



# Enhanced super-radiant emission of FEM near waveguide-cutoff and near zero-slippage conditions

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## Abstract

We report on super-radiance obtained from the TAU FEM just above waveguide cutoff and near grazing intersection.

Grazing intersection (or “Zero Slippage”) is defined as the point at which the two synchronous frequencies merge to one frequency. In this case, the radiated power frequency can be tuned over a very wide band by change of the pre-modulation frequency.

Near the lower synchronous frequency, the super-radiance power is much greater and the spectral width is much narrower than those at the higher synchronous frequency.

The super-radiance emission near cutoff (lower synchronous frequency) and near to the upper synchronous frequency was measured and compared to those predicted by an analytical model for a wide range of frequencies. © 2002 Published by Elsevier Science B.V.

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## 1. Introduction

Super-radiant electromagnetic radiation power may be obtained from a pre-modulated (bunched) electron beam which passes through a waveguiding structure located in a magnetic undulator. The radiation frequency  $\omega$  is the same as the e-beam pre-bunching frequency. It can build up to intense amplitude only at frequency bands near the “synchronous frequencies”. The “synchronous

frequencies” are given by the two possible intersections between the e-beam line and the waveguide dispersion curve (Fig. 1).

In previous experiments we characterized the radiation emission at the high synchronous frequency [1,2]. The emission at the lower frequency is now studied by us. For the nominal parameters of the TAU FEM ( $E = 70$  keV) the upper synchronous frequency is about 4.9 GHz and the lower one is just above the waveguide cutoff at about 3.153 GHz (see Fig. 1 line (b)). Both an analytical model [3] and computer simulations [4] predict that super radiance power near waveguide cutoff is much greater than the radiated power at

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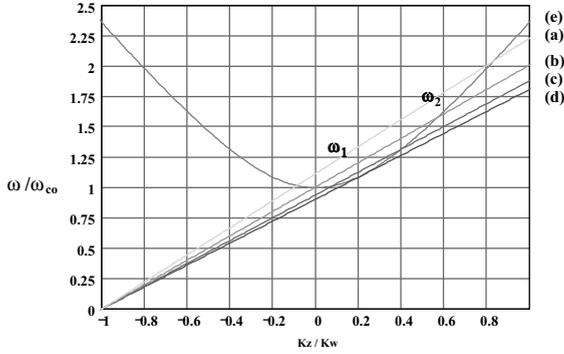


Fig. 1. Intersections between the waveguide dispersion curve (e) and the e-beam line for several e-beam energies: (a)  $E = 90$  keV (both forward and backward wave); (b)  $E = 70$  keV (near cutoff  $f_1 \cong f_{co} = 3.152$  GHz); (c)  $E = 60$  keV (near “grazing”); (d)  $E = 55$  keV (“Grazing”  $f_1 = f_2$ ).

the higher synchronous frequency. It also predicted a spectral width of the radiated power near cutoff which is much narrower.

If the electron beam energy is reduced, the slope of the e-beam line is also reduced (see Fig. 1 lines (c) and (d)). In this case, the two synchronous frequencies come closer to each other and can coincide to one frequency (d). This is the case of “Zero Slippage” also called “Grazing Intersection”. In this case, the radiated power frequency can be tuned over a very wide frequency band, much wider than in the case of the two well separated synchronous frequencies [5,6].

In this paper we report experimental results of FEM operation near waveguide cutoff and near “Zero Slippage”.

## 2. Theory of super-radiance in a waveguide FEL

The radiation obtained from a periodically pre-modulated e-beam traversing a waveguiding structure, located in a magnetic undulator of the type employed in Free Electron Lasers/Masers has been the subject of several recent investigations [7–12]. The analytical treatment in Ref. [3] takes into account space charge effects, current density and velocity modulation and characterizes the radiated power in the low and high gain regimes for both

collective and tenuous e-beam regimes. Modification of this model for a FEM employing a waveguide structure was developed in Ref. [8].

