



# Power, bistability and post-saturation optimization in a pre-bunched free electron laser

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## Abstract

Prebunching of the electron beam in a free-electron laser enables very high output powers and energy conversion efficiencies ( $> 90\%$ ). The output power is ultimately limited only by the cavity losses. The analysis based upon the pendulum equation predicts two stable steady states. The lower steady state appears due to a new saturation mechanism caused by open phase-space trajectories. The existence of 2 stable saturation powers leads to a bistability in the output power of the pre-bunched FEL oscillator. In fact, when the power in the oscillator reaches the lower steady state there is no further growth. A new technique is proposed to obtain the upper steady-state level. In this approach, a post-saturation dynamical readjustment of the FEL parameters is used to reach the upper stable level of oscillator power emission. © 2000 Published by Elsevier Science B.V. All rights reserved.

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## 1. Introduction

In this article we study the operation of pre-bunched beam FEL in the saturation regime. We identify a gain mechanism which would allow the FEL to operate with the highest efficiency.

The principle of the proposed gain mechanism of a pre-bunched beam is depicted in Fig. 1(top). The drawing displays phase-space evolution of an electron beam bunch trapped inside the ponderomotive potential traps.

The physics is that of a swinging pendulum – as is known the pendulum equation is a good approxi-

mation [3,4] in the non-linear regime. The trapped electrons move along closed orbits. Their energy loss equals the energy difference between the final and the initial values.

In our scheme the electrons are initially bunched, i.e. injected at almost identical phases. Therefore if they enter just below the top of the separatrix and exit the interaction region after half a cycle at the bottom of the separatrix (Fig. 1, top) then the overall energy loss by the electrons is maximal.

What distinguishes the prebunched operation [1,2] is that all the electrons in the beam lose the same amount of energy  $mc^2\Delta\gamma$ . Consequently, no energy spread is generated in the beam thus the oscillator design can be adjusted to maximize energy extraction. The electrons can also be collected efficiently without energy waste in a single stage depressed collector.

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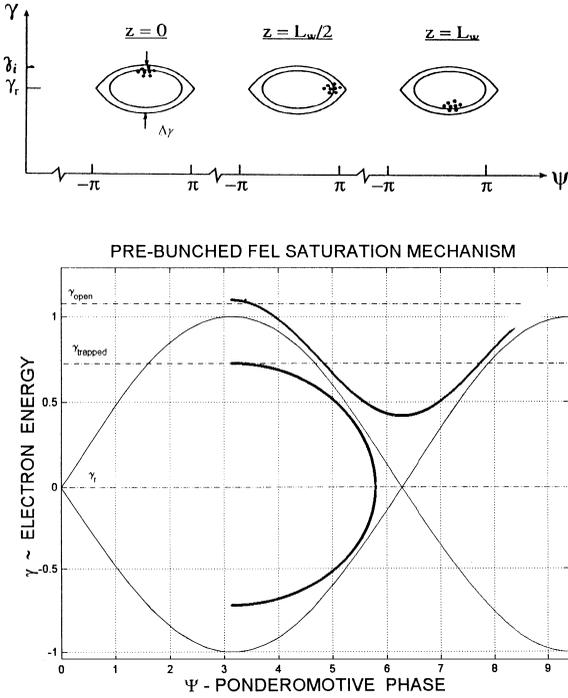


Fig. 1. (top) Operation scheme for a pre-bunched FEL. (bottom) Saturation mechanisms for closed and open trajectories. The closed trajectory (inside the separatrix) gives higher energy difference than an open trajectory.

