Enhancement of FEM radiation by prebunching of the e-beam (stimulated super-radiance)

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Abstract

An electron beam (e-beam) prebunched at the synchronous FEM frequency and traversing through a waveguide, located coaxially with a magnetic undulator, emits coherent radiation at the bunching frequency. Introduction of both a premodulated e-beam and a radio-frequency (r.f.) signal at the same frequency at the input of the waveguide can lead to more efficient interaction, and thus more power can be extracted from the electron beam. In order to achieve this, the density modulation of the electron beam should be at an appropriate phase with respect to the r.f. signal.

We report a first experimental demonstration of the influence of the phase difference between the r.f. input signal and the fundamental component of the density modulation of the e-beam on the radiated power in a Free-Electron Maser (FEM). Our experimental system allows control of the current density modulation, of the r.f. input power level, in the undulator region and of the phase between that r.f. input and the modulation of the e-beam.

A comparison between measured radiation power with that predicted by theory for various phase differences, current density modulation, and r.f. signal levels, was made. Good correlation was obtained. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

In the usual FEL operation, an electron beam interacts with an electromagnetic (EM) wave in an undulator. A bunching force resulting from FEL interaction density modulates the electron beam as it transits the undulator. In order to extract considerable energy from the e-beam, electron bunches must be located in the decelerating regions of the ponderomotive EM wave. The EM wave is thus amplified and coherent FEL radiation is obtained [1]. It was shown that coherent radiation from an e-beam can also be obtained if the e-beam is modulated prior to its entrance to the interaction region, even when no EM wave is introduced at the input to the wave interaction region [2–7]. This radiated power is called “Super-radiance” (or “Prebunched Beam power”—PB). The radiation process can be substantially enhanced under certain conditions if a modulated e-beam and an EM wave (r.f. signal) are introduced simultaneously into the interaction region. This process is called “Stimulated Super-radiance” (or “Stimulated Prebunched Beam power”—SPB). In order to obtain maximal radiation power, the r.f. signal and the modulation of the e-beam must be
phase-matched [2–4]. The radiated power in a Free-Electron Maser (FEM) as a function of such a phase matching is reported in the following. As far as we know such an experimental demonstration has not been reported previously.

2. Analytical model

In the general case, the total radiated power is the sum of the three radiation processes described above [2,3]:

\[
P(L) = P_s |C(L)|^2 = P(0)F_{\text{FEL}}(\tilde{\theta}, \tilde{\theta}_{pr}) + P_B F_{\text{PB}}(\tilde{\theta}, \tilde{\theta}_{pr}) + \sqrt{P(0)P_B F_{\text{SPB}}(\tilde{\theta}, \tilde{\theta}_{pr}, \phi)}
\]

(1)

where

- \(P_s\) is the normalization power of the waveguide mode,
- \(C(L)\) is the amplitudes of the EM field at \(z = L\),
- \(P(0)\) is the r.f. signal input power,
- \(P_B\) is the e-beam power parameter,
- \(Z_{\text{mode}}, A_{\text{em}}\) are the impedances and effective area of the EM mode, respectively,
- \(I_o, L\) are the e-beam DC current and interaction length, respectively.

The three detuning functions in Eq. (1) (assuming zero velocity modulation at the input wiggler) are defined as [2]

\[
F_{\text{FEL}} = 1 + \tilde{Q} \frac{d}{d\tilde{\theta}_{pr}} \left( \frac{\tilde{\theta} + \tilde{\theta}_{pr}}{2} \right) - \frac{\tilde{Q}}{2\tilde{\theta}_{pr}} \left( \frac{\tilde{\theta} + \tilde{\theta}_{pr}}{2} \right)
\]

(2)

\[
F_{\text{PB}}(\tilde{\theta}, \tilde{\theta}_{pr}) = M_j^2 \frac{\tilde{\theta}_{pr}^2}{(\tilde{\theta}^2 - \tilde{\theta}_{pr}^2)^2}
\times \left( 1 + \cos^2 \tilde{\theta}_{pr} + \left( \frac{\tilde{\theta}_{pr}}{\tilde{\theta}} \right)^2 \sin^2 \tilde{\theta}_{pr} - 2 \cos \tilde{\theta} \cos \tilde{\theta}_{pr} - 2 \tilde{\theta}_{pr} \sin \tilde{\theta}_{pr} \sin \tilde{\theta} \right)
\]

(3)

\[
F_{\text{SPB}}(\tilde{\theta}, \tilde{\theta}_{pr}, \phi) = M_j \left\{ \frac{\sin \left( \frac{\tilde{\theta} + \tilde{\theta}_{pr}}{2} \right)}{\cos \left( \frac{\tilde{\theta} + \tilde{\theta}_{pr}}{2} + \phi \right)} + \frac{\sin \left( \frac{\tilde{\theta} - \tilde{\theta}_{pr}}{2} \right)}{\cos \left( \frac{\tilde{\theta} - \tilde{\theta}_{pr}}{2} + \phi \right)} \right\}
\]

(4)

where

- \(\tilde{\theta} = \left( \frac{\omega}{v_e} - k_x - k_w \right) L\) is the normalized detuning parameter,
- \(\tilde{\theta}_{pr} = \left( \frac{\alpha}{v_e} \right) L\) is the normalized plasma frequency parameter,
- \(\tilde{Q}\) is the normalized gain parameter,
- \(M_j e^{i\phi} = \tilde{J}(0)/J_0\) is the current modulation index
- \(\phi\) is the phase of the fundamental component of current density modulation with respect to the ponderomotive wave phase.