Assume that the pre-modulated e-beam has a sinusoidal variation of the form

$$i(t) = I_0[1 + M_j \cos(\omega t)] \quad (1)$$

where  $I_0$  is the average (DC) current and  $M_j$  is the modulation amplitude:  $M_j = |\tilde{J}_{1z}(0)|/J_0$ ;  $\tilde{J}_{1z}(0)$  is the prebunching current complex amplitude at the wiggler entrance ( $z = 0$ ).  $J_0$  is the DC current density of the beam. Applying the analytical model of Ref. [3] for the case of a tenuous e-beam ( $\bar{\theta}_p \ll \pi$ ) without an external electromagnetic wave launched into the interaction region ( $C_s(0) = 0$ ), and neglecting the velocity modulation ( $M_v = 0$ ), results in the basic expression for super-radiant power at the exit of the interaction region (the wiggler of length  $L_w$ ) [3,7–9]:

$$P_{sr}(L_w) = P_B M_j^2 \sin^2[\bar{\theta}(\omega)/2] \quad (2)$$

where  $\bar{\theta}(\omega) = (\omega/V_{0z} - k_z(\omega) - k_w)L_w$  is the normalized detuning parameter and the prebunching power parameter is

$$P_B = \frac{1}{32} I_0^2 L_w^2 \left( \frac{a_w}{\gamma_0 \beta_{0z}} \right)^2 \frac{Z_{mode}}{A_{em}} \quad (3)$$

where  $A_{em}$  is the effective area of the excited waveguide mode. In the TAU FEM the operating mode is the  $TE_{10}$  mode, for which  $A_{em} = ab/2$ . The impedance of the excited mode is

$$Z_{mode} = Z_0 \frac{k_0}{k_z}$$

where  $k_0, Z_0$  are the wave number and wave impedance in free space

$$k_z(\omega) = \frac{1}{c} \sqrt{\omega^2 - \omega_{co}^2}$$

where  $\omega_{co}$  is the cutoff frequency.

## 3. Radiation emission near waveguide cutoff

For frequencies near waveguide cutoff the wavenumber  $k_z(\omega) \rightarrow 0$ ; thus, the impedance of the excited mode  $Z_{mode}$  and the phase velocity of a  $TE_{10}$  mode tends to infinity. Observing that the super-radiant power is proportional to the

impedance of the excited mode, an infinite radiated power is predicted near cutoff, which indicates a singularity in FEM behavior near waveguide cutoff. To avoid singularity ( $Z_{\text{mode}} = Z_0(k_0/k_z) \rightarrow \infty$ ) the waveguide losses were taken into consideration [13,14], using the following expression for the complex propagation constant of the TE<sub>10</sub> mode:

$$k_z^2 = k_{10}^2 + \sqrt{\frac{2}{k_0 Z_0 \sigma} \frac{\pi^2}{a^2 b} \left( k_{10}^2 + \frac{2b}{a} + 1 \right)} (1 + j) \quad (4)$$

$$k_{10} = \sqrt{k_0^2 - (\pi/a)^2}$$

where  $\sigma$  is the waveguide wall conductivity

#### 4. Experimental setup and measurements

The compact prebunched beam FEM developed at Tel-Aviv University was described in Ref. [2]. It was operated as an oscillator which permits mode selection and single frequency operation [15], and was used to demonstrate efficiency enhancement

by selection of an appropriate eigenfrequency [16]. Its operation as a stimulated superradiant source was also described [1].

The experimental setup for measuring prebunched beam radiation is shown in Fig. 2. The bunched e-beam is derived from a traveling-wave prebuncher. The e-beam modulation frequency (continuously adjustable over more than an octave bandwidth) is simply the input frequency to the traveling wave prebuncher. The prebunching modulation level  $M_j$  can be controlled by adjustment of the prebuncher RF input power ( $P_{\text{in TWT}}$ ) in the linear region of the prebuncher [17].

The premodulated e-beam derived from the prebuncher traverses a rectangular WR-187 waveguide located in the “wiggler” section (cutoff frequency at 3.152 GHz). At 70 keV beam energy the low synchronous frequency in the TE<sub>10</sub> mode is near cutoff at about 3.153 GHz (see Fig. 1(b)). Therefore, a modification of the RF system was required and was made in order to allow measurements near cutoff. The straight waveguide section, together with the waveguide “knee”, were not changed. Linear transitions to double-ridged