## 2. Ultimate power and efficiency at steady state

The phase space evolution of the electrons is described by the pendulum equation [3,4]

$$\frac{d^2\psi}{dz^2} = -\Omega^2 \sin \psi. \quad (1)$$

The ponderomotive phase is defined as  $\psi(z) = \int_0^z \theta(z) dz$  here  $\theta(z) = (k_{s,z} + k_w(z)) - \omega_s/v_z(z)$  is the electron detuning relative to the ponderomotive wave,  $\omega_s$  is the signal frequency,  $k_{s,z}$  is the  $z$ -component of the signal wave number,  $k_w = 2\pi/\lambda_w$  is the wiggler ‘wave number’ for period  $\lambda_w$ ,  $v_z(z)$  is the  $z$ -dependent electron axial velocity and  $\Omega$  is the characteristic synchrotron wave number

$$\Omega = \frac{2k_s \sqrt{a_w a_s}}{\gamma \gamma_z} \quad (2)$$

the dimensionless vector potentials are  $a_s = eA_s/m_e c$  and  $a_w = eA_w/m_e c$ .

We analyze the electron trajectories in the  $\theta$  versus  $\psi$  phase space. Let  $\theta_0 = \theta(z=0)$ . For all values of the parameters the solutions are periodic functions of  $z$ . It is significant though that for  $4\Omega^2 > \theta_0^2$  the solutions  $\psi(z)$  are bounded trapped trajectories while for  $4\Omega^2 < \theta_0^2$  the solutions  $\psi(z)$  are unbounded open trajectories. For  $2\Omega = \theta_0$  the solutions form a separatrix in phase space

$$\theta(\psi) = \pm 2\Omega \cos(\psi/2) \quad (3)$$

which separates the open trajectories from the trapped (see Fig. 1, bottom).

The maximal energy loss of the electron occurs if it swings half a cycle – from the top of the trap as it enters the wiggler to the bottom of the trap at the end of the wiggler. This is illustrated in Fig. 1 (bottom). The power extracted from the electrons to the radiation field  $\Delta P$  is a function of the initial detuning  $\theta_0$  and the injected power  $P_0$  which determines the parameter  $\Omega \propto P^{1/4}$ . Shown in Fig. 2 is a contour plot of the generated power  $\Delta P(\theta_0, \Omega)$ . The region of high power  $\Delta P$  corresponds to trapped trajectories  $2\Omega > \theta_0$  and that of low power extraction  $2\Omega < \theta_0$ . Dividing them is a valley of extremely low  $\Delta P$ .

From the definition of the separatrix (3), it follows that the energy difference between the top and the bottom of the separatrix is

$$\Delta\gamma_{\text{max}} = K(\Delta\theta_{\text{max}}) = 4K\Omega \quad (4)$$

here  $K$  is a proportion coefficient. Since  $\Delta P \propto \Delta\gamma$ , while as mentioned above  $\Omega \propto P^{1/4}$ , it follows that in the optimal pre-bunched operation

$$\Delta P \propto P^{1/4}, \quad \frac{\Delta P}{P} \propto P^{-3/4}. \quad (5)$$

As the power in the resonator builds up the relative gain  $\Delta P/P$  falls until it equals the combined transmission and losses from the cavity. This determines the steady-state saturation power.

We denote the transmission coefficient by  $T_m$ . Let the dissipative loss coefficient be  $\mathcal{L}$ . To each transmission coefficient  $T_m$  corresponds to a certain saturation power. For a fixed loss coefficient  $\mathcal{L}$  there exists an optimal mirror transmission  $T_m$  giving *maximum power* output. However higher generation efficiency  $\eta = P_{\text{out}}/\Delta P$  is also desirable.

