Optical frequency shot-noise suppression in electron beams: Three-dimensional analysis

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A predicted effect of current shot-noise suppression at optical-frequencies in a drifting charged-particle-beam and the corresponding process of particles self-ordering are analyzed in a one-dimensional (1D) model and verified by three-dimensional numerical simulations. The analysis confirms the prediction of a 1D single mode Langmuir plasma wave model of longitudinal plasma oscillation in the beam, and it defines the regime of beam parameters in which this effect takes place. The suppression of relativistic beam shot noise can be utilized to enhance the coherence of free electron lasers and of any coherent radiation device using an electron beam. © *2010 American Institute of Physics*. [doi:10.1063/1.3388385]

I. INTRODUCTION

The electron beam (e-beam) current shot-noise is the source of optical radiation emission in incoherent radiation sources based on spontaneous radiation emission from free charged particles. These include synchrotron and undulator radiation sources, Cerenkov radiation sources, Smith–Purcell radiation, transition radiation,[1,2] and self amplified spontaneous emission (SASE) free electron laser (FEL).[3,4] This incoherent spontaneous emission radiation is present also in coherent radiation sources, such as FEL oscillators[5] and amplifiers (seed injected FELs[6,7]). In this context, the spontaneous radiation emission of the beam is considered "radiation noise" and similar to conventional laser oscillators (as first pointed out by Schawlow and Townes[8]), it limits the coherence of the laser.

In relativistic accelerated e-beam, the beam noise is normally dominated by current shot-noise.[3] It is, therefore, widely believed that the intensity of SASE FEL radiation is solely determined by the e-beam current shot-noise, and that the coherence of seed-injected FELs will be fundamentally limited by the beam current shot-noise. Therefore, in radiation seeded FELs the intensity of the injected coherent seed radiation should significantly exceed the radiation-equivalent input power of the beam current shot-noise in order to attain coherence.[9] A similar requirement exists in prebunched and high gain harmonic generation FELs.[10]

It is, thus, of both fundamental and application interest, that in a common charged-particle beam, having certain geometrical and beam quality parameters, there is a beam-dynamic collective interaction process of particles self-ordering, that makes it possible to control and suppress the current shot-noise level.[11] This may enable control over the spontaneous radiation emission in free electron optical radiation devices and suppression of its incoherent radiative emission below the classical shot-noise limit level.[12]

The present theory for the effect is based on an extended one-dimensional (1D) (single mode Langmuir plasma wave) small signal linear model.[11] According to this model, due to the granularity of the charge distribution in the beam, longitudinal Coulomb-collective interaction takes place along the transport line of an intense high brightness e-beam in a particular range of beam transport parameters. For short collective interaction lengths, this process is the cause of intensity enhancement and spatial coherence of optical transition radiation (OTR) emission. Such coherent OTR (COTR) was measured in Linac Coherent Light Source (Ref. 13) and other laboratories,[14] and it is also the cause of microbunching instabilities in the dispersive sections of beam transport lines.[15]

The enhancement in COTR radiation power is explained as a two stage process. In the first stage the Coulomb interaction randomly modulates the longitudinal velocity distribution of the e-beam. In a second stage, this longitudinal velocity (energy) modulation (velocity noise) turns into increased current-noise in a dispersive magnet section. If the e-beam passes through a foil, this shot-noise enhancement is expressed by the COTR effect. Contrary to other papers on charge granularity and microbunching dynamics in e-beams, our model applies also to long interaction lengths. In a long interaction length the velocity modulation of the beam caused by the longitudinal space charge field Coulomb forces, is no longer linear as a function of propagation distance. In this process random longitudinal plasma oscillation start taking place. While the longitudinal velocity modulation of the beam (the velocity noise) continues to grow, the beam density non uniformities smear out, and consequently the beam current shot noise is suppressed.

