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The general velocity and current modulation linear transfer matrix of FEL and control over SASE power in the collective regime

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ABSTRACT

We present the general linear transfer matrix of FEL in the collective regime in terms of a single-mode radiation field amplitude and e-beam current and velocity modulation parameters. The formulation is useful to account for composite configurations of the FEL, including non-radiating sections, and is employed to demonstrate the possibility to control the SASE radiation power (including its substantial reduction for facilitating coherent emission with seed radiation amplification) by varying the space-charge parameter in a drift section.

1. Introduction

A general description of the FEL as a linear response problem is possible after modal expansion of the coupled Maxwell-plasma equations [1–5]. In the one-dimensional limit and in the case of single radiation and plasma-mode excitation, it is possible to obtain explicit analytical expressions for the transfer matrix in the frequency regime in terms of the small signal variables of the single-mode FEL problem: the radiation-mode amplitude $\tilde{a}_q(z)$, the current density modulation $\tilde{J}_z(z)$, and the beam velocity modulation $\tilde{V}(z)$. Our approach thus relates to the general FEL start-up problem, studied in numerous other works [6–8]. For useful applications we calculate explicit expressions for all matrix parameters, including the excitation parameters of the beam current and velocity modulation, and also specify operating in the practical cold-beam regime [9].

The velocity modulation noise parameters, which are often ignored, are correlated to the beam current modulation through the Poison equation. We contend that their inclusion in the transfer matrix is important for proper description of the noise amplification process in FEL when collective plasma effects in the wiggler or in other beam transport sections are non-negligible. The derivation of the explicit FEL linear response matrix, presented in this article, would be useful for characterization of coherent and incoherent radiation in various FEL configurations in all gain regimes. In particular, it enables to explore the effect of beam transport sections before and along the wiggler (acceleration, drift-free and dispersive sections).

2. General transfer matrix formulation

Under the small-signal assumption we can express all parameters of the electron fluid plasma equations as the sum of a time-averaged part and a time-varying part—whose amplitude is much smaller than the time-averaged part. In conformity with the use of a small signal model, we neglect all cross-products of two time-varying parameters (producing time-independent and second harmonic quantities). Using such approximations, we can write all quantities as a set of linear equations:

$$n(\mathbf{r},t) = n_0 + \frac{1}{2}(\tilde{N}(\mathbf{r})e^{-\mathrm{i}\omega t} + \mathrm{c.c})$$

$$\mathbf{V}(\mathbf{r},t) = \mathbf{V_0} + \frac{1}{2}(\tilde{\mathbf{V}}(\mathbf{r})e^{-\mathrm{i}\omega t} + \mathrm{c.c})$$

$$\mathbf{j}(\mathbf{r},t) = -en(\mathbf{r},t)\mathbf{V}(\mathbf{r},t) = \mathbf{J_0} + \frac{1}{2}(\tilde{\mathbf{J}}(\mathbf{r})e^{-\mathrm{i}\omega t} + \mathrm{c.c})$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r}) + \frac{1}{2}(\tilde{\mathbf{E}}(\mathbf{r})e^{-\mathrm{i}\omega t} + \mathrm{c.c})$$

where $n(\mathbf{r},t)$, $\mathbf{V}(\mathbf{r},t)$, $\mathbf{j}(\mathbf{r},t)$, and $\mathbf{E}(\mathbf{r},t)$ are the beam density, the beam velocity, the beam current, and electric field, respectively.

It is convenient to use the frequency domain in order to find the radiative emission from devices employing e-beams. We use a formulation in which the traveling wave spectral radiation fields are expanded in terms of a complete set of transverse modes q (the beam propagation is entirely in the z-direction):

$$\mathbf{E}_{\mathrm{rad}}(z,t) = \mathrm{Re}\left[\sum_q \tilde{a}_q(z,\omega)\tilde{\mathbf{E}}_{q\perp}(r_\perp)e^{-\mathrm{i}\omega t}\right]$$