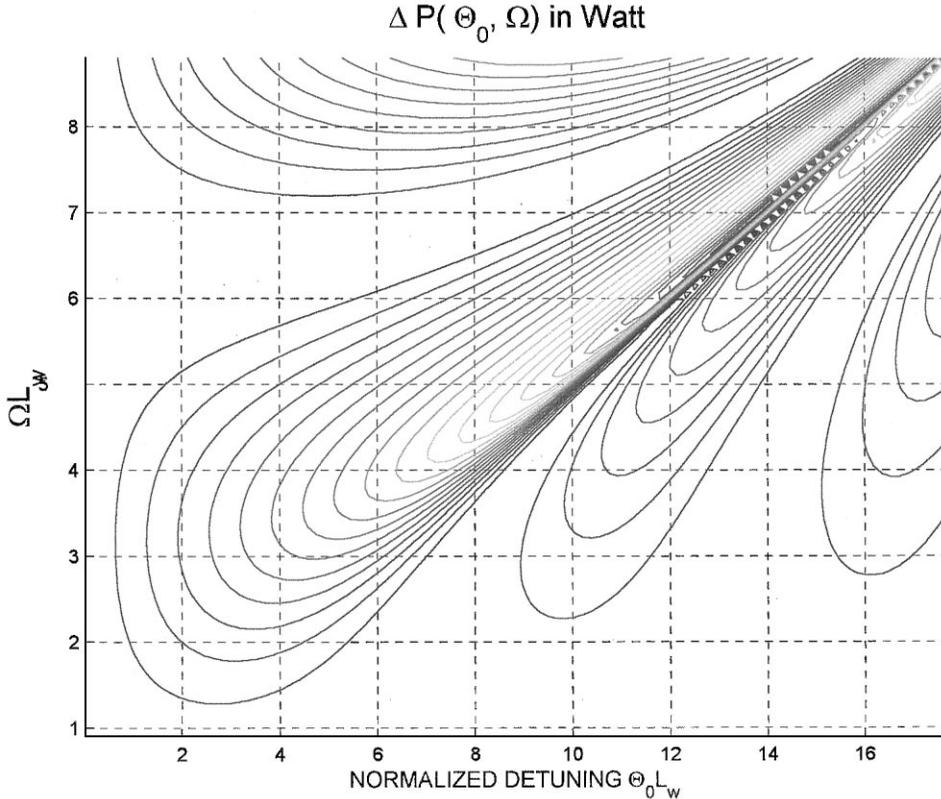


Fig. 2. Contour plot of the power extraction  $\Delta P$  as a function of the initial detuning  $\theta_0$  and the synchrotron parameter  $\Omega \propto P^{1/4}$ .

If full energy retrieval is obtained from the wasted bunched electron beam, then this efficiency factor is the ultimate efficiency of the FEL as a system.

In Fig. 3 we plot the saturation power and efficiency  $\eta$  as a function of the mirror transmission  $T_m$  for a fixed resonator loss  $\mathcal{L} = 1\%$ . There exists a range of  $T_m$  values for which both  $\eta > 90\%$  and  $P_{\text{out}}$  is close to its maximum.

### 3. Power buildup and bistability in the pre-bunched FEL

In the pre-bunched regime of operation there exist some effects not present in the common FEL operation. Specifically, we show below that two different saturation power levels are possible in this case.

We have used the FEL 3D code [5] to simulate a prebunched FEL operation. Fig. 4 shows the exit power  $P(L)$  generated at the end of the interaction

region as a function of the initial power  $P(0)$  present at the beginning of the wiggler (single pass amplification). To locate the steady-state operation points a graph of the steady-state oscillator condition  $P(L) = P(0)/R$  ( $R$  is the mirror reflectivity) is also plotted for  $R = 0.95$ . Fig. 4 reveals three intersections. The lower and the upper ones are stable equilibria, while the middle intersection is an unstable point. The existence of two different steady-states results in a bistable operation of the oscillator as shown in Fig. 5. The lower and upper saturation points relate, respectively, to the open and closed trajectories in Fig. 1 (bottom).

### 4. Enhancement of the saturation power by electron energy ramping

When power in the FEL oscillator grows from zero it will continue to increase, and the relative

