Coherence Limits of Free Electron Lasers

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Abstract—The e-beam and radiation wave dynamics in the radiating and nonradiating beam transport sections of free electron lasers (FELs) are analyzed in the collective regime by use of a single transverse mode linear response formulation. This is employed to derive conditions for coherent operation of seeded high-gain FELs. It is shown that the level of incoherent selfamplified spontaneous emission radiation power can be controlled by adjusting the plasma oscillation phase in the nonradiating beam transport sections preceding the FEL, and that at short wavelengths the FEL coherence is limited by energy noise (rather than current shot-noise), and ultimately by quantum noise.

Index Terms—Accelerator beams, bunched particle beams, electron beams, free electron lasers (FEL), particle beam dynamics, shot noise.

I. INTRODUCTION

E XCITING progress in the technology of free electron lasers (FEL) holds promise for the development of highbrightness X-ray radiation sources at wavelengths as short as 1 Å, and below with brightness 6–10 orders of magnitude higher than that of other radiation sources in this wavelength regime [1]. These capabilities have just been demonstrated at a wavelength of 1.5 Å in LCLS [2] Important applications in all fields of science are expected to open up with these radiation sources, notably in the biological sciences, where schemes for coherent imaging of single macromolecules are anticipated [3].

Because coherent X-ray radiation sources were not available for use as an input radiation signal in a high-gain FEL amplifier, and because X-ray mirrors were not available for use in the resonator of an FEL oscillator, the leading concept in X-ray, FEL development has been until now self amplified spontaneous emission (SASE) FEL. In this configuration, the amplified signal is the incoherent synchrotron undulator radiation emitted in the first stages of the FEL undulator, or alternatively stated—the current fluctuations associated with the random distribution of charged particles in a beam (shotnoise). Naturally, the temporal coherence of such sources is limited, but they are still extremely bright due to a significant feature of e-beam optical guiding, which makes it possible to establish transverse spatial coherence of the radiation wave in the high-gain operating regime [4].

It would be highly desirable to operate the laser with temporal coherence and high *spectral* brightness. However, this is hampered by the temporal incoherence of SASE, which

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is considered to be ultimately limited by the current shot-noise [5], [6]. In recent years, a number of schemes were developed to overcome the coherence limitation of FELs due to shot-noise. These include schemes of seed radiation injection, which have been demonstrated at the IR [7] and recently at UV wavelengths [8] based on high-harmonic generation of an intense femtoSecond laser beam in a gas. Another seeding scheme is based on prebunching the e-beam by consecutive harmonic generation and high-gain amplification (HGHG) in wiggler structures, which has been demonstrated in the visible [9]. In these schemes, coherence is expected to be achieved if the coherent harmonic signal (of radiation or current modulation) is strong enough to significantly exceed the shot noise (SASE) power.

In this context, as efforts persist to produce temporally coherent X-UV FELs of extremely high-spectral brightness, SASE radiation is no longer a desired FEL output, but rather a source of noise which hinders the attainment of full temporal coherence with a coherently seeded FEL amplifier. In this paper, we address this problem using a linear response formulation to describe the e-beam noise evolution in the beam transport line preceding the FEL [10]. The linear model is also employed to describe the subsequent coherent and incoherent radiation power-generation in the FEL wiggler [11]-[13]. Using the combined analysis of the accelerator transport line and the FEL wiggler we present a scheme for suppression of SASE radiation noise based on controlling the input current shot-noise of the e-beam at the entrance to the wiggler. The formulation results in the conditions for suppression of the radiation noise and the expressions for the ultimate coherence limits achievable in FEL.

II. SINGLE TRANSVERSE MODE LINEAR RESPONSE FORMULATION

The schemes for reduction of beam noise below the current shot-noise level is based on "smoothing" the e-beam current (or density) fluctuations by means of space-charge force repulsion. This occurs around the point where a quarter period of the e-beam plasma oscillation takes place during the transit time along the e-beam transport line [10]: $\omega'_{pr}L/v_{z0} = \pi/2$, where we define the longitudinal beam plasma oscillation frequency as [11]

$$\omega_{pr}^{\prime 2} = r_p^2 e^2 n_0 / \varepsilon_0 m \gamma_0 \gamma_{z0}^2. \tag{1}$$

Here, $r_p \leq 1$ is the finite width beam plasma reduction factor, *e* and *m* are the electron charge and its mass, respectively, n_0 is the electron density in the e-beam, γ_0 is the beam relativistic Lorenz factor, γ_{z0} is the axial (average) Lorenz factor.

