ACCURATE DETERMINATION OF *Q* FACTORS OF A QUASI-OPTICAL RESONATOR

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ABSTRACT: *This paper presents a novel approach to measuring the* Q *factors of a quasi-optical resonator used in a free electron maser (FEM) operating at 100 GHz. Estimation of the* Q *factors is based on measuring the reflection coefficient in the vicinity of resonance frequencies. The unknown parameters (attenuation constant and coupling inductance) are determined from the numerical solution of a nonlinear system of two equations. As an example, the* Q *factors of a quasi-optical resonator, based on the shorted-line waveguide section, have been determined using the proposed method.* © 2003 Wiley Periodicals, Inc. Microwave Opt Technol Lett 36: 303–306, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.10749

Key words: *mm-wave measurements; Q* factors; quasi-optical resonators

1. INTRODUCTION

Loaded and unloaded *Q* factors are important characteristics of a resonator used in high-power mm-wave sources [1]. However, a critical problem in quasi-optical resonators is the measurement of these parameters in real operation conditions by means of exciting elements. These may consist of a mode exciter, mirror, and polarizing grid, etc. Figure 1 demonstrates a configuration of a quasi-optical resonator used in the mm-wave FEM at Tel Aviv University [2]. The resonator is composed of parallel curved-plate waveguide sections [3], shorted at the end, with entrance and exit holes for transporting an electron beam through the resonator. In order to study the resonator's characteristics, it is excited from an external source, such as a network analyzer. The polarizing grid coupler is illuminated by a free-space Gaussian beam, formed by means of a mode exciter and a mirror, as described in [4].

Diffraction and ohmic losses within the exciting system may themselves create return loss up to 10 dB. As a result, the amplitude response of a reflection-type resonator is considerably distorted, thus causing problems in accurate *Q*-factors measurement and preventing application of conventional measuring techniques. To solve this problem, different calibration procedures have been suggested to take coupling losses into consideration [5–9]. However, they cannot be accurately made with quasi-optical components in the mm-wave range.

To model this system, an equivalent 2-port circuit with known parameters can be introduced [10] that allows the FEM resonator's interior parameters to be determined in terms of lumped characteristics L-C-R. Based on this approach and curve-fitting approximation, the loaded and unloaded *Q* factors have been estimated for the resonator shown in Figure 1. One of the drawbacks of this technique is the difficulty of separating external coupling elements from internal resonating elements. In order to do such a separation, a distributed model of the FEM resonator based on shorted-line representation is preferable. Such a model provides some advantages:

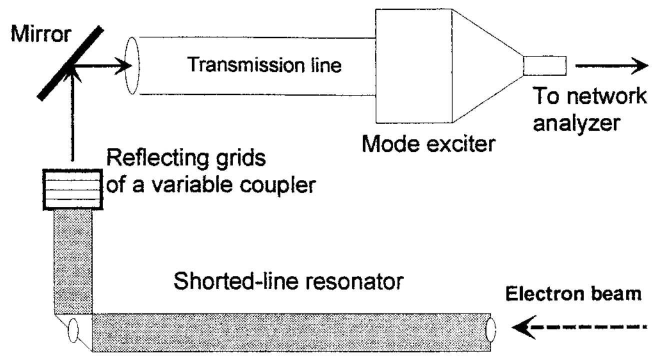

Figure 1 Configuration of the shorted-line FEM resonator

- a possibility to simulate all longitudinal modes resonating within a specified frequency range;
- elements responsible for resonance or coupling effects can be easily separated;
- a possibility to estimate the dispersion nature of the FEM resonator.

2. MODEL DESCRIPTION

The FEM resonator can be presented by the equivalent schematic shown in Figure 2 [11]. It consists of a shorted-line section of length *l* corresponding to the physical length of the FEM resonator. The equivalent inductor *L* corresponds to the polarizing grid's coupler-shunt input.

A distributive model of the FEM resonator allows the *Q* factors to be extracted in a relatively easy way. The formulation can be done using the inductively coupled shorted-line resonator model [11]. The input impedance of the resonator can be calculated as follows:

$$Z_{in} = Z_L Z_{sh}/(Z_L + Z_{sh}), \tag{1}$$

where $Z_L = i2\pi fL$ is the inductor resistance, $Z_{sh} = Z_{01}\tanh[(\alpha + i\beta)l]$ is the impedance of the shorted-line of length l, Z_0 and Z_{01} are the impedances of the network analyzer waveguide, resonator waveguide mode α is the attenuation constant, and $\beta = 2\pi f/c$.