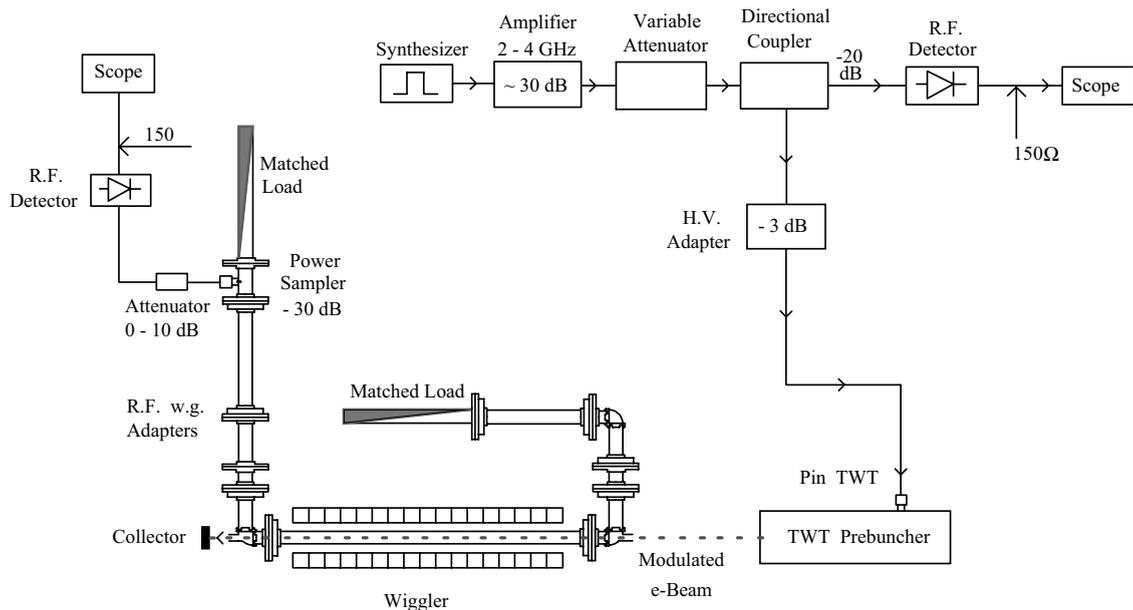


Fig. 2. Schematic of the experimental setup for prebunched e-beam radiation measurements near waveguide cutoff and near “Zero Slippage”.

waveguide (WRD-250), having a lower cutoff frequency and a wider frequency band, were used. The input and output RF ports of the FEM were terminated in matched loads.

### 5. Experimental measurements near cutoff

The dashed curve in Fig. 3a shows the dependence of the measured super-radiant power on frequency. The use of the theoretical expressions (2) to describe such experiment is not valid, especially near the singular region near cutoff. For this reason the theoretical curve (continuous line in Fig. 3a) was recalculated from the field

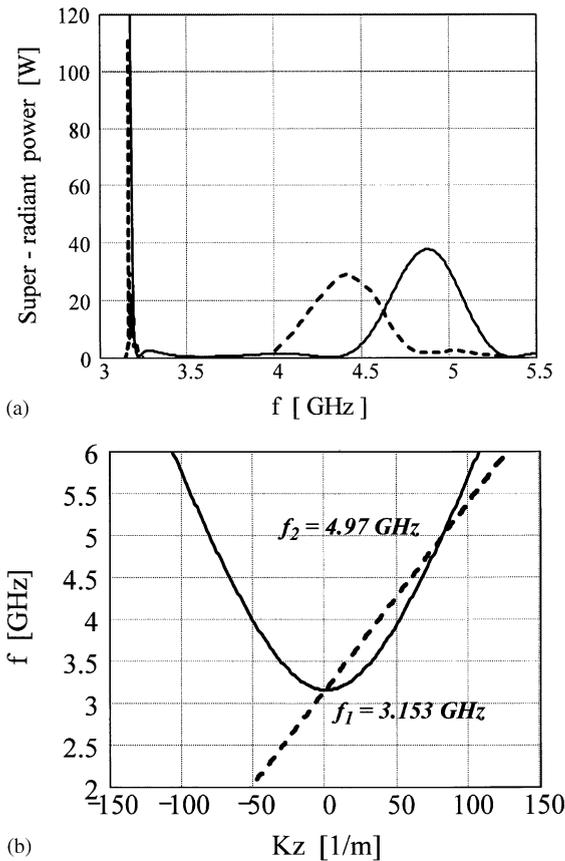


Fig. 3. (a) Comparison of measured (dashed) super-radiance power vs. frequency with calculations (solid) for e-beam energy 70 keV—near waveguide cutoff conditions. (b) The dispersion diagram for this case.

