Superradiant and Stimulated Superradiant Emission in a Prebunched Beam Free-Electron Maser

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An electron beam, prebunched at the synchronous free-electron laser frequency and passing through a magnetic undulator, emits coherent (superradiant) synchrotron undulator radiation at the bunching frequency. If an external electromagnetic wave is introduced into the interaction region, at the same frequency and at a proper phase, the radiation process will be stimulated (stimulated prebunched beam radiation). We report first experimental measurements of stimulated superradiant emission in a prebunched free-electron maser. Measurements are in good agreement with theory.

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Radiation sources arranged in space (bunched), such that their radiation field wave packets add up constructively (i.e., are of the same phase), produce superradiant emission.

In the case of an electron beam passing through an undulator, superradiance is obtained if the electron beam is prebunched before it enters the undulator at a frequency within the band of its synchrotron-undulator radiation emission. In conventional synchrotron-undulator radiation with a uniform, randomly distributed electron beam [1], the radiation field is only partially coherent and its power is proportional to the beam current. By contrast, in the superradiant synchrotron-undulator emission ("prebunched beam radiation"—PB) [2–9] the radiation field is fully coherent, and its power is proportional to the square of the beam current.

If in addition to the prebunched e-beam an external electromagnetic wave is launched into the interaction region (the undulator) at the prebunching frequency and is properly phased, the wave will be amplified and the bunched electrons will be stimulated to emit high power radiation which is more intense than the superradiant emission. This "stimulated superradiant emission" (or "stimulated prebunched beam radiation"—SPB) can also be more intense than the stimulated radiation emission of an unbunched e-beam (conventional free electron laser—FEL [10]), since it "spares" the bunching process required in the FEL in the first sections of the undulator. Thus, in the SPB radiation process "negative work" is performed by the wave on the electron bunches, stimulating them to radiate throughout the entire undulator length. This is essentially the radiative emission process in the radiation section of the "optical klystron" [11].

The processes of superradiance and stimulated superradiance in FELs are closely related to the radiation concept of synchrotron amplified spontaneous emission (SASE) [12], which is presently of great experimental interest [13]. In the latter, the initial density and velocity bunchings are *random* (corresponding to shot noise and beam energy spread, respectively). In some suggested schemes [14], a partially coherent radiation field (generated by filtering the undulator synchrotron radiation) is also reinjected into the SASE undulator; consequently, this radiation process depends also on the initial conditions of the radiation field. As was shown by Haus [15], the SASE process can be analyzed by means of a spectral-statistical extension of the solution to the coherent (single frequency) initial-conditions problem. In both cases, the radiation field along the undulator is expressed in terms of the three initial conditions at the entrance to the undulator: the radiation field amplitude and the amplitudes of the current bunching and velocity bunching. Thus, study of the radiation processes in a prebunched beam FEL may help to understand new schemes of SASE FEL.

In [2] a general solution for the prebunched FEL radiation field was derived by solving the well-known cubic (Pierce) dispersion equation [16] using the aforementioned three initial conditions. A general linear expression for the radiation field amplitude $C(z)$ along the undulator was obtained:

$$C(z) = \sum_{j=1}^{3} [A_j C(0) + B_j \tilde{J}(0) + C_j \tilde{\nu}(0)] \exp(ik_j z), \tag{1}$$

where $\tilde{J}(0)$, $\tilde{\nu}(0)$ are the complex amplitudes of the initial current density and axial velocity modulations of the e-beam, and k_j are the three complex roots of the cubic dispersion equation.

The radiation power at the end of the undulator at $z = L$ is $P(L) = |C(L)|^2$. Using (1) and expressions given in [2] results in three general terms:

$$P(L) = P(0)F_{\mathrm{FEL}}(\overline{\theta}) + P_B F_{\mathrm{PB}}(\overline{\theta}) + [P(0)P_B]^{1/2} F_{\mathrm{SPB}}(\overline{\theta}, \phi), \tag{2}$$

corresponding, respectively, to three radiation processes: stimulated emission (normal FEL), prebunched beam radiation (superradiance), and stimulated prebunched beam radiation. In (2), $\overline{\theta} \equiv (\omega/\nu_z - k_z - k_w)L$ is the detuning parameter, where $k_z(\omega)$ is the radiation mode wave

TABLE I. Detuning functions in the regimes of low gain and high gain [2]; $\mathrm{sinc}(x) = \sin(x)/x$.