The reflection coefficient S_{11} measured by the scalar network analyzer is:

$$S_{11} = |(Z_{in} - Z_0)/(Z_{in} + Z_0)| \tag{2}$$

Substituting actual values of the resonator's parameters in Eq. (2) we can write the final expression needed to simulate the a reflection coefficient as

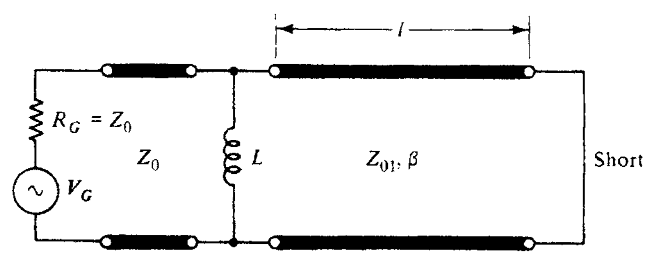

Figure 2 Equivalent schematic of an inductively coupled shorted-line resonator [11]

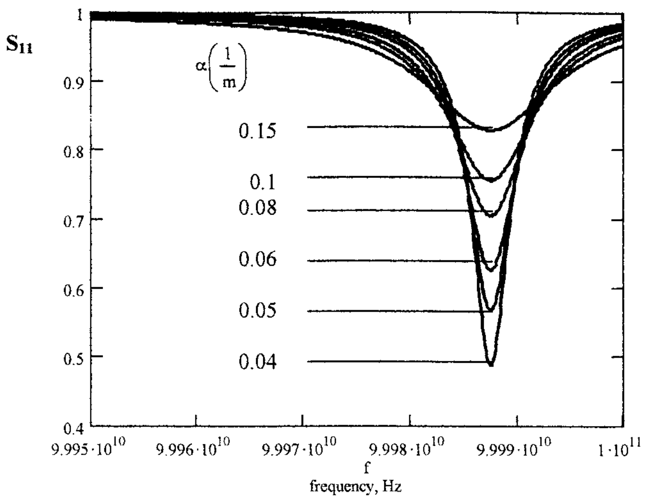

Figure 3 Behavior of local minima of S_{11} near-resonance frequency for different α and specified resonant modes

$$S_{11}(\alpha, L, \omega) = \left| \frac{\left(\frac{j\omega L}{Z_0} - 1\right)\frac{Z_{01}}{Z_0}\tanh[(\alpha + j\beta)l] - \frac{j\omega L}{Z_0}}{\left(\frac{j\omega L}{Z_0} + 1\right)\frac{Z_{01}}{Z_0}\tanh[(\alpha + j\beta)l] + \frac{j\omega L}{Z_0}} \right|,$$

$$\omega = 2\pi f. \quad (3)$$

In a typical situation, the parameters Z_0, Z_{01}, l, and β are known, so that Eq. (3) can be used for solving the inverse problem, namely, reconstructing an attenuation constant α and inductance L based on the measured magnitude of S_{11}. It should be stressed that if a scalar network analyzer is used for measuring S_{11}, an infinite number of points in the space of $\{\alpha, L\}$ may satisfy Eq. (3) for a fixed S_{11}. Each of them can be characterized by their own local minimum of S_{11}. Figures 3 and 4 illustrate this feature for different α and L with the following parameters of the resonator: l = 1.56091 m, $Z_0 = 467\Omega$, $Z_{01} = 377\Omega$. Both depth of minima

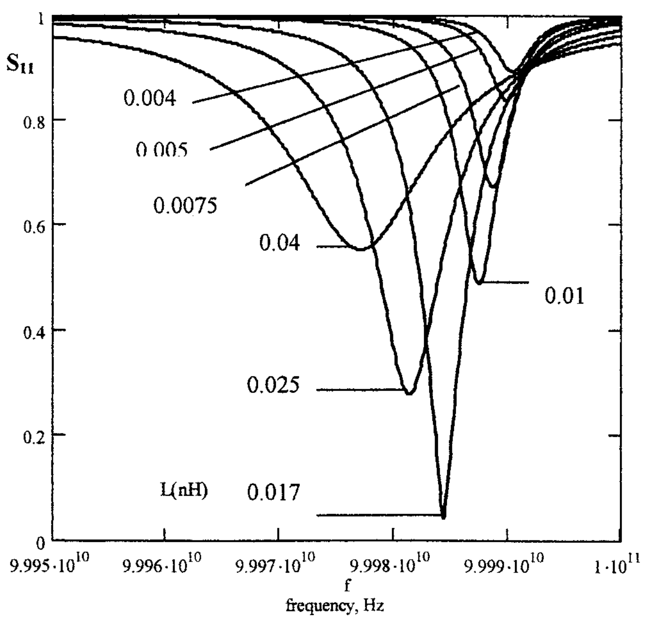

Figure 4 Behavior of local minima of S_{11} near-resonance frequency for different L and specified resonant modes

