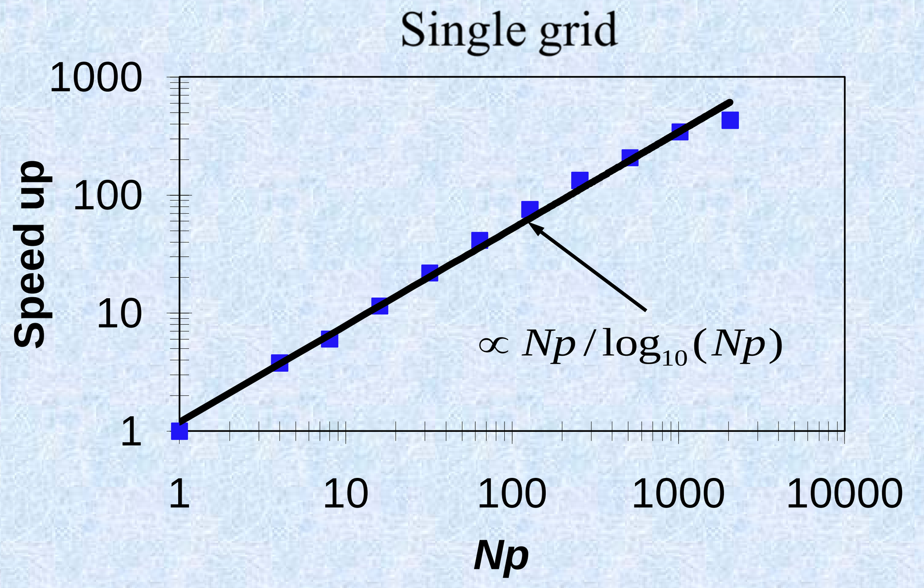
Data communication principle

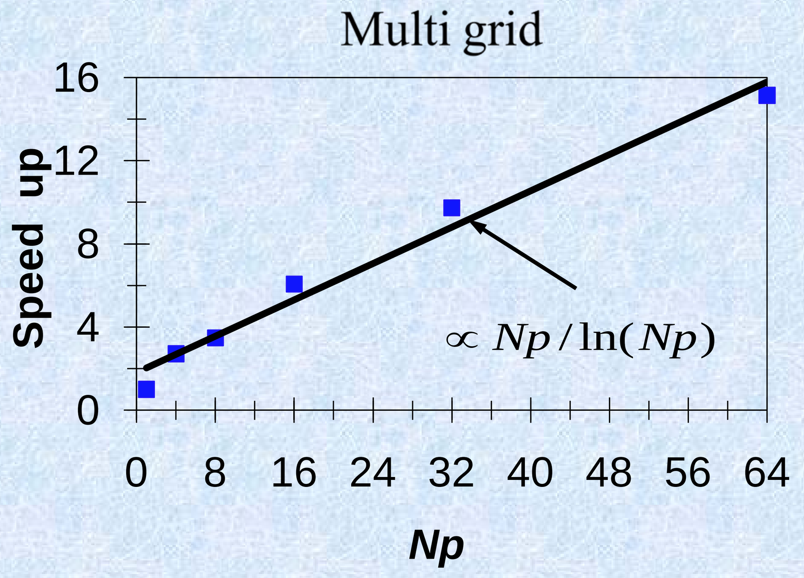


All volumes located at the sub -volume vertical edges exchange data with diagonal neighbors