	Low gain	High gain
F_{FEL}	$1 + \overline{Q}\,\frac{d}{d\overline{\theta}}\,\mathrm{sinc}^2(\overline{\theta}/2)$	$\frac{1}{9}\exp(\sqrt{3}\,\overline{Q}^{1/3})$
F_{PB}	$M_J^2\mathrm{sinc}^2(\overline{\theta}/2)$	$\frac{M_J^2}{\overline{Q}^{1/3}}\exp(\sqrt{3}\,\overline{Q}^{1/3})$
F_{SPB}	$2M_J\mathrm{sinc}(\overline{\theta}/2)\cos(\overline{\theta}/2 + \phi)$	$2\,\frac{M_J}{\overline{Q}^{1/3}}\exp(\sqrt{3}\,\overline{Q}^{1/3})\cos\left(\phi + \frac{5\pi}{6}\right)$

number, k_w is the undulator wave number, and P_B is an e-beam power normalization parameter:

$$P_B = \frac{1}{32} I_0^2 L^2 \left(\frac{a_w}{\beta\gamma}\right)^2 \frac{Z_{\mathrm{mode}}}{A_{em}},$$

where I_0 is the beam current, a_w is the undulator parameter, Z_{mode} is the electromagnetic mode impedance, and A_{em} is its effective area.

The detuning functions $F_{\mathrm{FEL}}(\overline{\theta})$, $F_{\mathrm{PB}}(\overline{\theta})$, $F_{\mathrm{SPB}}(\overline{\theta}, \Phi)$ are, in general, bilinear expressions of $\tilde{J}(0)$ and $\tilde{\nu}(0)$. They were calculated in [2] for a general case, including collective and high gain regimes. For simplicity, we summarize in Table I the expressions for these functions for the case of negligible collective effects and absence of velocity modulation [$\tilde{\nu}(0) = 0$].

In Table I, $\overline{Q} \propto I_0 B_w^2 L^3$ is the FEL gain parameter [16] [$\overline{Q} = (4\pi N_w \rho)^3$, where ρ is sometimes referred to as the Pierce parameter [12]]. It characterizes the gain regime of the FEL: $\overline{Q} \ll \pi$ corresponds to low gain; $\overline{Q} \gg \pi$ corresponds to high gain [16]. The bunching index is

$$\tilde{M}_J = M_J \exp(i\phi) = \tilde{J}(0)/J_0 \tag{3}$$

(assuming $M_J \ll 1$), where $\tilde{J}(0)$ is the prebunching current complex amplitude at the wiggler entrance ($z = 0$), and its phase ϕ is measured relative to the phase of the radiation field amplitude $C(0)$ at this point. J_0 is the dc current density of the beam. In Table I, the third column corresponds to the high gain regime, relevant for SASE schemes. These high gain detuning functions are the maximum gain expressions evaluated at the optimal detuning value $\overline{\theta} \equiv 0$. Note that the only term which depends on phase ϕ is the F_{SPB} term. It is periodic with ϕ in both the low and the high gain regimes.

The magnitude of each of the three terms in Eq. (2) for a given set of FEL and e-beam parameters is a function of the detuning parameter $\overline{\theta}$ and the input power $P(0)$. In the low-gain limit the FEL gain is at a maximum for $\overline{\theta} = -2.6$ and is proportional to $P(0)$. The PB term is at a maximum for $\overline{\theta} = 0$ and is independent of $P(0)$; the SPB has its maximum at $\overline{\theta} = \varphi = 0$ and is proportional to $\sqrt{P(0)}$. Figure 1 shows the scaling of the maximum values of these three terms as a function of $P(0)$ for the parameters of Table II corresponding to our PB-FEM experiment [4].

Some experimental studies of various radiation processes occurring in a prebunched beam FEL were carried out earlier [4–9]. We developed a tabletop prebunched beam FEM apparatus (Fig. 2), which has been used to demonstrate mode selection [4] and efficiency enhancement [8] in an oscillator configuration. We recently employed it to study the characteristics of PB radiation [9] and SPB radiation. First results of experimental studies and demonstrations of SPB radiation are presented in Figs. 3 and 4.

In our experimental system, described in [4], prebunching of the beam was obtained by means of a traveling wave (TW) prebuncher. The prebunched beam was accelerated to 70 keV in a short dc acceleration gap. In SPB radiation experiments, the rf source power was split between the prebuncher input, P_{bunch}, and the FEL input, $P(0)$ (see Fig. 2). These input powers were controlled by two attenuators and the relative phase between their fields was controlled by means of a phase shifter. The nominal experimental parameters of the FEL are given in Table II. The bunching modulation index M_J was controlled by the input power to the prebuncher P_{bunch}. We have shown before [9,17] that in the small signal region ($P_{\mathrm{bunch}} < 2$ W)