This shot-noise suppression effect has never been yet observed experimentally at optical frequencies. To acquire confidence in the analytical 1D model, it is essential to show that the longitudinal 1D interaction process is not smeared out by three-dimensional (3D) effects. Moreover, it is necessary to define the limits of validity of such a model, and to determine if it can be satisfied, and in what frequency range, with practically attainable beam parameters. Finally, it should be shown that the effect occurs for realizable beam

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parameters with all three spatial components of the Coulomb force taken into account. We address these issues in this paper by using a full 3D particle simulation code [General Particle Tracer—(GPT)].

In Sec. I we briefly present the 1D (or rather—single transverse Langmuir plasma wave mode) model. The subsequent sections verify the predictions of the model numerically within the parameter range of the 1D model and confirm its validity limits.

II. LONGITUDINAL INTERACTION MODEL

In a uniform beam-drift section, the solution of the cold beam linearized plasma fluid equations in the frequency domain results in the following expression for the evolution of the current and kinetic voltage (energy) spectral components of the beam[11]

$$\breve{i}(L_d,\omega)=[\cos\varphi_p\breve{i}(0,\omega) - i(\sin\varphi_p/W_d)\breve{V}(0,\omega)]\exp(i\varphi_b(L_d)\ , \tag{1}$$

$$\breve{V}(L_d,\omega)=[-iW_d\sin\varphi_p\breve{i}(0,\omega) + \cos\varphi_p\breve{V}(0,\omega)]\exp(i\varphi_b(L_d)\ , \tag{2}$$

where L_d is the drift length in the laboratory frame, $\varphi_b = L_d\omega/v_z$ is the plasma wave optical-phase, $\varphi_p=\theta_{pr}L_d$ is the plasma longitudinal oscillation phase, $\theta_{pr}=r_p\omega_{pl}/v_0$ is the plasma longitudinal oscillation wave-number, $\omega_{pl} = (e^2n_0/m\varepsilon_0\gamma^3)^{1/2}$ is the relativistic longitudinal plasma oscillation frequency, $W_d=r_p^2(\mu_0/\varepsilon_0)^{1/2}/k\theta_{pr}A_e$ is the beam modulation impedance, r_p is the plasma reduction factor, $k=\omega/c$ is the optical wave-number, A_e is the e-beam area, and $\breve{V}(z,\omega)$ is the relativistic spectral kinetic voltage (following Chu[16])

$$\breve{V}(\omega)=-(m/e)\gamma_0^3v_0\breve{v}(\omega)=-(mc^2/e)\breve{\gamma}(\omega). \tag{3}$$

This single mode longitudinal interaction model is valid for a beam in the cold beam regime, namely, under the condition that the optical frequency modulation phase is not smeared out due to longitudinal velocity spread: $\Delta\varphi_b = kL_d\Delta(1/\beta_z)\ll\pi$ (where $\Delta\beta_z$ is the axial velocity spread). Also, it is required that higher order Langmuir plasma wave transverse modes are not excited in the beam, as such excitation would negate the single mode interaction process. These requirements set conditions on the beam (slice) energy spread and emittance parameters and on the beam cross-section dimensions.[11]

If these conditions are satisfied for a long enough distance, L_d, so that the beam charge performs quarter plasma oscillation: $\varphi_p=\theta_{pr}L_d=\pi/2$, we attain from Eqs. (1) and (2) that at this distance, full transformation of velocity noise into density noise and vice versa takes place:

$$\overline{|\breve{i}(L_d,\omega)|^2}=\overline{|\breve{V}(0,\omega)|^2}/W_d^2, \tag{4}$$

$$\overline{|\breve{V}(L_d,\omega)|^2}=\overline{|\breve{i}(0,\omega)|^2}W_d^2. \tag{5}$$

TABLE I. Simulation beam parameters.