For a tenuous e-beam (\(\tilde{\theta}_p \ll \pi\)), the detuning functions are reduced to the following form [2,3]:

\[
F_{\text{FEL}} = 1 + \tilde{Q} \frac{d}{d\tilde{\theta}_{pr}} \sin^2 \left( \frac{\tilde{\theta} + \tilde{\theta}_{pr}}{2} \right)
\]

(5)

\[
F_{\text{PB}} = M_j^2 \sin^2 \left( \frac{\tilde{\theta}}{2} \right)
\]

(6)

\[
F_{\text{SPB}} = 2M_j \sin \left( \frac{\tilde{\theta}}{2} \right) \cos \left( \frac{\tilde{\theta}}{2} + \phi \right).
\]

(7)

Eq. (5) expresses the FEL gain in the absence of prebunching. The prebunched FEM radiation (expressed by Eq. (6)) was investigated by us previously [5]. The SPB radiation given by Eq. (7) has a sinusoidal dependence on the phase.
difference $\phi$ between the current modulation and the injected r.f. signal.

For the set of parameters of our prebunched FEM (see Table 1) Eqs. (5)–(7) described the experimental scenario fairly well [5].

The total radiated power (Eq. (1)) is the sum of the three terms given by Eqs. (5)–(7). $F_{\text{FEL}}$ and $F_{\text{PB}}$ (Eqs. (5) and (6)) do not depend on phase $\phi$. Since the SPB term (Eq. (7)) is sinusoidal vs. phase $\phi$, the total radiated power (Eq. (1)) also varies sinusoidally with $\phi$ and has an average power determined by the amplified input signal (FEL power) and by the PB power (first two terms of Eq. (1)).

We investigated experimentally the influence of the relative phase between the fundamental frequency of current density modulation of the e-beam and the r.f. signal wave introduced to the waveguide input; i.e. we investigated the third term of Eq. (1) (the SPB power).

### 3. Experimental setup and measurements

The experimental demonstration was made with the aid of a table-top prebunched beam FEM developed at Tel-Aviv University. The use of prebunching in operation as an oscillator permits mode selection and single frequency operation [9].

Efficiency enhancement of the FEM oscillator was made possible by selection of an appropriate eigenfrequency as described in [10]. Possible uses as a frequency agile oscillator (on a pulse to pulse basis) were described in [11]. An experimental study of super-radiance both at the upper and at the lower synchronous frequency was reported in [5].

A schematic illustration of the experimental setup is shown in Fig. 1. The premodulated e-beam is derived from a traveling-wave-type prebuncher [8]. The fundamental component of the e-beam current modulation frequency is simply the input frequency to the traveling wave (TW) prebuncher. The prebunching modulation index "$M_f$" at the fundamental bunching frequency can be varied by the adjustment of the prebuncher r.f. input power ($P_{\text{bunch}}$).

The premodulated e-beam, derived from the prebuncher, traverses a rectangular waveguide located in the “wiggler” section. At 70 keV beam energy, the predicted upper synchronous frequency in the TE$_{10}$ mode is about 4.9 GHz. The r.f. output port of the FEM is terminated in a matched load. A power divider placed at the output of the r.f. signal generator splits the source power into two separate channels. One channel is used to provide the r.f. signal wave into the waveguide. The other channel is used to provide...
r.f. input power to the prebuncher. A variable attenuator in each channel allows the adjustment and r.f. power setting at the inputs to the waveguide and to the TW prebuncher. A directional coupler in each channel allows the measurement of these power levels. The calibrated phase-shifter allows the change of relative phase between the r.f. signal wave and the modulated e-beam (at least over a range of \(2\pi\) radians with a resolution of 0.36°). The operational parameters of the compact FEM are shown in Table 1.

Fig. 2 shows that the PB radiated power is nearly proportional to prebuncher input power in the linear regime \((P_{\text{bunch}} < 1\, \text{W})\) as per Eq. (6). Therefore, \(M_J^2\) evaluated from this curve is proportional to the prebuncher input power. Thus, by controlling \(P_{\text{bunch}}\) and the buncher input frequency we control “\(M_J^2\)” and the bunching frequency, respectively.

Fig. 3 shows measurements of the total radiated power vs. phase for an input r.f. power of 1 W to the waveguide and for two prebuncher input power levels: 0.25 W (curve a) and 0.8 W (curve b). For a constant input power \(P(0) = 1\, \text{W}\), the
average radiated power is greater for $P_{\text{bunch}} = 0.8 \text{ W} \ (M_J = 0.19)$ as compared to $P_{\text{bunch}} = 0.25 \text{ W} \ (M_J = 0.1)$. For the higher $M_J$ we note that the PB radiation power (proportional to $M_J^2$) is higher than the SPB radiation power (proportional to $M_J$). These experimental results agree with the predictions of the analytic model [2]. For other experimental r.f. input power levels, agreement with theory was also good.

4. Conclusion

We demonstrated experimentally the existence of a periodic variation of radiation power vs. phase difference between the EM wave introduced into the wiggler region and the phase of the fundamental current of a prebunched e-beam. The radiated power is maximized (for phase matching) and minimized (for antiphase conditions). The amplitude of the variable component of radiation power corresponds closely to that predicted theoretically. The maximum total radiated power also corresponds to theoretically predicted values and under optimal phase conditions, the FEM radiation power is enhanced considerably. The measured variation of radiated power as a function of phase difference, modulation index, and r.f. input power into the wiggler region corresponds well to the theoretical predictions confirming that both theory and experimental tests are valid.

References