Table 1  
Parameters of the prebunched FEM

Electron beam energy	70 keV
Electron beam current— $I_0$	0.7 A
Electron beam radius— $r_b$	3 mm
RF frequency— $f$	3.15–6.5 GHz
Current modulation index	$0 \leq M_J \leq 0.25$
Wiggler field	300 G
Wiggler period— $\lambda_w$	4.44 cm
Number of periods— $N_w$	17
Waveguide cross-section	2.215 cm $\times$ 4.755 cm
Mode	TE <sub>10</sub>

amplitude using the formulation of Ref. [3] (in the low gain limit) and using a complex expression for the wavenumber (Eq. 4) with  $\sigma \cong 5.82 \times 10^7 (\Omega \text{ m})^{-1}$  (for a copper waveguide). The other resonator and FEL parameters are listed in Table 1. For best match to the measured data at high frequencies we included in the calculations, in addition to the density modulation  $M_J = 0.25$ , an assumption of a velocity modulation  $M_v = 0.2\%$  and a phase difference of  $0.9\pi$  between the velocity and density modulation.

In the upper range the measured power level was in good agreement with theory except for some down shift in frequency. The experimental results in the lower frequency range near cutoff show higher power level and narrower spectral width than in the upper frequency range as expected theoretically. The super-radiant power peak near cutoff, measured around 3.158 GHz, is overlaps quite well the theoretical curve.

### 6. Experimental measurements near “Zero Slippage”

The e-beam energy was reduced to 60 keV in order to measure super-radiant power near “zero slippage” conditions. The dashed curve in Fig. 4a shows the dependence of the measured super-radiant power on frequency. The theoretical curve (continuous) was calculated based on Ref. [3] using the low gain expressions with a current modulation parameter  $M_J = 0.25$ , velocity modulation parameter  $M_v = 0.2\%$ , phase difference of

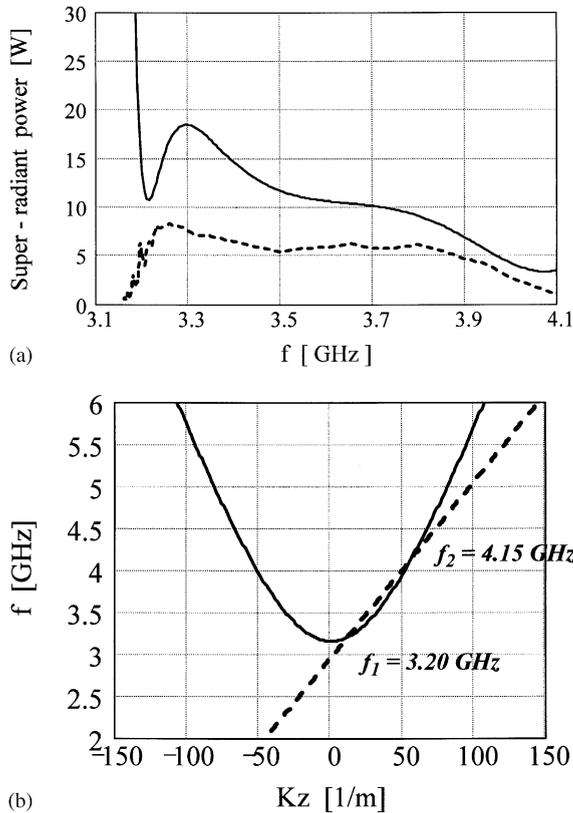


Fig. 4. (a) Comparison of measured (dashed) super-radiance power vs. frequency with calculations (solid) for e-beam energy 60 keV—near “Zero Slippage” conditions. (b) The dispersion diagram for this case.

$0.2\pi$  between the current and velocity modulation and the parameters of Table 1 except for the beam energy (60 keV). Although the measured power is

lower than the theoretical curve, Fig. 4a confirms very well the broad frequency bandwidth of the radiated power under these conditions.

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