All volumes located at the sub -volume faces exchange data with neighbors

SCALABILITY PROPERTIES

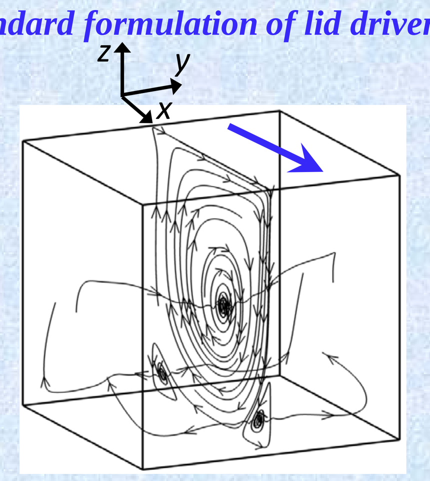




NUMERICAL RESULTS

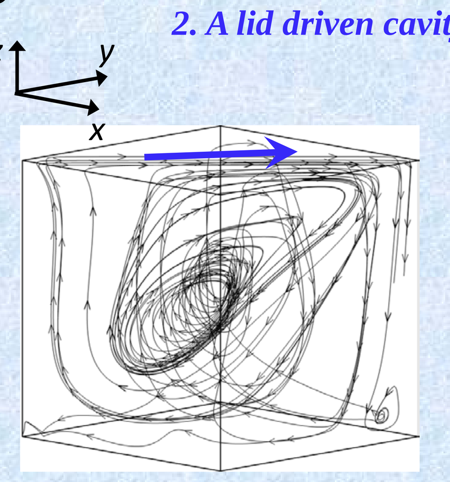
Steady state solution

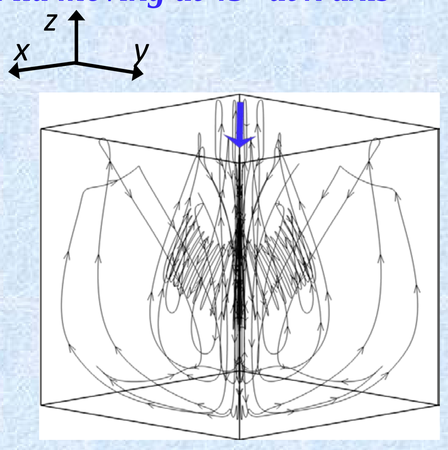
1. A standard formulation of lid driven cavity



Re=1900

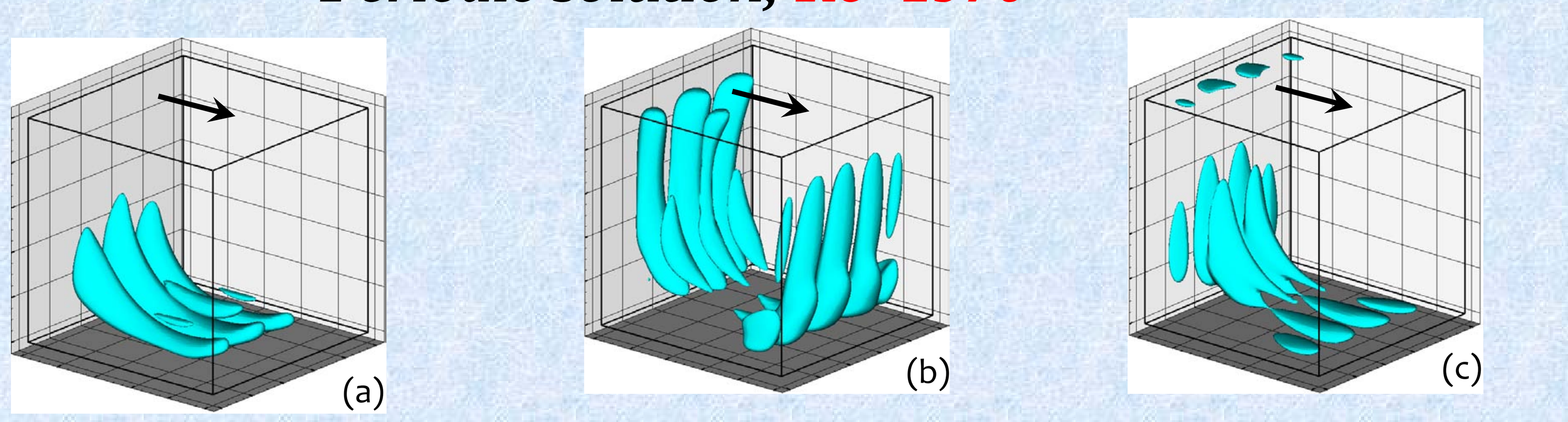
2. A lid driven cavity with lid moving at 45° at x axis





Re=1000

Periodic solution, Re=1970



Iso-surface of oscillations amplitude, 104^3 grid, Re=1970: (a) v_x component; (b) v_z component; (c) spanwise v_y component.

CONCLUSION

A parallelized version of multigrid algorithm based on PAR-ASA-CLGS smoother for time integration of incompressible Navier-Stokes equations on staggered grids has been developed and successfully verified using the lid driven cubic cavity flow benchmark problem. The scalability properties of the algorithm were studied for up to 2048 cores running in parallel reaching the overall speedup of ≈450. It was found that the described multigrid approach is efficient only for those problems that exhibit a slow convergence. When the convergence is fast or initial guess can be chosen close to the solution, the single-grid approach becomes preferable. An example of the latter is time-integration when the current result is used as the initial guess for the next time step.