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In order to explore the e-beam noise reduction scheme, one need to employ an inclusive frequency transfer matrix of the beam and radiation wave parameters along the entire e-beam transport line from the e-gun cathode to the wiggler exit, including acceleration, drift-free regions, and magnetic dispersive sections. The FEL linear interaction between the radiation field and the beam current waves can be described in the fluid-plasma model in terms of small signal parameters

$$f(\mathbf{r},t) = f_0(\mathbf{r}) + Re\left(\tilde{f}(\mathbf{r})e^{-i\omega t}\right)$$
(2)

where f represents the beam plasma parameters: n is the density, v_z is the velocity, J_z is the current density and E_z is the axial space-charge field; and in terms of the mode expansion coefficients of the radiation fields

$$\mathbf{E}_{rad}\left(\mathbf{r},t\right) = \operatorname{Re}\left[\sum_{q} \tilde{C}_{q}\left(z\right) \tilde{\boldsymbol{\varepsilon}}_{q}\left(\mathbf{r}_{\perp}\right) e^{ik_{z_{q}}z - i\omega t}\right].$$
 (3)

In the present formulation, the radiation modes are normalized to mode power P_q . For a set $\tilde{\boldsymbol{\epsilon}}_q(\mathbf{r}_{\perp})$ of TEM modes: $\frac{1}{2}\sqrt{\mu_0/\epsilon_0}\int |\tilde{\boldsymbol{\epsilon}}_q(\mathbf{r}_{\perp})|^2 d^2r_{\perp} = P_q$. For such normalization, the single frequency power per mode is given at any point by $P(z, \omega) = P_q |\tilde{C}_q(z, \omega)|^2$, where $\tilde{C}_q(z, \omega)$ is a phasor harmonic component, and the multifrequency spectral power (for positively defined frequencies) is given by $\frac{dP(z,\omega)}{d\omega} = \frac{2}{\pi}P_q |\tilde{C}_q(z,\omega)|^2$, where $\tilde{C}_q(z,\omega)$ is a Fourier spectral component. The Fourier decomposition is defined here by $\check{f}(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt$.

The radiation and plasma wave parameters are coupled to each other by the fluid plasma equations (force equation, continuity equation) and the Maxwell equations, including the Poisson space-charge equation. The synchronous coupling between the "slow" Longmuir space-charge waves (eigensolutions of the finite cross-section e-beam problem) and the "fast" radiation modes is made possible due to the longitudinal ponderomotive force resulting from the $\mathbf{v}_{\mathbf{w}}(\mathbf{r}) \times \mathbf{B}_{rad}(\mathbf{r},t)$ term in the Lorentz force equation. The wiggler wavenumber k_w , which originates from the electron transverse wiggling velocity, $\mathbf{v}_w(\mathbf{r}) = \operatorname{Re}\left(\tilde{\mathbf{v}}_w e^{-ik_w z}\right)$ makes it possible to synchronize the radiation wave with the e-beam and bunch it. The synchronized interaction of the bunched e-beam and the radiation field provides the FEL gain [11], [13].

We assume a model of a single beam-plasma wave Longmuir mode (characterized by a plasma reduction factor) and a single radiation mode q. It is assumed that the electron beam of cross-section area A_e is narrow, so that the radiation mode field is nearly uniform across the electron beam. The effective cross-section area of the mode is defined by

$$A_{em} = 2P_q Z_0 \Big/ \left| \tilde{\varepsilon}_q(\mathbf{r}_{\perp e}) \right|^2$$

where $\mathbf{r}_{\perp e}$ are the transverse coordinates of the beam, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the free space wave impedance.

In the high-gain FEL regime, we assume the mode q to be the fundamental e-beam guided mode [14] which characterized by a filling factor parameter $FF = A_e/A_{em}$. This formulation is a single-mode model extension of the basic 1-D FEL theory.

For the analysis of the e-beam modulation and noise dynamics we introduce here a relativistic extension of Chu's kinetic voltage parameter [10], [15], [16]

$$\tilde{V} = -\tilde{v}_z \frac{m}{e} \frac{d\gamma_0(z)}{dv_{z0}} = -\frac{m}{e} \gamma_0 \gamma_{z0}^2 v_{z0} \tilde{v}_z = -\left(\frac{mc^2}{e}\right) \tilde{\gamma}(z).$$
(4)

This expansion near the average beam parameters is valid also for sections of axial acceleration and sections with transverse magnetic force: $\gamma_{z0}^2(z) = \gamma_0^2(z)/(1 + a_{\perp}^2(z)), a_{\perp}(z) = -(e/mc) \int_0^z B_{\perp}(z') dz'.$

The solution of the FEL linear response problem can be expressed in terms of the radiation mode amplitude $\tilde{C}_q(z)$, the beam-current $\tilde{i}(z) = \int \tilde{J}_z(\mathbf{r}) d^2 \mathbf{r}_{\perp}$, the kinetic voltage $\tilde{V}(z)$ and a general frequency transfer matrix

$$\begin{pmatrix} C_q(z) \\ \tilde{i}(z) \\ \tilde{V}(z) \end{pmatrix} = \underline{\tilde{\mathbf{H}}}_{FEL}(z) \begin{pmatrix} \tilde{C}_q(0) \\ \tilde{i}(0) \\ \tilde{V}(0) \end{pmatrix}.$$
(5)