Energy	100 MeV
Pulse current, duration	80 A, 9 pS
Beam radius	1 mm
Drift length	48 m

If the beam is dominated by current shot noise before entering the collective interaction region (which is usually the case in relativistic high current e-beams), namely: $|\breve{i}(0,\omega)|^2\gg|\breve{V}(0,\omega)|^2/W_d^2$, then, the current noise gain is much smaller than unity:

$$f_i=\frac{\overline{|\breve{i}_{out}(\omega)|^2}}{|\breve{i}(0,\omega)|^2}=\frac{\overline{|\breve{V}(0,\omega)|^2}/W_d^2}{|\breve{i}(L_d,\omega)|^2}\ll 1. \tag{6}$$

III. 3D NUMERICAL SIMULATIONS

The 1D (single mode) model of the noise suppression process needs to be verified within its range of validity by a 3D study. We report here the results of such a study based on full 3D particle simulations. In the 3D simulations, the beam dynamics in the collective interaction region was computed in the rest frame of the e-beam (which moves relatively to the laboratory frame with velocity v_0), by solving the motion equations of all sample particles considering the Coulomb field forces exerted to them by all other particles in a finite dimensions bunch of electrons (long enough to regard the bunch as a caustic beam and ignore coherent edge effects).

The simulations were carried out in the beam rest frame using the GPT code. The starting condition was a uniform random distribution of sample particles in a pencil shaped charge bunch. The positions and velocities $(\boldsymbol{r}',\boldsymbol{v}')$ were calculated for each particle (j) as a function of time (t'). In the post processing these variables were transformed to the laboratory frame $(t,\boldsymbol{r},\boldsymbol{v})$, using Lorentz transformation. They were calculated as a function of the position of the center of the bunch $z=v_0t=v_0\gamma_0t'$. These were used to calculate the laboratory frame current and velocity noise as a function of z

$$|\breve{i}(\omega,z)|^2=(q_eA_e)^2\left|\sum_{j=1}^{N}\exp[i\omega t_j(z)]\right|^2, \tag{7}$$

$$|\delta\breve{v}(\omega)|^2=\frac{1}{(n_0v_0A_e)^2}\left|\sum_{j=1}^{N}(v_j-\bar{v}_{0j})\exp[i\omega t_j(z)]\right|^2, \tag{8}$$

where q_e is the charge of each macroparticle, $\bar{v}_{0j}$ is the average velocity of the electrons within a wavelength range around the jth particle (this local average is used in order to eliminate the effect of the average energy chirp along the bunch length due to space charge). The summation is performed on all the macroparticles within the pulse.

A. Beam parameters and simulation method

The simulations of the drifting beam are based on parameters of the FERMI@Elletra facility (Table I). We assumed a flat top current density distribution beam.

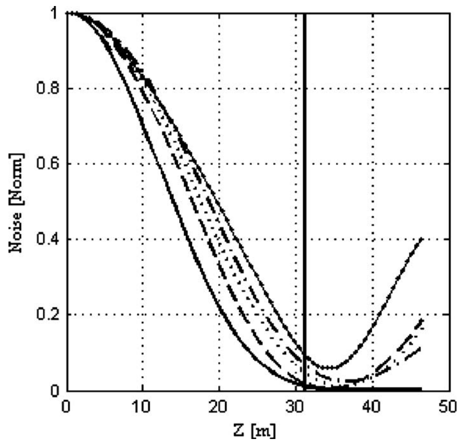

FIG. 1. Current shot noise of 5 μm. Curves obtained from 30 000 macroparticles GPT simulations for five different initial random particle distribution sets. The vertical line represents the theoretical quarter plasma oscillation length (in laboratory frame)-$z=\pi c/2\omega_{pl}$.

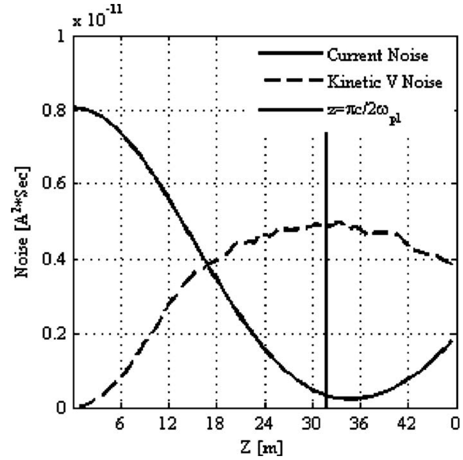

FIG. 2. Spectral current noise and kinetic voltage noise (normalized by the beam impedance) of 5 μm. The curves were obtained from a 30 000 macroparticles GPT simulation. The vertical line represents the theoretical quarter plasma oscillation length (in laboratory frame)—$z=\pi c/2\omega_{pl}$.

