Single and Multi-Grid Solution of Incompressible Navier-Stokes Equations on Massively Parallel Supercomputers

Yuri Feldman and Alexander Yu. Gelfgat

School of Mechanical Engineering, Faculty of Engineering, Tel-Aviv University, Ramat Aviv, Israel

INTRODUCTION

Reliable and robust numerical solution of unsteady three-dimensional fluid dynamics problems at large Reynolds numbers remains a challenging task of modern computational fluid dynamics. In this context a parallel implementation of fully pressure-velocity coupled multigrid solver based on analytical solution accelerated (ASA-CLGS) approach has been developed and successfully parallelized for running on massively parallel platforms. The parallelized algorithm is characterized by enhanced scalability taking advantage of an existence of analytical solution for the entire row of control volumes. The parallel performances and speedups are presented for up to 2048 processors for both single- and multigrid approaches. The developed parallelized algorithm is applied for analysis of a time-dependent three-dimensional incompressible lid-driven cavity flow. As an example of fully 3D flow the lid-driven cubic cavity with the lid moving in parallel at 45° relatively to its lateral boundaries is considered.

GEOMETRY AND NUMERICAL MODEL

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Governing equations</th>
<th>Boundary conditions</th>
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<tbody>
<tr>
<td></td>
<td>( \nabla \cdot \mathbf{u} = 0 )</td>
<td>Non-slip velocities on all boundaries</td>
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<td>( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} )</td>
<td>No boundary conditions for pressure</td>
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</tbody>
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PARALLELIZATION

Parity partitioning of the whole domain into subdomains.

SCALABILITY PROPERTIES

Numerical results for steady state solution:

1. A standard formulation of lid driven cavity
2. A lid driven cavity with lid moving at 45° at x axis

Numerical results for periodic solution, Re=1970:

Iso-surface of oscillations amplitude, 104 grid, Re=1970: (a) \( \zeta_x \) component; (b) \( \zeta_y \) component; (c) spanwise \( \zeta_z \) component.

CONCLUSION

A parallelized version of parallel multigrid algorithm based on PAR-ASA-CLGS smoother for time integration of incompressible Navier-Stokes equations on staggered grids has been developed and successfully verified using the lid driven cubic cavity flow benchmark problem. The scalability properties of the algorithm were studied for up to 2048 cores running in parallel reaching the overall speedup of ≈450. It was found that the described multigrid approach is efficient only for those problems that exhibit slow convergence. When the convergence is fast or initial guess can be chosen close to the solution, the single-grid approach becomes preferable. An example of the latter is time-integration when the current result is used as the initial guess for the next time step.