We define at the wiggler exit $\underline{\tilde{\mathbf{H}}}_{FFL}(z = L_w) = \underline{\tilde{\mathbf{H}}}_{FFL}$

$$\underline{\underline{\tilde{H}}}_{FEL} = \begin{pmatrix} H^{EE} & H^{Ei} & H^{EV} \\ H^{iE} & H^{ii} & H^{iV} \\ H^{VE} & H^{Vi} & H^{VV} \end{pmatrix}.$$
(6)

The explicit expression for the components of the transfer matrix of a uniform wiggler section can be derived in all linear gain regimes [12]. In the high-gain regime, the FEL transfer matrix (6) can be written as

$$\widetilde{\underline{\mathbf{H}}}_{FEL} = \widetilde{\mathbf{h}}(L_w) \exp\left(-\frac{(\omega - \omega_0)^2}{2\Delta\omega_{HG}^2} \frac{\sqrt{3} + i}{3}\right) \\
\exp\left(\frac{\sqrt{3} + i}{2}\Gamma L_w\right) \exp\left(\frac{-i\pi}{12}\right)$$
(7)

where [see (8) at the bottom of the page]

$$\tilde{\mathbf{h}}(L_w) = \frac{1}{3} \begin{pmatrix} 1 & -\frac{\tilde{\varepsilon}_q^*}{8P_q} \frac{a_w}{\gamma\beta_z\Gamma} \exp\left(i\frac{\pi}{3}\right) \frac{i\tilde{\varepsilon}_q^*}{8P_q} \frac{a_w}{\gamma\beta_z\Gamma^2} \frac{\theta_{prw}}{W_w} \exp\left(i\frac{\pi}{6}\right) \\ -i\tilde{\varepsilon}_q \frac{a_w}{2\gamma\beta_z} \frac{\theta_{prw}}{W_w\Gamma^2} \exp\left(i\frac{\pi}{6}\right) & 1 & -\frac{i\theta_{prw}}{W_w\Gamma} \exp\left(i\frac{\pi}{3}\right) \\ \tilde{\varepsilon}_q \frac{a_w}{2\gamma\beta_z\Gamma} \exp\left(i\frac{\pi}{3}\right) & i\frac{W_w\Gamma}{\theta_{prw}} \exp\left(i\frac{\pi}{6}\right) & 1 \end{pmatrix}.$$
(8)

 $\theta_{prw} = \omega'_{pr}/v_{oz}$ is the beam plasma wavenumber, $a_w = eB_w/mk_w$ is the wiggler parameter, $W_w = r_p^2 Z_0 / (k\theta_{prw}A_e)$ is the wiggler e-beam impedance, $\Delta \omega_{HG} = 3^{3/4} \sqrt{\Gamma/L_w \omega_0/k_w}$ is the FWHM of the gain curve for the high-gain FEL and $\Gamma^3 = \frac{\theta_{prw}}{32A_{em}} \left(\frac{a_w}{\gamma \beta_z}\right)^2 \frac{Z_0}{W_w}$ is the gain parameter [11]. The gain parameter Γ related to the so called Pierce parameter [18] through

$$\Gamma = 2k_w\rho. \tag{9}$$

III. ACCELERATOR AND BEAM TRANSPORT SECTION

In nonradiating nondissipative sections we postulate, following Chu [10], [15], [16], a relativistic kinetic power conservation theorem, which is applicable also in the acceleration and drift beam transport sections, including drift through transverse magnetic fields

$$\operatorname{Re}\left(\tilde{i}\left(z\right)\tilde{V}\left(z\right)^{*}\right) = const.$$
(10)

From here, it is shown (Appendix A) that the general transfer matrix of such a nondissipative transport section has the following symmetry

$$\underline{\tilde{M}} = \begin{pmatrix} K\cos\phi & -\frac{i}{W}\sin\phi \\ -iW\sin\phi & \frac{1}{K}\cos\phi \end{pmatrix} \exp(i\varphi_A)$$
(11)

where ϕ , K and W are real functions.

For particular cases, it is possible to solve analytically the plasma equations and get explicit expressions. In general, $\varphi_A = \phi_b = \omega \int_{z_i}^{z} dz' / v_z (z')$, which is the optical phase accumulated by the e-beam modulation along the interaction length [12].

In a section of fast acceleration (no plasma wave oscillation) dynamics, the off-diagonal elements of (11) vanish and $\tilde{i}(z) = \tilde{i}(z_i)$, $\tilde{V}(z) = \tilde{V}(z_i)$. Consequently the transfer matrix (11) applies with $\phi = 0$, K = 1 ($\underline{\tilde{M}}_{acc} = \underline{I}_{acc}$ a unit matrix). In a drift section of constant parameters, it is found [10]

In a drift section of constant parameters, it is found [10] that K=1, $W = W_d = r_p^2 Z_0 / (k\theta_{pr}A_e)$ is the e-beam wave impedance, $\phi = \phi_p = \theta_{pr}(z - z_i)$ is the phase of the plasma wave, $\theta_{pr} = \omega'_{pr}/v_{z0}$ is the plasma wavenumber, ω'_{pr} is the plasma frequency (1), all evaluated for the drifting beam parameters γ_{z0} , γ_0 , v_{z0} .