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where $\tilde{a}_q$ and $\tilde{\mathbf{E}}_{q\perp}$ are the fast-varying amplitude and the transversal profile of an electromagnetic mode q, respectively. The mode amplitudes $\tilde{a}_q(z)$ may grow along the wiggler interaction length in accordance to the mode excitation equation:

$$\left(\frac{\mathrm{d}}{\mathrm{d}z}-\mathrm{i}k_{qz}\right)\tilde{a}_q(z)=\frac{-1}{4P_q}e^{-\mathrm{i}k_{qz}z}\iint\tilde{J}(x,y,z)\cdot\tilde{\mathbf{E}}_q(x,y)\,\mathrm{d}x\,\mathrm{d}y \tag{1}$$

where $P_q=-\frac{1}{2}\mathrm{Re}\iint\tilde{E}_q\times\tilde{\mathbf{H}}_q^*\cdot\hat{e}_z\,\mathrm{d}x\,\mathrm{d}y$ is the mode normalized power, k_{qz} is the wave number of mode q, and $\tilde{J}$ is the bunching current component at frequency $\omega\cdot$ The current modulation ($\tilde{J}$) is phase matched to the radiation wave ($\tilde{a}_q$) and must be calculated self-consistently from the electron force equations. With these definitions, the power in mode q is given by $|\tilde{a}_q|^2P_q$.

In this work we do not treat average particle acceleration or deceleration, and the time-independent part of the electric field is null. Therefore, the force equation for relativistic motion and the continuity equation only have a time-dependent part:

$$m\gamma_0\gamma_{0z}^2\left(-\mathrm{i}\omega+V_{0z}\frac{\mathrm{d}}{\mathrm{d}z}\right)\cdot\tilde{\mathbf{V}}(z)=-e\tilde{\mathbf{E}}_{\mathbf{pm}}(z)-e\tilde{\mathbf{E}}_{\mathbf{sc}}(z) \tag{2.1}$$

$$\hat{\mathbf{e}}_{\mathbf{z}}\cdot\frac{\mathrm{d}\tilde{\mathbf{J}}}{\mathrm{d}z}=-\mathrm{i}\omega e\tilde{N}(z) \tag{2.2}$$

where m is the electron mass, γ_0 is the electron Lorenz factor, $\gamma_{0z}=\gamma_0/(1+a_w^2/2)$ is the average axial Lorenz factor. The two axial force components are the ponderomotive field [1,5]:

$$\tilde{E}_{\mathrm{pm}}(z)=\tilde{a}_{\mathrm{q}}(z)E_{\mathrm{pm}}e^{\mathrm{i}k_{\mathrm{w}}z} \tag{3.1}$$

and the beam space-charge field:

$$\mathrm{div}\tilde{E}_{\mathrm{sc}}=-\frac{e\tilde{N}(z)}{\varepsilon_0} \tag{3.2}$$

The linearized axial beam current density is

$$\tilde{J}_z(z)=-e(V_{0z}\tilde{N}(z)+n_0\tilde{\mathbf{V}}(z)) \tag{4.1}$$

From Gauss law (Eq. (3.2)) and the continuity equation (Eq. (2.2)), we obtain:

$$\tilde{E}_{\mathrm{sc}}(z)=-\frac{\mathrm{i}}{\omega\varepsilon_0}\tilde{J}_z(z)+\mathrm{const.} \tag{4.2}$$

Using $\mathrm{rot}\,\mathbf{H}=\mathbf{J}-\mathrm{i}\omega\varepsilon_0\mathbf{E}$ and neglecting transverse field variation in the single mode or the one-dimension approximation ($\mathrm{d}/\mathrm{d}x=\mathrm{d}/\mathrm{d}y=0$), there is no z-component of $\mathrm{rot}\,\mathbf{H}$ and the constant has been set zero.

The force equation (Eq. (2.1)), the excitation equation (Eq. (1)) and the continuity equation (Eq. (2.2)) (with field definitions (Eqs. (3.1) and (4.1))) constitute a self-consistent set of linear differential equations. By using Laplace transform, these differential equations are converted from real space (Z) to linear algebraic equations in Laplace space (s). After some algebraic transformations we obtain the known expression for the electromagnetic amplitude [1] and the corresponding expressions for the beam dynamic parameters, which may be written in the matrix form:

$$\begin{pmatrix}\tilde{a}_q(s+\mathrm{i}k_{qz})\\ \hat{J}(s+ik_{qz}+\mathrm{i}k_{\mathrm{w}})\\ \hat{V}(s+ik_{qz}+\mathrm{i}k_{\mathrm{w}})\end{pmatrix}=\frac{1}{\Delta(s)}\begin{bmatrix}(s-\mathrm{i}\theta)^2+\theta_{\mathrm{p}}^2 & \sqrt{\dfrac{S_{\mathrm{pb}}}{P_q}}\dfrac{(s-\mathrm{i}\theta)}{J_0} & -\mathrm{i}\sqrt{\dfrac{S_{\mathrm{pb}}}{P_q}}\dfrac{\omega}{V_{0z}^2}\\ -\mathrm{i}QJ_0\sqrt{\dfrac{P_q}{S_{\mathrm{pb}}}} & s(s-\mathrm{i}\theta) & -\mathrm{i}J_0\dfrac{\omega}{V_{0z}^2}s\\ -Q\dfrac{V_{0z}^2}{\omega}\sqrt{\dfrac{P_q}{S_{\mathrm{pb}}}}(s-\mathrm{i}\theta) & -\dfrac{\mathrm{i}\theta_{\mathrm{p}}^2V_{0z}^2}{J_0\omega}s & s(s-\mathrm{i}\theta)\end{bmatrix}\times\begin{pmatrix}\tilde{a}_q(0)\\ \tilde{J}(0)\\ \tilde{V}(0)\end{pmatrix}$$

where $S_{\mathrm{pb}}=(I_{\mathrm{b}}^2\sqrt{\mu/\varepsilon}/32)(a_{\mathrm{w}}/\gamma\beta_z)^2(1/A_{\mathrm{em}})$, $I_{\mathrm{b}}=J_0A_{\mathrm{e}}$ is the total beam current, $\theta=(\omega/V_{0z})-k_{qz}-k_{\mathrm{w}}$ is the detuning parameter, $\theta_{\mathrm{p}}=(-e/\gamma_zV_{0z})(\sqrt{n_0/\varepsilon_0m\gamma})$ is the relativistic longitudinal plasma wavenumber, $\Delta(s)=s((s-\mathrm{i}\theta)^2+\theta_{\mathrm{p}}^2)-\mathrm{i}Q$—FEL dispersion function (Pierce equation) and Q is the gain factor.

The inverse Laplace transform of the general transfer matrix (Eq. (5)) may be calculated by a standard procedure for a polynomial fraction: the method of residues. Let S_j ($j=1,\ldots,3$) be the roots of the FEL dispersion function. Finally, the transfer matrix can be derived in real space as

$$\begin{pmatrix}\tilde{a}_q(z)\\ \tilde{J}(z)\\ \tilde{V}(z)\end{pmatrix}=e^{-\mathrm{i}k_{qz}z}\begin{bmatrix}H^{\mathrm{EE}} & H^{\mathrm{EJ}} & H^{\mathrm{EV}}\\ H^{\mathrm{JE}} & H^{\mathrm{JJ}} & H^{\mathrm{JV}}\\ H^{\mathrm{VE}} & H^{\mathrm{VJ}} & H^{\mathrm{VV}}\end{bmatrix}\begin{pmatrix}\tilde{a}_q(0)\\ \tilde{J}(0)\\ \tilde{V}(0)\end{pmatrix} \tag{6}$$

where

$$H^{\mathrm{EE}}(\omega)=\sum_{j=1}^{3}Res\left(\frac{(s-\mathrm{i}\theta)^2+\theta_{\mathrm{p}}^2}{\Delta(s)}\right)e^{S_jz}$$

$$H^{\mathrm{EJ}}(\omega)=\frac{1}{J_0}\sqrt{\frac{S_{\mathrm{pb}}}{P_q}}\sum_{j=1}^{3}\mathrm{Res}\left(\frac{s-\mathrm{i}\theta}{\Delta(s)}\right)e^{S_jz}$$

$$H^{\mathrm{EV}}(\omega)=\frac{-\mathrm{i}\omega}{V_{0z}^2}\sqrt{\frac{S_{\mathrm{pb}}}{P_{\mathrm{q}}}}\sum_{j=1}^{3}Res\left(\frac{1}{\Delta(s)}\right)e^{S_jz}$$

$$H^{\mathrm{JE}}(\omega)=-\mathrm{i}QJ_0\sqrt{\frac{P_{\mathrm{q}}}{P_{\mathrm{pb}}}}\sum_{j=1}^{3}Res\left(\frac{1}{\Delta(s)}\right)e^{S_jz-\mathrm{i}k_{\mathrm{w}}z}$$