The current and velocity spectral noise parameters were calculated from Eqs. (7) and (8) in a frequency (wavelengths) range satisfying the conditions[11]

$$n_0 A_e \lambda \beta_0 \gg 1, \tag{9}$$

$$\lambda \gamma_0 \beta_0 \sim 2r_b. \tag{10}$$

The first is an obvious requirement—having a multitude of particles (in the simulations—macroparticles) per wavelength. The second condition is to assure operation in the regime of single transverse mode Langmuir wave. The simulations were run with GPT for several sets of random starting distributions of 30 000 and 60 000 macroparticles. The shown simulation results correspond to zero initial velocity spread. Simulations with initial axial velocity spread up to $\Delta\beta'=0.002$, corresponding to $\Delta\gamma/\gamma=0.002$ in the laboratory frame, produced similar results. This is in agreement with the cold beam condition (11) $\Delta\gamma/\gamma \ll \beta_0^3\gamma_0^2\lambda/2L_d$.

B. Noise suppression results

The current noise variation as a function of drift distance is shown in Fig. 1 for different random initial distributions. Because of the randomness of the sampled beam shot-noise, the initial shot noise level in each set was different, and, therefore, in order to show the characteristics of the noise evolution, the initial values of the curves are normalized.

Despite the variance between the different random starting particle distribution sets, it is evident that there is noise suppression in all cases. Moreover, it is clear that the noise minima occur at a distance slightly greater, but very close to the calculated quarter plasma oscillation length $\pi c/2\omega_{pl} = 31m$.

We also calculated the velocity noise [Eq. (8)] and the corresponding kinetic voltage noise for one of the sets (Fig. 2). It reached its maximum value at the same place that the current noise reaches its minimum (quarter plasma oscillation time). This too provides good confirmation to the analytical linear single mode theory. The shown simulations were performed with an initially cold beam.

C. 3D effects and the plasma reduction factor

In the parameter range $\lambda\gamma_0\beta_0 > 2r_b$, the single mode model holds, however, due to the finite dimensions of the beam, the plasma wave frequency deviates from the 1D plasma frequency due to the plasma reduction factor $r_p < 1$. The reason for this is the fringing of the microbunching space charge field lines at the periphery of the beam cross-section. This reduces the effective strength of the longitudinal space-charge field and the correspondent longitudinal plasma oscillation frequency. Therefore, in this case, the quarter-plasma oscillation length is longer. Yet, this operating regime is the desirable operating regime, since in the opposite limit (for which $r_p=1$) there is excitation of higher order Langmuir plasma wave modes and the transverse coherence of the bunching breaks down. In this short wavelengths range, it was shown by Venturini,[17] that even at short interaction length, 3D effects wash out any transverse coherence of the bunching.

These theoretical observations are well confirmed by the current noise evolution as a function of transport length, which was calculated at different wavelengths for a particular particles simulation set (Fig. 3). The results confirm that

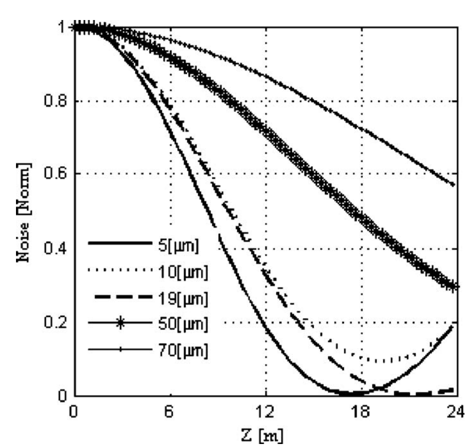

FIG. 3. Current shot noise at different wavelengths—(from 5 to 70 μm). Results obtained from a particular 30 000 macroparticles GPT simulation.