In a nondissipative section, there is no interaction between the e-beam and radiation wave, and the 3×3 transfer matrix of the beam transport section is

$$\underline{\underline{\tilde{H}}}_{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \\ 0 & \underline{\tilde{M}}_{T} \end{pmatrix}.$$
 (12)

If the transport section is composed only of fast acceleration and drift sections, $\underline{\tilde{M}}_T = \underline{\tilde{M}}_{DRIFT}$ and the 3×3 transfer matrix from the cathode to the FEL wiggler may be modeled by

$$\underline{\tilde{H}}_{T} = \begin{pmatrix} 1 & 0 & 0\\ 0 & \cos \phi_{p} & -i \sin \phi_{p} / W_{d}\\ 0 & -i W_{d} \sin \phi_{p} & \cos \phi_{p} \end{pmatrix} \exp(i\varphi_{b}).$$
(13)

In our previous analysis of noise dynamics in a drifting electron beam [10] we showed, based on Eq. 13, that at a drift distance

$$L_d = \pi / 2\theta_{pr} \tag{14}$$



Fig. 1. FEL system composed of a free drift e-beam transport section and a radiating wiggler section.

the initial beam velocity noise turns into current noise and vice versa

$$\overline{\left|\breve{i}(L_d,\omega)\right|^2} = \overline{\left|\breve{V}(z_i,\omega)\right|^2} / W_d^2$$
(15)

$$\overline{\left|\breve{V}(L_d,\omega)\right|^2} = \overline{\left|\breve{i}(z_i,\omega)\right|^2} W_d^2 \tag{16}$$

where for any physical variable f we define $\check{f}(\omega) \equiv \langle \check{f}(\omega) \rangle_T / T$ as an ensemble average over statistical random variables of the spectral parameter, where $\check{f}(\omega) \equiv \int_{-T/2}^{T/2} f(t) e^{-i\omega t} dt$, T is an averaging time period longer than the radiation coherence time.

Usually, the noise of high-quality relativistic electron beams, that are used in FELs, are dominated by current shot-noise. Namely

$$N^{2} \equiv \overline{\left|\breve{V}(z_{i},\omega)\right|^{2}} / \overline{\left|\breve{i}(z_{i},\omega)\right|^{2}} W_{d}^{2}; N^{2} \ll 1.$$
(17)

Under this condition, transporting the beam through a quarter plasma wavelength oscillation length (14) reduces the current shot-noise by a factor N^2 (17), and since the SASE radiation power in FEL is believed to be dominated by the input current shot-noise, this would enable suppression of SASE radiation power. However, since there is continued noise evolution dynamics also within the FEL interaction region, it is necessary to solve for the noise dynamics evolution in the combined system of the beam transport section and the wiggler in order to get more accurate expressions for the radiation noise suppression and the optimal drift length for SASE output minimization power.

IV. COHERENT AND INCOHERENT RADIATION IN THE COMBINED E-BEAM TRANSPORT AND FEL SECTIONS

Consider now a general FEL structure that consists of a nonradiative section (acceleration and drift sections) and a radiating wiggler (Fig. 1). Based on the solution of the FEL linear response problem (5) for *coherent radiation* seed injection and for *coherent beam prebunching* schemes we get, respectively, the following:

$$[P_s(L_w)]_{coh} = P_q \left| \tilde{C}_q(L_w) \right|^2 = P_q \left| \tilde{H}_{FEL}^{EE} \right|^2 \left| \tilde{C}_q(0) \right|^2$$
(18)

$$[P_{s}(L_{w})]_{prebunch} = P_{q} \left| \tilde{H}_{FEL}^{Ei}(\omega) \tilde{i}(0) + \tilde{H}_{FEL}^{Ev}(\omega) \tilde{V}(0) \right|^{2}$$
(19)

where z = 0 is the wiggler entrance point and L_w its length.