$$H^{\mathrm{JJ}}(\omega)=\sum_{j=1}^{3}Res\left(\frac{s(s-\mathrm{i}\theta)}{\Delta(s)}\right)e^{S_jz-\mathrm{i}k_{\mathrm{w}}z}$$

$$H^{\mathrm{JV}}(\omega)=-\mathrm{i}J_0\frac{\omega}{V_{0z}^2}\sum_{j=1}^{3}Res\left(\frac{s}{\Delta(s)}\right)e^{S_jz-\mathrm{i}k_{\mathrm{w}}z}$$

$$H^{\mathrm{VE}}(\omega)=-\frac{\mathrm{i}QV_{0z}^2}{\omega}\sqrt{\frac{P_{\mathrm{q}}}{P_{\mathrm{pb}}}}\sum_{j=1}^{3}\mathrm{Res}\left(\frac{s-\mathrm{i}\theta}{\Delta(s)}\right)e^{S_jz-\mathrm{i}k_{\mathrm{w}}z}$$

$$H^{\mathrm{VJ}}(\omega)=-\frac{\mathrm{i}\theta_{\mathrm{p}}^2V_{0z}^2}{J_0\omega}\sum_{j=1}^{3}Res\left(\frac{s}{\Delta(s)}\right)e^{S_jz-\mathrm{i}k_{\mathrm{w}}z}$$

$$H^{\mathrm{VV}}(\omega)=\sum_{j=1}^{3}Res\left(\frac{s(s-\mathrm{i}\theta)}{\Delta(s)}\right)e^{S_jz-\mathrm{i}k_{\mathrm{w}}z}.$$

3. Drift-free propagation

Evidently, in free space there is no coupling between the beam current and an electromagnetic wave. So the ponderomotive force in the force equation (Eq. (2.1)) is equal to zero and the axial Lorenz factor (γ_z) can be replaced by the classic Lorenz factor (γ_0). The set of differential equations for the e-beam propagation in a drift-free section is then given by

$$m\gamma_0^3\left(-\mathrm{i}\omega+V_0\frac{\mathrm{d}}{\mathrm{d}z}\right)\cdot\tilde{\mathbf{V}}(z)=-e\tilde{\mathbf{E}}_{\mathbf{sc}}(z)$$

$$\hat{\mathbf{e}}_{\mathbf{z}}\cdot\frac{\mathrm{d}\tilde{\mathbf{J}}}{\mathrm{d}z}=-\mathrm{i}\omega e\tilde{N}(z)$$

$$\tilde{E}_{\mathrm{sc}}(z)=-\frac{\mathrm{i}}{\omega\varepsilon_0}\tilde{J}(z)$$

In the Laplace space:

$$\hat{J}(s)=\frac{(s-(\mathrm{i}\omega/V_0))\tilde{J}(0)-J_0(\mathrm{i}\omega/V_0^2)\tilde{V}(0)}{(s-(\mathrm{i}\omega/V_0))^2+\theta_{\mathrm{p_d}}^2}$$

$$\hat{V}(s)=\frac{(s-(\mathrm{i}\omega/V_0))\tilde{V}(0)+(\theta_{\mathrm{p_d}}^2V_0^2/\mathrm{i}\omega J_0)\tilde{J}(0)}{(s-(\mathrm{i}\omega/V_0))^2+\theta_{\mathrm{p_d}}^2}$$

where $\theta_{\mathrm{p_d}}=(-e/\gamma V_0)(\sqrt{n_0/\varepsilon_0 m\gamma})$ is the plasma wavenumber in free space.

Finally, the transfer matrix for the free-drift propagation region can be derived in real space as

$$\begin{pmatrix}\tilde{a}_q(L_\mathrm{d})\\ \tilde{J}(L_\mathrm{d})\\ \tilde{V}(L_\mathrm{d})\end{pmatrix}=e^{\mathrm{i}\varphi_\mathrm{B}}\begin{bmatrix}t_\mathrm{R}e^{\mathrm{i}(\varphi_\mathrm{R}-\varphi_\mathrm{B})} & 0 & 0\\ 0 & \cos(\theta_{\mathrm{p_d}}L_\mathrm{d}) & -\frac{\mathrm{i}J_0\omega}{V_0^2\theta_{\mathrm{p_d}}}sin(\theta_{\mathrm{p_d}}L_\mathrm{d})\\ 0 & \frac{\theta_{\mathrm{p_d}}V_{0z}^2}{\mathrm{i}\omega J_0}\sin(\theta_{\mathrm{p_d}}L_\mathrm{d}) & \cos(\theta_{\mathrm{p_d}}L_\mathrm{d})\end{bmatrix}\times\begin{pmatrix}\tilde{a}_q(0)\\ \tilde{J}(0)\\ \tilde{V}(0)\end{pmatrix} \quad (7)$$