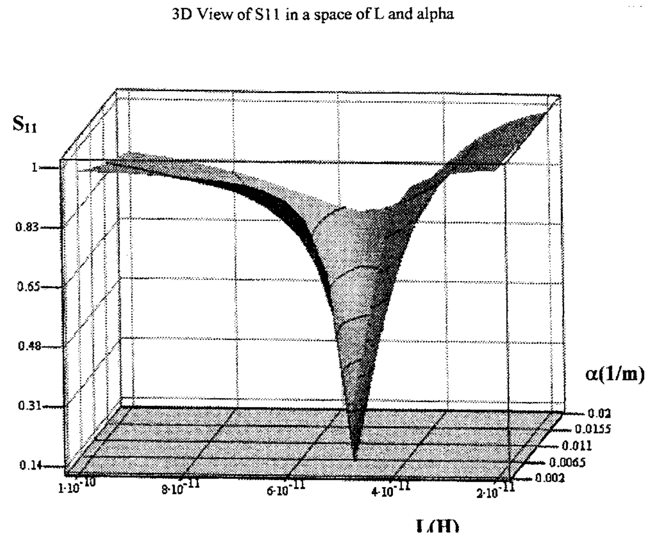

Figure 5 Behavior of the magnitude of S_{11} in a space of $\{\alpha, L\}$ near-resonance frequency f_0 = 99.99 GHz

and width of resonance curves depend on values of α and L, but the coupling inductor also shifts the resonance frequency. Also, adjusting L, we can vary the coupling coefficient and form the under-coupled ($L < 0.017$ nH), critical ($L = 0.017$ nH) and over-coupled ($L > 0.017$ nH) regimes. The 3D view shown in Figure 5 demonstrates the behavior of the magnitude of S_{11} in the space of $\{\alpha, L\}$, clarifying the distribution of the above mentioned local minima and indicating the presence of a global minimum that corresponds to critical coupling with the resonator's near-resonance frequency f = 99.99 GHz.

3. RECONSTRUCTING ALGORITHM

Consider local minima in the space of $\{\alpha, L\}$ placed near the resonance. To form the algorithm for reconstructing α and L, two measurements of S_{11} amplitude must be done: the first is at the resonance frequency f_0 and the second is at the frequency near resonance f_1. Assuming that these magnitudes are A_0 and A_1, respectively, the system of two nonlinear equations can be written for unknown α and L:

$$S_{11}(\alpha, L, \omega_0) = A_0$$
$$S_{11}(\alpha, L, \omega_1) = A_1. \quad (4)$$

The system of Eq. (4) can be solved numerically by iterations. When solving equations numerically, it is necessary to define values from which the solver should start the search for a solution. To create a required guess value, some preliminary study of the reflection coefficient in the space $\{\alpha, L\}$ is required near the resonance frequency. This can be done, for instance, by comparing results calculated from Eq. (3) and measured values of return loss. The unknown parameters α and L are varied to find the best curve-fitting results, and then the intervals for each variable are established, namely:

$$\alpha_{low} < \alpha < \alpha_{up};\ L_{low} < L < L_{up}, \quad (5)$$

where α_{low}, α_{up}, and L_{low}, L_{up} are lower and upper values, respectively, of parameters determined for proper guess values. Afterwards, standard procedures can be employed to find true

TABLE 1 Results of Measurements and Reconstructing *Q* Factors for Under-Coupled Regime

Guess Values
$\alpha = 0.05$ (l/m) $L = 0.1$ (nH)
Solutions of System (4)
$\alpha = 0.0844$ (l/m) $L = 0.053$ (nH)
Q Factors
$Q_{un} = 12393$ $Q_e = 261883$ $Q_{load} = 11833$ $f_0 = 99.99$ GHz $f_1 = 99.985$ GHz $S_{11} = 0.9099$ $S_{11} = 0.96039$

TABLE 2 Results of Measurements and Reconstructing *Q* Factors for Critically Coupled Regime

Guess Values
$\alpha = 0.12$ (l/m) $L = 0.15$ (nH)
Solutions of System (4)
$\alpha = 0.087$ (l/m) $L = 0.249$ (nH)
Q Factors
$Q_{un} = 11979$ $Q_e = 13829$ $Q_{load} = 6419$ $f_0 = 99.5978$ GHz $f_1 = 99.596$ GHz $S_{11} = 0.11885$ $S_{11} = 0.21135$

solutions. The results reported below were obtained via the MinErr procedure of the MathCad-2000 Solver.