For the *incoherent* radiation power calculation we need to keep a transfer matrix that includes both the FEL ($\underline{\tilde{\mathbf{H}}}_{FEL}$) and accelerator ($\underline{\tilde{\mathbf{H}}}_{T}$) sections (starting from the "cathode" position $z = z_c$ or more correctly—from the drift section entrance point)

$$\underline{\underline{\tilde{H}}}_{TOT} = \underline{\underline{\tilde{H}}}_{FEL} \underline{\underline{\tilde{H}}}_{T}.$$
(20)

The total incoherent spectral power at the FEL output is then

$$\left(\frac{dP(L_w,\omega)}{d\omega}\right)_{incoh} = \frac{2P_q}{\pi} \left|\overline{\check{C}_q(L_w,\omega)}\right|^2 = \frac{2P_q}{\pi} \left\{ \left|\tilde{H}_{FEL}^{EE}\right|^2 \left|\overline{\check{C}_q(0,\omega)}\right|^2 + \left|\tilde{H}_{TOT}^{EV}(\omega)\right|^2 \left|\overline{\check{V}(z_c,\omega)}\right|^2 + \left|\tilde{H}_{TOT}^{EV}(\omega)\right|^2 \left|\overline{\check{V}(z_c,\omega)}\right|^2 + 2\operatorname{Re}\left(\tilde{H}_{TOT}^{EI}\tilde{H}_{TOT}^{EV*}\right)\operatorname{Re}\left(\overline{\check{i}\check{V}_c^*}\right) - 2\operatorname{Im}\left(\tilde{H}_{TOT}^{Ei}\tilde{H}_{TOT}^{EV*}\right)\operatorname{Im}\left(\overline{\check{i}\check{V}_c^*}\right)\right\}.$$

$$(21)$$

Our model is specified to a caustic beam case, namely the beam charge bunch is uniform and long enough relative to the optical wavelength, so that front and end coherent emission edge effects are negligible. At the entrance to the drift section, we assume that various dissipative processes that increase the e-beam energy spread and emittance render the velocity noise to be *uncorrelated* with the electron current shot noise [10]. At this no-correlation point, $\text{Im}\left(\tilde{i}\tilde{V}_c^*\right) = 0$, and single particle analysis produces the following expressions for the beam noise parameters

$$\left|\check{i}\right|^2 = eI_b \tag{22}$$

$$\left|\breve{V}\right|^{2} = \left(mc^{2}\delta\gamma_{ef}\right)^{2}/eI_{b}$$
⁽²³⁾

$$\check{i}\check{V}^* = mc^2\delta\gamma_{ef}\delta\beta_{zc}/\beta_{zc}.$$
(24)

Here, $\delta \gamma_{ef} \equiv \gamma_{z0c}^2 \gamma_{0c} \beta_{z0c} \delta \beta_{zc}$, where $\beta_{z0c} = \langle \beta_{zj} \rangle_j$ and $(\delta \beta_{zc})^2 = \langle (\beta_{zj} - \beta_{z0c})^2 \rangle_j$ are averages over the electron beam axial velocity b_{z0j} distribution at the start point $(\delta \beta_{zc} < \beta_{z0c})$. The beam energy spread is fundamentally limited by the cathode temperature (for thermionic cathode): $\delta \gamma_{ef} = \delta \gamma_c = k_B T_c / mc^2$, where T_c is the cathode temperature and k_B —Botlzman constant. However, in practice, after transport through the e-gun and accelerator sections the energy spread is increased or even intently heated [17]. The effective ("slice") energy spread $\delta \gamma_{ef}$ after acceleration is hard to measure, but is at least 3 orders of magnitudes larger than the cathode temperature energy spread.

To deal further with the incoherent FEL radiation, instead of the output power (21), it is convenient to define an incoherent (noise) effective radiation input power, which lumps all effective incoherent input signal sources, and is composed of the e-beam shot-noise contributions (of current, velocity and kinetic power) and the quantum spontaneous emission and background black-body radiation at the FEL entrance (see Fig. 2)

$$\left(\frac{dP_{in}}{d\omega}\right)^{eff} = \left(\frac{dP(L_w)}{d\omega}\right)_{incoh} / \left|\tilde{H}_{FEL}^{EE}\right|^2.$$
(25)



Fig. 2. Coherent and incoherent radiation and beam modulation input signals at the FEL amplifier input. The amplified signals output power is partially coherent.

V. CONVENTIONAL FEL THEORY

In conventional FEL theory, it is customary to assume $\underline{\tilde{\mathbf{H}}}_{TOT} = \underline{\tilde{\mathbf{H}}}_{FEL}$ (no plasma oscillation dynamics and no velocitycurrent noise correlation process in the beam transport sections preceding the FEL). Substituting then (22)–(24) into (21), (25), one obtains that the e-beam effective radiation input noise power (see Fig. 2) is composed of three contributions (current noise, velocity noise and kinetic power noise)

$$\left(\frac{dP_{in}}{d\omega}\right)_{conv}^{i} = \frac{eI_b Z_0}{16\pi A_{em}} \left(\frac{a_w}{\gamma \beta_z \Gamma}\right)^2$$
(26)

$$\left(\frac{dP_{in}}{d\omega}\right)_{conv}^{V} = \left(\frac{mc^{2}\delta\gamma_{ef}}{eI_{b}}\right)^{2} \left(\frac{\theta_{pr}}{W_{w}\Gamma}\right)^{2} \left(\frac{dP_{in}}{d\omega}\right)_{conv}^{i}$$
(27)

$$\left(\frac{dP_{in}}{d\omega}\right)_{conv}^{iv} = \frac{2}{\pi}\sqrt{3}mc^2\delta\gamma_{ef}\delta\beta_{zc}/\beta_{zc}.$$
 (28)