where φ_R is the accumulated optical phase of the radiation mode in the drift section and $\varphi_\mathrm{B}=L_\mathrm{d}\omega/V_0$. Drift sections are present in many FEL system designs before and between wiggler sections. In this form we save the EM-wave transfer factor (t_R). It may contain phase-shift and amplitude decay of the EM-signal in the optical beam propagation path. If the radiation beam is not re-injected to the wiggler after the drift-free region, t_R is simply set to be zero.

In this paper we also use drift sections as a model for magnetic dispersion sections which have form similar to Eq. (7) in the space-charge regime.

4. Application to SASE noise reduction in FEL

While SASE FEL is based on amplification of shot-noise, in a coherently seeded FEL, SASE radiation is a noise source that compromises the coherence of the FEL. To obtain coherence in a seeded or pre-bunched FEL, the amplified coherent power should exceed the SASE power considerably. In order to achieve this goal it is desirable to reduce the SASE power as much as possible. We employ the transfer matrix formulation to demonstrate reduction of the SASE noise by proper adjustment of the propagation length of plasma wave excitations in a drift region positioned after a short wiggler section and preceding the long wiggler of the FEL (Fig. 1). The total transfer matrix of a system like this is given by

$$\underline{\underline{\hat{\mathbf{H}}}}_{\mathbf{tot}}=\underline{\underline{\hat{\mathbf{H}}}}_{\mathbf{wig2}}\cdot\underline{\underline{\hat{\mathbf{H}}}}_{\mathbf{d}}\cdot\underline{\underline{\hat{\mathbf{H}}}}_{\mathbf{wig1}}$$

where $\underline{\underline{\hat{\mathbf{H}}}}_{\mathbf{wig1}}$ and $\underline{\underline{\hat{\mathbf{H}}}}_{\mathbf{wig2}}$ are defined by Eq. (6) with $z=L_{\mathrm{w1}}$ and L_{w2}, respectively, and $\underline{\underline{\hat{\mathbf{H}}}}_{\mathbf{d}}$ is defined by Eq. (7).

To use the transfer matrix formulation for incoherent signal, we need to interpret the frequency domain parameters as Fourier components, namely

$$E_{\mathrm{rad}}(z,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\sum_q\tilde{a}_q(z,\omega)\tilde{E}_{q\perp}(r_\perp)e^{-\mathrm{i}\omega t}\right]\mathrm{d}\omega.$$

In this case, the radiation field amplitude is varying over a range of frequencies, and it needs to be described in terms of spectral power:

$$\frac{\mathrm{d}P}{\mathrm{d}\omega}=\frac{2P_q}{\pi}\frac{\langle|\tilde{a}_q(\omega,L)|^2\rangle_T}{T}$$

Here T is an averaging time over which the beam statistics remains stationary.

As an input excitation for SASE FEL we use current shot-noise. For simplicity of the model we assume pure current shot-noise at the entrance to the first wiggler and neglect any other input noise:

$$\frac{\mathrm{d}P^{(J)}(\omega,L)}{\mathrm{d}\omega}=\frac{2P_\mathrm{q}}{\pi}|H_{\mathrm{tot}}^{\mathrm{EJ}}(\omega)|^2\frac{\langle|\tilde{I}(0,\omega)|^2\rangle}{TA_\mathrm{e}^2}$$

where $\tilde{I}(0,\omega)=\tilde{J}(0,\omega)A_\mathrm{e}$ and [10]

$$\frac{\langle|I(0,\omega)|^2\rangle}{T}=eI_\mathrm{b}$$

The total SASE power is then calculated as

$$P_{\mathrm{tot}}=\int\frac{\mathrm{d}P}{\mathrm{d}\omega}\mathrm{d}\omega=\frac{2P_\mathrm{q}}{\pi A_\mathrm{e}}eI_\mathrm{b}\int|H_{\mathrm{tot}}^{\mathrm{EJ}}(\omega)|^2\,\mathrm{d}\omega$$