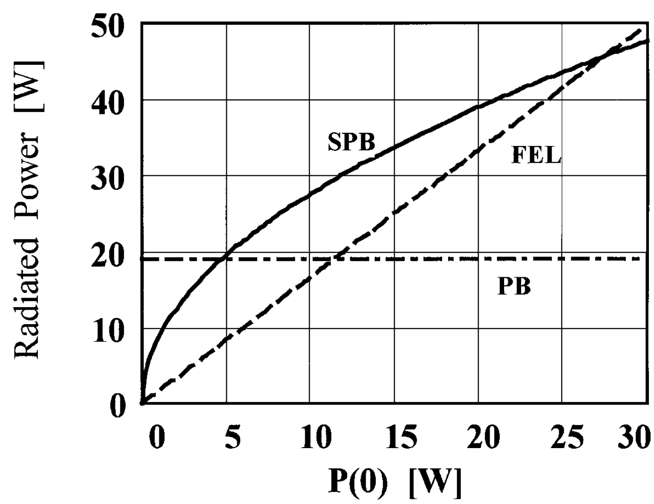

FIG. 1. The dependence of the maximum FEL, PB, and SPB radiation power [Eq. (2)] on input power $P(0)$ for the parameters of Table I with $M_J = 0.19$.

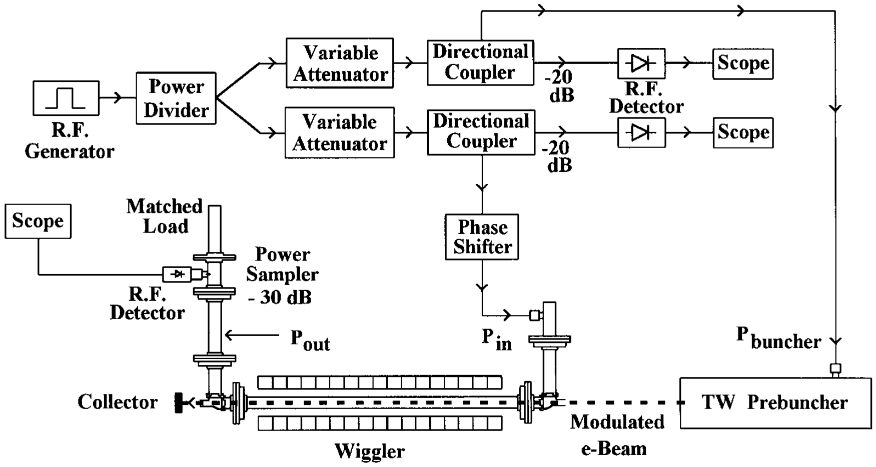

FIG. 2. Experimental setup for study of prebunched beam FEM radiation.

the superradiant power and, consequently, M_J^2 are proportionate to P_{bunch}.

The FEL characteristic parameters [2,16] can be evaluated from Table II: The space-charge parameter value is $\overline{\theta}_{pr} = r_p \cdot \overline{\theta}_p = r_p \omega_p' L/v_z \cong 0.8$ (for a longitudinal plasma frequency $\omega_p' = 1.87 \times 10^9$ rad/s and a reduction parameter value $r_p = 0.1$ [5]); the gain parameter value is $\overline{Q} \cong 2.5$; the prebunching power parameter [9] is $P_B = 524$ W. These parameters correspond to an intermediate regime of moderate collective effects and moderate gain. Nevertheless, the low gain tenuous beam gain expressions (Table I, column 2) describe fairly well the experimental scenario. We had verified earlier [5] that in the absence of prebunching the FEL gain curve [first term in Eq. (2)] agrees well with the theoretical S-shaped low gain detuning curve $F_{\mathrm{FEL}}(\overline{\theta})$. We had verified also that in the superradiant limit (no input signal, only prebunching) the PB emission [the second term in Eq. (2)] follows well the predicted $P_{\mathrm{PB}}(\overline{\theta}) \propto I_0^2 \mathrm{sinc}^2(\overline{\theta}/2)$ dependence [9].

In the following, we report an experimental demonstration of the scaling laws of the stimulated superradiant emission process. The dashed curve in Fig. 3 shows the dependence of the measured FEL total output power on the phase shift between the prebunching rf field and the FEL input signal (see Fig. 2). The theoretical curve was calculated from expression (2) using the low gain functions of Table I for $\overline{\theta} = 0$, $M_J = 0.19$, and $P(0) = 5.3$ W. Both the average and the periodic components of the theoretical curve match well the experimental data. This confirms the validity of the PB and SPB expressions, i.e., the second and third terms of Eq. (2) (note that at $\overline{\theta} \cong 0$ the FEL gain is zero and $F_{\mathrm{FEL}} = 1$). Even though it is not possible to determine from the measured curve the absolute phase ϕ of Eq. (3), we conclude from the functional dependence of $F_{\mathrm{SPB}}(\overline{\theta}, \phi)$ (Table I) that one of the peaks of the periodic curve in Fig. 3 corresponds to $\phi = -\overline{\theta}/2$. Noting that the only ϕ dependent term in Eq. (2) is the third (SPB) term, we can identify the amplitude of the measured periodic component of the curve of Fig. 3 as the amplitude of the periodic stimulated superradiance term:

$$P_{\mathrm{SPB\ max}} = 2[P(0)P_B]^{1/2} M_J \,\mathrm{sinc}(\overline{\theta}/2). \tag{4}$$

Figure 4 shows the measured amplitude of the periodic component of the FEL output power vs input power $P(0)$

TABLE II. Parameters of the PB-FEM.