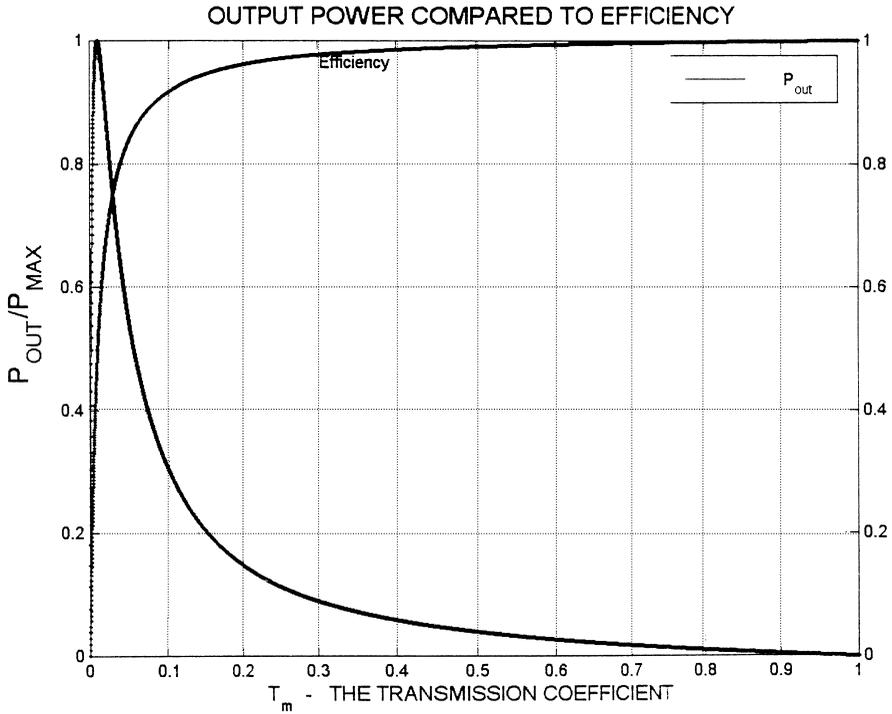


Fig. 3. The normalized power output  $P_{out}/P_{out,max}$  and efficiency  $\eta = P_{out}/\Delta P$  versus the transmission coefficient  $T_m$ . The losses are  $\mathcal{L} = 1\%$ .

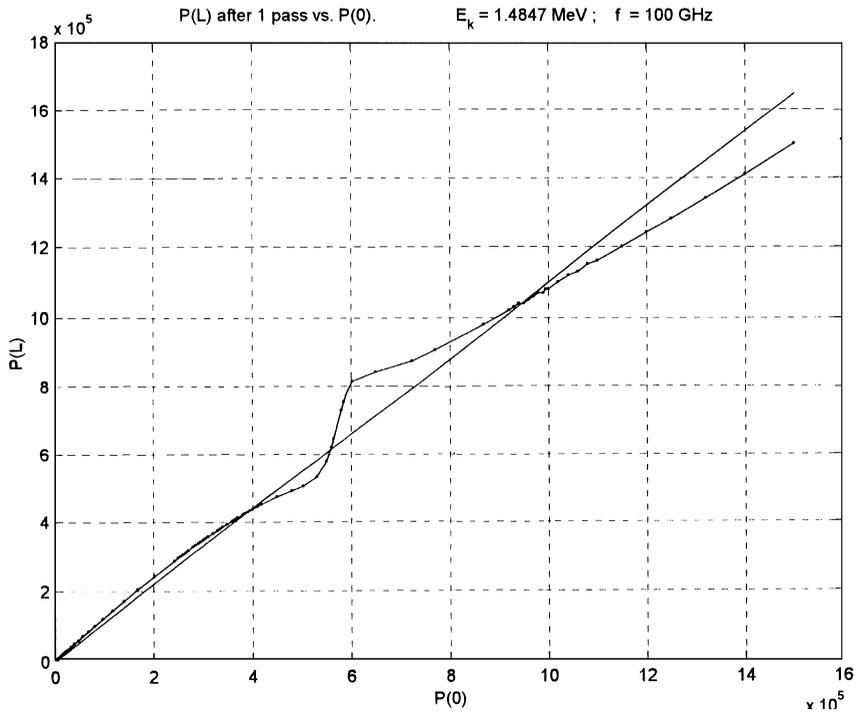


Fig. 4. The power at the end of the wiggler  $P(L)$  as a function of the initial power  $P(0)$ . Single-pass amplification.