5. Numerical example

We employ the formulation above to calculate the SASE radiation spectral power and total power at the end of the second wiggler of the High Gain FEL configuration as shown in Fig. 1. For the beam and wiggler parameters we assume exemplary values based on the 4GLS FEL design [11] (see Table 1). For simplicity we assume that single-mode optical guiding is maintained along the

Table 1
XUV-FEL parameters

Energy	750 MeV
Bunch charge	1 nC
RMS bunch duration	266 fs
e-Beam radius	77 μm
Undulator parameters	
Length of wiggler 1	2 m
Length of wiggler 2	10 m
Wiggler parameter a_w	1
Period	45 mm

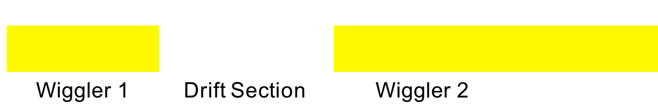

Fig. 1. Scheme of a High Gain FEL with an incorporated drift-free section.

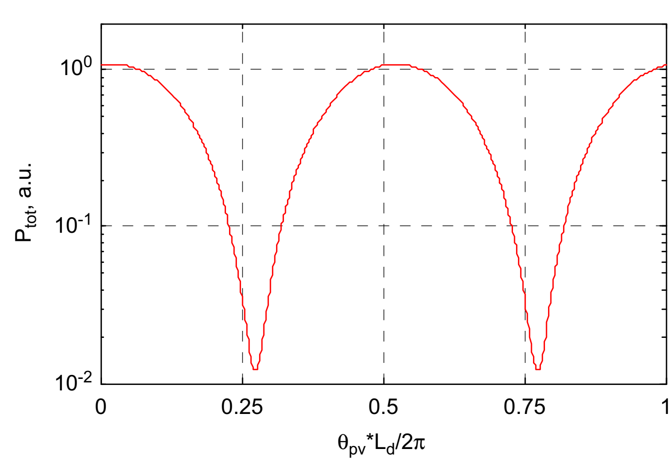

Fig. 2. Total radiated power after the second wiggler section as function of the normalized drift length.

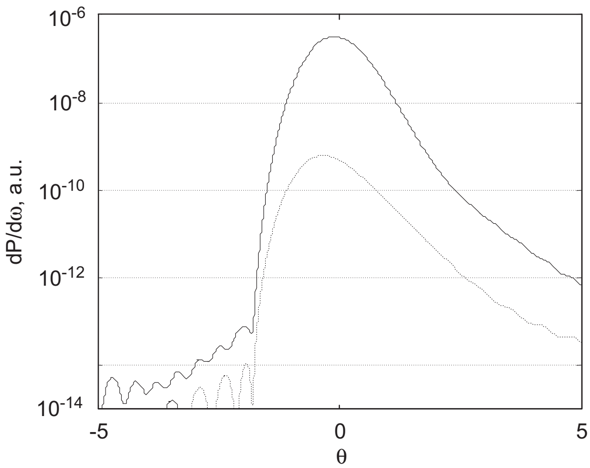

Fig. 3. Spectral power radiation for zero length of drift section (solid curve) and in minimum shot-noise radiation (dashed curve).

entire system, the mode cross-section area is equal to the beam area ($A_{em} = A_e$), and $T_R = 0$.

Fig. 2 displays in a logarithmic scale the significant control of the SASE noise level that can be exercised by varying the length of the drift-free region L_d. An effect of 100 times decrease in the SASE power is attained when the plasma oscillation phase is $\theta_{P_d} L_d \approx \pi/2$ for which the current shot-noise nearly vanishes at the end of the drift sections (see element H_d^{IJ} in Eq. (7)). For the parameters of Table 1 this happens for $L_d \sim 12\,m$. In practice this can be accomplished with a dispersive magnet section of much shorter length.

Fig. 3 shows comparatively the SASE radiation spectral power for the cases of zero-drift region length (solid curve) and maximum noise reduction point (dashed curve). It is seen that the SASE spectrum remains similar in shape but reduced in amplitude by a big factor.

6. Conclusions

A general linear transfer matrix formulation for FEL is presented in the collective regime. This is employed to calculate the SASE emission in an FEL configuration that incorporates a drift section of controlled length. The numerical computation indicates that it is possible to control the SASE emission level, and decrease it by orders of magnitude due to space charge wave effects. It is suggested that this concept can be adopted for enhancing the coherence of future seeded coherent FELs.

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