In addition, there may be an effective input radiation noise due to spontaneous emission and finite temperature (T) background black-body radiation [19]

$$\left(\frac{dP}{d\omega}\right)_{in}^{E} = \hbar\omega / \left(1 - e^{-\hbar\omega/k_{B}T}\right).$$
⁽²⁹⁾

FELs operate in practice only in the cold beam regime [11] $\delta\beta_{z0}/\beta_{z0} \ll \Gamma/k_0$, where $\delta\beta_{z0} = \delta\gamma_{ef}/\gamma_{z0}^2\gamma_0\beta_{z0}$ is the axial velocity spread in the FEL. Therefore (27), (28) are negligible relative to (26). This justifies in retrospect the nonobvious common neglect of velocity noise in conventional SASE-FEL theory.

Thus, under the assumptions of conventional FEL 1-D linear theory, neglecting all noise contributions except current shotnoise, the FEL coherence condition for seed radiation and prebunching schemes simplify to

$$[P_s(0)]_{coh} \gg \frac{eI_b Z_0}{16\pi A_{em}} \left(\frac{a_w}{\gamma \beta_z \Gamma}\right)^2 \Delta \omega \tag{30}$$

$$\left|\tilde{i}_{s}(0)\right|^{2} \gg eI_{b}\Delta\omega. \tag{31}$$

Here, $\Delta \omega$ is the frequency bandwidth of the incoherent power [11]. If filtering is employed, then $\Delta \omega$ is the filter bandwidth. In a pulse of duration t_p the bandwidth is Fourier transform limited: $\Delta \omega \approx \pi/t_p$.

VI. SASE POWER CONTROL AND SUPPRESSION

We now show that by proper control of the e-beam plasma dynamics in the nonradiating sections of the transport line it may be possible to reduce the current shot noise below the velocity noise, so that the incoherent power of the FEL would not be limited by shot noise, but by the slice energy spread of the beam.

Assume there is a nonradiative section between the starting point (noncorrelation) and the wiggler entrance that contains a fast acceleration section to any middle energy level (γ_{0d}), a long enough drift section and a second fast acceleration stage, which accelerates the beam to the energy level, (γ_0) required for the wiggler radiation (Fig. 1). In this case, the effective input power derived from (21) and (25) is modified due to the dynamics of energy transfer and correlation between the beam dynamic parameters (\check{i} and \check{V}) in the drift section. The modified current, kinetic voltage and kinetic power noise effective radiation input power can be written then in terms of the corresponding conventional (no drift section) effective input radiation noise power (26)–(28). Using (8) and (13), one obtains

$$\left(\frac{dP_{in}}{d\omega}\right)^{i} = \left(\frac{dP_{in}}{d\omega}\right)^{i}_{conv} \left|\cos\phi_{pd} - e^{i\pi/6}S\sin\phi_{pd}\right|^{2}$$
(32)

$$\left(\frac{dP_{in}}{d\omega}\right)^{V} = \left(\frac{dP_{in}}{d\omega}\right)_{conv}^{V} \left|\frac{1}{S}\sin\phi_{pd} + e^{-i\pi/6}\cos\phi_{pd}\right|^{2} \quad (33)$$

$$\left(\frac{dP_{in}}{d\omega}\right)^{iV} = \left(\frac{dP_{in}}{d\omega}\right)^{iV}_{conv}.$$
(34)

Here, the noise suppression parameter S is

$$S = \frac{W_d}{W_w} \frac{\theta_{prw}}{\Gamma} \approx \left(\gamma_{0d}^3 \beta_{0d}^3 / \gamma_0 \gamma_{0z}^2 \beta_{0z}^3\right)^{\frac{1}{2}} \left(\theta_{prw} / \Gamma\right). \tag{35}$$

Equations (32), (33) can be written as

$$\left(\frac{dP_{in}}{d\omega}\right)^{i} = \left(\frac{dP_{in}}{d\omega}\right)^{i}_{conv} \frac{1+S^{2}+\sqrt{1+S^{2}+S^{4}}\cos\left(2\phi_{pd}+\varphi\right)}{2}$$
(36)

$$\left(\frac{dP_{in}}{d\omega}\right)^{V} = \left(\frac{dP_{in}}{d\omega}\right)^{V}_{conv} \frac{1 + S^{2} - \sqrt{1 + S^{2} + S^{4}}\cos\left(2\phi_{pd} + \varphi\right)}{2S^{2}}$$
(37)

where $\varphi = \arccos \frac{1-S^2}{\sqrt{1+S^2+S^4}}$. In the limit $\phi_{pd} = 0$, these equations reduce to the conventional theory expressions (26)-(27). Normally $\gamma_{0d} < \gamma_0$, and since we assumed operating in the high-gain tenuous beam regime of FEL $\theta_{prw} \ll \Gamma$, the inequality, $S \ll 1$ is always satisfied. Therefore, the effective input radiation power originating from the current shot-noise (32) attains its is minimum at

$$\phi_{pd} = \frac{\pi}{2} - \frac{\sqrt{3}}{2}S.$$
 (38)