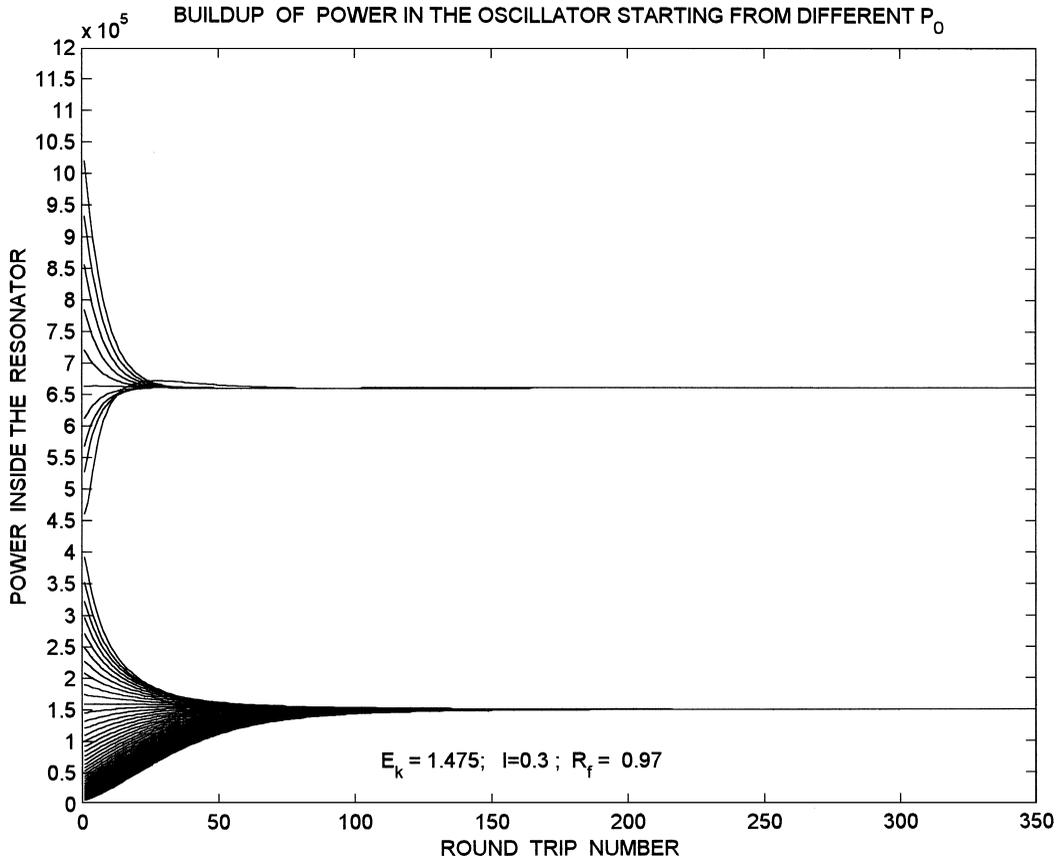


Fig. 5. The evolution of power in the oscillator as a function of round-trip number. Different curves correspond to different initial powers. Two stable steady-states give the saturation powers of the oscillator. The lower steady-state corresponds to open trajectories.

gain  $\Delta P/P$  to decrease, according to Eq. (5) until the steady-state condition

$$P(L) = \frac{P(0)}{R} \quad (6)$$

is met. Unfortunately, the lower saturation power will be the first to be reached. For higher circulating powers – between the 2 lower stable states – there is a negative net power balance  $\Delta P - T \cdot P(L) < 0$  after each single round-trip thus obviously leading to a drop in power. The practical consequence is that the buildup of power to the higher steady state is prevented.

This low steady state can be avoided by post-saturation ramping of the electrons energies. By

ramping-up the electrons injection energies the electrons initial detuning is increased. This would correspond to following the ridge of high extraction power in Fig. 2. Such a scheme provides a way to reach the maximum power extraction from the pre-bunched electron beam.

## 5. Conclusions

In this study we have investigated the properties of the FEL operation in the pre-bunched regime. The pendulum equation model gives a comprehensive picture of the power generation in this case. A new feature found here is the bistability of the FEL oscillator as a function of the starting power.

Electron energy ramping provides a method to obtain the ultimate loss-limited power output.

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