The unloaded Q_{un} and external Q_e factors are calculated using formulas (9.23) and (9.28) from [11]:

$$Q_{un} = \beta/2\alpha, \tag{6}$$

$$Q_e = [B_L + \phi(1 + B_L^2)]Z_0/2Z_{01}, \tag{7}$$

where $B_L = Z_{01}/2\pi f_0 L$ and $\phi = 2\pi f_0 l/c$.

4. RESULTS

In the experiments described below, a variable coupler consisting of three grids has been used, through which the external source's signal was coupled with the resonator's input. All polarizing grids have parallel thin wires and are placed ⅛ of a wavelength from each other. The inner grid can be rotated to change the angular position of the wires relative to the other external grids. Hence, changing the angular position of the input coupler's inner grid can vary the coupling coefficient.

Two examples of Q-factor measurements are discussed below, as an illustration of the proposed technique:

- excitation of the resonator in the regime of weak coupling corresponding to the typical experimental parameters of the FEM (under-coupled regime, $Q_e > Q_{un}$);
- excitation of the resonator in the regime near to critical coupling (critical regime, $Q_e \approx Q_{un}$)

4.1 Under-Coupled Regime, $Q_e > Q_{un}$

Weak coupling of the resonator can be realized by setting small angles between the inner polarizing grid and the external ones (about 10°). The measured magnitudes of S_{11} at the resonance frequency f_0 and at the frequency near the resonance f_1 are given in Table 1. For preliminary determined guess values of α and L, the solution to the system of Eq. (4) has been found (see Table 1).

4.2 Critically Coupled Regime, $Q_e \approx Q_{un}$

Preliminary experiments have proved that critical coupling of the resonator with a waveguide can be realized for medium angles between the inner polarizing grid and external ones (about 30°). Properly measured and calculated results, as well as guess values of α and L, are given in Table 2.

Based on values of α and L determined from the solution to the system of Eq. (4) we can reconstruct frequency dependencies of S_{11} for both situations given in Tables 1 and 2. Figures 6 and 7 depict both reconstructed and measured behavior of S_{11} as a function of frequency. Rather good agreement between these data is observed.

It should be pointed out that a proper choice of the guess values is important to accomplish the reconstructing process. Hence, the convergence problem needs more detailed study. For the situations considered, a deviation about 30% to 50% from the original guess values can provide a stability of the MinErr process. However, there is no guarantee that the same deviation is valid for other input data. In any case, it is strongly recommended to estimate the stability before applying the MinErr procedure. Sometimes instability is caused by incorrect guess values due to a small shift in resonance frequencies. Their small correction is recommended to

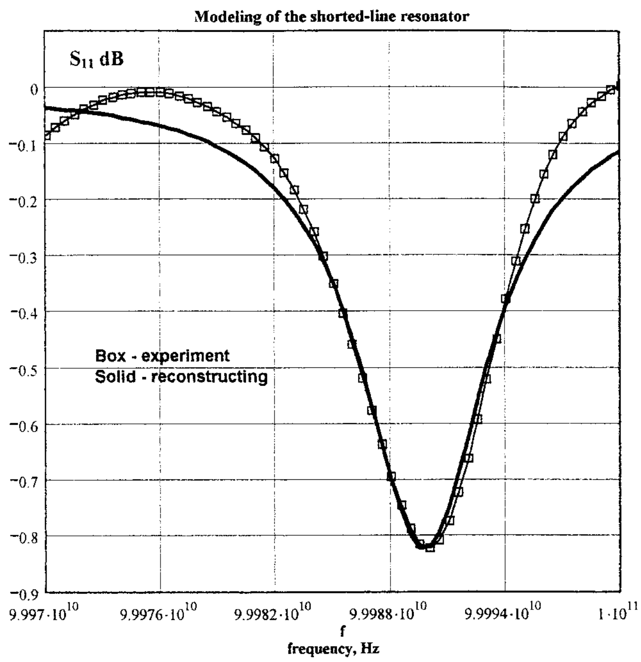

Figure 6 Comparison measured (boxed line) and reconstructed (solid line) of reflection coefficients for the under-coupled regime of a quasi-optical resonator