This suppressed input radiation power is then

$$\left(\frac{dP_{in}}{d\omega}\right)^{i} = \left(\frac{S}{2}\right)^{2} \left(\frac{dP_{in}}{d\omega}\right)^{i}_{conv}.$$
(39)

However, at the same time the effective input radiation power originating from the kinetic voltage noise attains a maximum value

$$\left(\frac{dP_{in}}{d\omega}\right)^{V} = \left(\frac{1}{S}\right)^{2} \left(\frac{dP_{in}}{d\omega}\right)^{V}_{conv}.$$
(40)

Both effective radiation input power parameters (39), (40) are smaller than the conventional (no drift) input radiation power. To weigh their importance we define the parameter D which is the ratio between these two terms

$$D \equiv \left(2N/S\right)^2 \tag{41}$$

where $N^2 \ll 1$ is defined in (17).

A. Dominance of Suppressed Radiation Noise Due to Shot-Noise

Since both $N \ll 1$ and $S \ll 1$, we need to examine separately two cases. If $2N \ll S$, then $D \ll 1$, and consequently

$$\left. \left(\frac{dP_{in}}{d\omega} \right)^i \right|_{\min} \gg \left(\frac{dP_{in}}{d\omega} \right)^{iV} \gg \left(\frac{dP_{in}}{d\omega} \right)^V.$$
(42)

In this case, the dominant effective input radiation power term is the contribution of the conventional current shot-noise, which is reduced by a factor $(S/2)^2 \ll 1$ relative to the conventional effective radiation power (see (39). Its explicit expression [substituting (26) and (35)] is

$$\left(\frac{dP_{in}}{dw}\right)^{eff} = \frac{2}{\pi} \frac{\theta_{prw}}{\Gamma} \frac{W_d^2}{W_w} eI_b.$$
(43)

B. Dominance of Suppressed Radiation Noise Due to Velocity Noise (Energy Spread)

By choice of $\gamma_{0d} \ll \gamma_{0w}$, the current noise reduction in (39) can be substantial, but at the same time the effective input radiation power originating from the cathode velocity noise (33) increases by about the same factor and may become dominant. If $2N \ll S$ then $D \gg 1$ and the inequalities (42) are reversed. The dominant noise contribution in this case is due to the cathode velocity noise

$$\left(\frac{dP_{in}}{d\omega}\right)^{V} = N^{2} \left(\frac{dP_{in}}{d\omega}\right)^{i}_{conv}.$$
(44)

Also, in this latter case, substantial radiation noise suppression is attained, a factor $N^2 \ll 1$ (17) relative to the conventional SASE radiation power (26). After substitutions we find out that the effective input radiation noise of the FEL is determined in this case by the initial beam energy spread (and *not* by its shot-noise)

$$\left(\frac{dP_{in}}{d\omega}\right)^{eff} = \frac{2}{\pi} \frac{\Gamma}{\theta_{prw}} \frac{W_w}{W_d^2} \frac{\left(mc^2 \delta \gamma_{eff}\right)^2}{eI_b}.$$
 (45)

Satisfying the conditions for this case and attaining effective incoherent input power limited by the beam velocity noise (namely, by its energy spread) is realizable with present accelerator parameters at optical (perhaps up to near UV) frequencies.

C. Suppressed Radiation Noise in the Quantum Noise Limit

The Bose-Einstein radiation input noise term (29) assumes the value k_BT at low frequencies, up to the THz regime (for ambient temperature T = 300 K). At higher frequencies it results in the quantum limit radiation noise expression $\left(\frac{dP}{d\omega}\right)_{in}^{E} = \hbar\omega$. In the high (X-UV) frequency regime, this term can be quite large. Yet it has never been expected that FEL coherence may be governed by this quantum limit, and the fundamental FEL coherence limit has been considered always to be the current shot-noise (30). Theoretically, at high frequencies the quantum radiation input noise $\hbar\omega$ become dominant over all other terms (43), (45), and the FEL coherence would become quantum limited

$$\left(\frac{dP}{d\omega}\right)_{in}^{eff} = \hbar\omega. \tag{46}$$

With present day accelerator technology the validity conditions of the analysis cannot yet be satisfied at this limit.

VII. DISCUSSION AND CONCLUSION

The conditions for coherent operation of seeded high-gain FEL and for operating such FELs with coherence beyond the shot-noise limit were derived. The analysis is based on single Longmuir mode fluid plasma linear response theory employed in the wiggler and in a drift section preceding the wiggler.