Electron beam energy	70 keV
Electron beam current I_0	0.7 A
Electron beam radius r_b	3 mm
rf frequency f	4.5 GHz
Current modulation index	$0 \leq M_J \leq 0.26$
Wiggler field	300 G
Wiggler period λ_w	4.44 cm
Number of periods N_w	17
Waveguide cross section	2.215 cm × 4.755 cm
Mode	TE_{10}

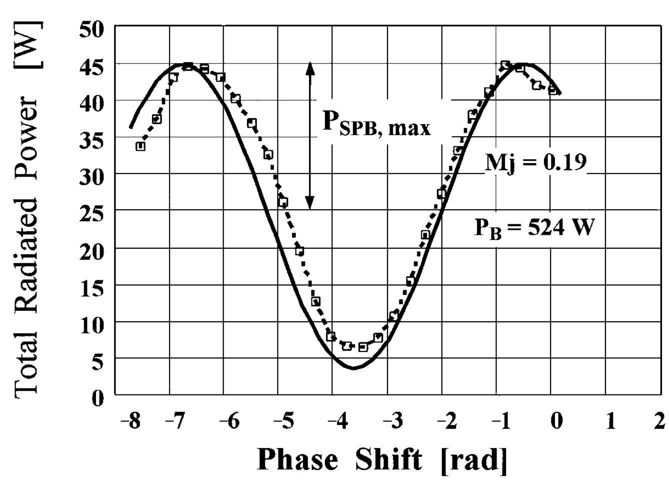

FIG. 3. Experimental and theoretical curves of prebunched FEL total output power dependence on the phase delay between the bunching and the FEL input fields.

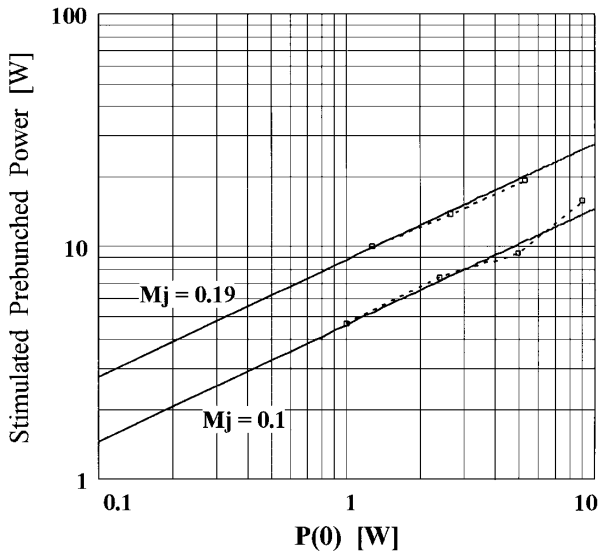

FIG. 4. Experimental and theoretical curves of stimulated superradiance amplitude (SPB) dependence on FEL input power and on current modulation index.

for two buncher input powers $P_{\mathrm{bunch}} = 0.25$ W, 0.8 W. The theoretical expression (4) (for $\overline{\theta} = 0$) is drawn on top of the data points for best fit bunching index values $M_J = 0.1$, 0.19, respectively. There is excellent agreement between the experimental measurements and the analytic expression. Specifically, Fig. 4 confirms very well the square root scaling of the SPB power with $P(0)$. The square root scaling of M_J with P_{bunch} is confirmed fairly well.

In conclusion, superradiance (PB) and stimulated superradiance (SPB) in FEL were investigated experimentally. The predicted dependence of SPB radiation power on input radiation power, bunching index, and relative phase between the current bunching and the radiation field were confirmed quantitatively in the linear low gain regime.

We note that in several new schemes, suggested for enhancing the gain of the SASE and high gain FEL [14,18], partially coherent SPB processes take place. Considering that in SASE FEL collective effects may play a role (e.g., for the set of experimental parameters given in [13], $\overline{\theta}'_p \cong 11 > \pi$), it appears that further experimental investigation of these radiation processes should be carried out. This can be done with the aid of our tabletop FEM in operating regimes (including collective effects and high-gain) which are relevant also for short wavelength FELs, and were not investigated thoroughly as yet.

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