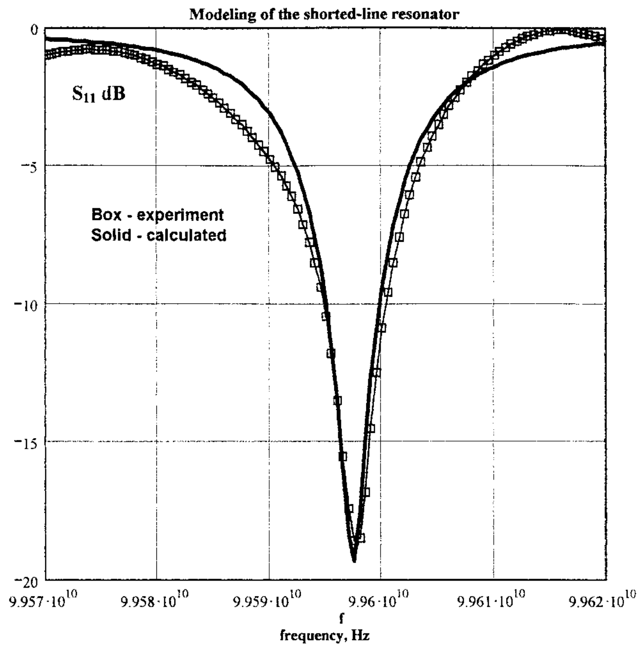

Figure 7 Comparison measured (boxed line) and reconstructed (solid line) of reflection coefficients for the critically-coupled regime of a quasi-optical resonator

avoid this effect. A final checking of the behavior of S_{11} near the resonance frequency is also recommended.

5. CONCLUSION

A consistent approach for determination of Q factors was presented, applicable to mm-wave quasi-optical FEM resonators with high values of Q and proper coupling elements. The model is based on an inductively coupled shorted-line section substituting the FEM resonator. To reconstruct the value of the actual Q factors, a system of two nonlinear equations is solved for preliminarily determined values of attenuation constant and coupling inductance. Only two measurements of reflection coefficient amplitude near the resonance are needed. The examples have demonstrated a validity of this technique in different coupling regimes of the FEM waveguide resonator.

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A TRANSMISSION LINE MODEL APPLIED TO SHAPED SLOT DIELECTRIC-FILLED WAVEGUIDE ANTENNAS

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ABSTRACT: *This paper presents an extension of the so-called transmission line model method (TLMM) applied to shaped slot dielectric-filled waveguide antennas (SDFWAs), especially the L-shaped slot. This radiating longitudinal slot is fed by a transversal slot used as a coupling element; their association is shaped like the letter L.* © 2003 Wiley Periodicals, Inc. Microwave Opt Technol Lett 36: 306–310, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.10750

Key words: *antenna; shaped slot; dielectric-filled waveguide*

INTRODUCTION

Waveguide-fed slot arrays have numerous applications in microwave communications and radar systems, especially when narrow-beam and shaped-beam radiation patterns are required. This variety of applications can be explained by the resonant arrays' very low losses and low crossed-polarization characteristics, typically in millimeter waves. Given these features, the radiation characteristics of longitudinal antennas in the broad face of a rectangular waveguide are determined by the slot's length and its displacement from the center of the waveguide. Slots used in waveguide array are spaced one-half-guide wavelength at the design frequency, which requires them to alternate in order to compensate for the current phase. Weights are obtained by controlling the slot's displacement. One consequence of the design is an increase in cross polarization and the half-power aperture in E-plane increments. The L-shaped slot can reduce these problems. Indeed, the longitudinal part, used as a radiating element, is centered on the broad face of the waveguide and fed by the transversal part. The power intensity is then controlled by the transversal slot's length. Before using this kind of slot in array, its characteristics must be determined.

This paper presents a method which has already been used to compute longitudinal SDFWA [1, 2] and various printed slot antenna shapes [3]. The slot impedance in the slot's plane, found via the TLMM, is computed in the waveguide by using a transformation ratio that corresponds to the voltage discontinuity introduced by the transversal slot.

To validate this method, first the computed results are compared with those of the CST Microwave software [4], in the millimeter-wave band, for several transversal slots' length. Second, a comparison with measurements in the X band, for one transversal length, is done with a metallized foam waveguide [5].