We obtained two new expressions for the lower limits of the suppressed beam-noise equivalent radiation input power. In one case, the incoherent radiation (SASE) power is reduced by a factor corresponding to the ration between the initial velocity noise and current shot-noise. In the other case, the FEL incoherent radiation is limited by the velocity noise (or initial energy spread) only. We also identified a theoretical limit in which the incoherent radiation of FEL is limited by quantum noise.

In practice, a variety of effects can limit the validity of the model and may impede attainment of the theoretically predicted coherence limits. Various e-beam instabilities and aberrations in the accelerator and in electron-optical components, electron Coulomb collisions (at low energies-the Boersch effect [20]) and wake-field interactions along the transport line may increase the electron axial velocity spread, and may interfere in the plasma wave oscillation process. Also, excitation of multiple transverse plasma waves may make it difficult to control the SASE power with a single parameter ϕ_p . As discussed in [10], the single transverse mode Longmuir plasma wave condition may be attained by proper design of the drift section parameters. However, at any operating regime it is necessary to verify that electron-optical and beam transport imperfections do not corrupt the collective noise suppression process in the drift section, and that 3-D effects do not overshadow the process [21].

At the present technological state of the art it would be hard to attain optical current shot-noise suppression at short wavelength (X-UV) and use it to enhance the coherence of *seed radiation* injected FELs. However, in *prebunching* schemes like HGHG, the noise suppression scheme may be still effective even at such short wavelengths, since the main contribution to the high-frequency shot-noise in this scheme originates from HGHG of the shot-noise at the *fundamental harmonic frequency*, where shot-noise reduction is possible.

We conclude that theoretical considerations permit very high coherence and *spectral* brightness of FEL operating at optical frequencies, and such enhancement may possibly be attainable in the future in X-UV FELs. Appropriate design and technological improvements of the nonradiating sections of the FEL transport line can provide control over SASE shot noise power. Once coherence beyond the current shot-noise limit is attained, the FEL coherence is limited by the beam velocity noise determined by its energy spread. The ultimate coherence limit of FELs is the *quantum noise limit*. It is theoretically attainable at X-UV frequency, but its attainment is yet beyond the present technological state of the art. However, its definition can serve presently as a fundamental limit yard-stick, similarly to the Schawlow–Towns limit for atomic laser oscillators [22], [23].

APPENDIX A

In a small signal 1-D or single transverse mode Langmuir plasma wave model, the beam plasma equations reduce to a set of linear equations relating $\tilde{i}(z)$ and $\tilde{V}(z)$ at any point to their initial value at z_i [10], [16], [21]. Consequently the small signal current and kinetic voltage at any distance relate to their initial conditions through a 2 × 2 transfer matrix

$$\begin{pmatrix} \tilde{i}(z) \\ \tilde{V}(z) \end{pmatrix} = \underbrace{\tilde{\mathbf{M}}}_{=} \begin{pmatrix} \tilde{i}(z_i) \\ \tilde{V}(z_i) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \tilde{i}(z_i) \\ \tilde{V}(z_i) \end{pmatrix}.$$
(A-1)

We substitute this general relation in the extended Chu's theorem (10).

Since the ABCD-matrix elements are independent of the beam parameters (\tilde{V} and \tilde{i}), and one can choose arbitrary values of these parameters (include zero), we get that the matrix elements must satisfy three conditions

$$A^*C + C^*A = 0 (A-2)$$

$$B^*D + D^*B = 0 (A-3)$$

$$C^*B + D^*A = 1.$$
 (A-4)

Presenting the ABCD-matrix element in terms of modulus and phase (for example $A = |A| \exp(i\varphi_A)$ the transfer matrix can be written as

$$\underline{\tilde{\mathbf{M}}} = \begin{pmatrix} |A| & -i |B| \\ -i |C| & |D| \end{pmatrix} \exp(i\varphi_A).$$
(A-5)

From (A-2) and (A-3), one can observe that the matrix $\underline{\tilde{\mathbf{M}}}$ satisfied $|\det \underline{\tilde{\mathbf{M}}}| = 1$. Therefore it is possible to express the terms of the matrix in terms of ordinary trigonometric functions and three independent real functions ϕ , *K* and *W*, which are defined by

$$|A| |D| = \cos^2(\phi) \tag{A-6}$$

$$|A|/|D| = K^2 \tag{A-7}$$

$$|C|/|B| = W^2 \tag{A-8}$$

where *K* and *W* are the positive functions: $0 < K < \infty$ and $0 < W < \infty$. Finally, the transfer matrix may be written in general as

$$M(z) = \begin{pmatrix} K\cos(\phi) & -\frac{i}{W}\sin(\phi) \\ -iW\sin(\phi) & \frac{1}{K}\cos(\phi) \end{pmatrix} \exp(i\varphi_A(z)). \quad (A-9)$$

This general symmetry applies to the transfer matrix of any nondissipative beam transport section, including drift, acceleration and dispersive transport sections. To find the values and z-dependence of the three independent matrix parameters the linear beam plasma equations must be solved specifically.

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