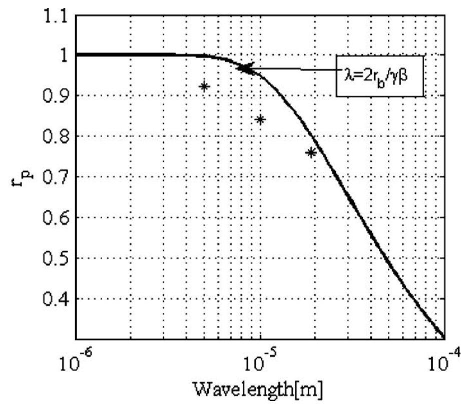

FIG. 4. Plasma reduction factor dependence on wavelength: following Venturini (Ref. 17) (solid curve) and from the minima of the current shot noise time (black dots).

the minimal noise point is proportionally shifted to longer drift distances for longer wavelengths.

Based on Venturini,[17] we estimate the plasma reduction factor of the Langmuir wave fundamental mode from

$$r_p^2 = 1 - (kr_b/\gamma)K_1(kr_b/\gamma). \tag{11}$$

Where $K_1(x)$ is the modified Bessel function and $k=2\pi/\lambda$ is the optical wave-number. This dependence is shown in Fig. 4 by taking the ratio between the 1D quarter plasma oscillation length $z=\pi c/2\omega_{pl}$ and the minimum current-noise drift length for three different wavelengths, we calculated the plasma reduction factor for these wavelengths (marked by black dots in Fig. 4). The calculated points fall quite close to the theoretical curve and confirm its go-down trend.

Another observation we made is that in the short wavelength range $\lambda \ll 2r_b/\gamma_0\beta_0$, 3D effects obscure the longitudinal self-ordering effect, and the simulations did not produce any significant noise reduction results. This observation agrees with the findings of Venturini.[17]

The conclusion of this discussion is that when condition (10) is satisfied, the single mode (1D) model of Gover and Dyunin[11] is valid, and 3D deterioration effects are negligible under conditions (9) and (10). When $\lambda \gg 2r_b/\gamma_0\beta_0$, the model is still valid but the reduction in r_p shifts the maximum noise suppression point to longer lengths (Fig. 3) which may be impractical or forbidden by beam quality limitations.

D. Charge homogenization effect

The spectral current shot-noise suppression effect in the laboratory frame is equivalent to spatial charge homogenization in the e-beam rest frame. This exceptional "self ordering" effect takes place over a wide range of spatial frequencies of the beam density random spatial fluctuations (granularity). This predicted homogenization effect is frequency dependent. At high frequencies it is limited by conditions (9) and (10) [condition (9) is a physical limitation for n_0 which is the particles density in the laboratory frame. It is a validity condition of the simulation procedure when n_0 represents the density of the sample particles]. In order to observe the density homogenization effect, it is desirable to view it through a transmitting filter $(\lambda_1 < \lambda < \lambda_2)$ in a range

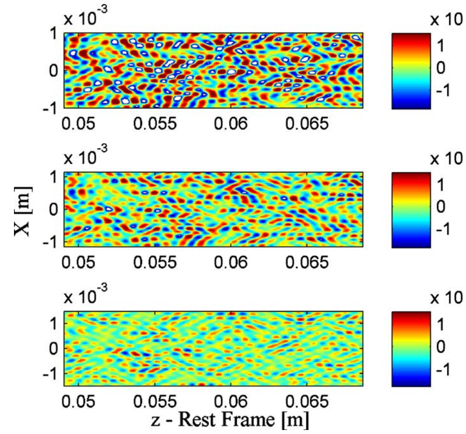

FIG. 5. (Color online) Three beam snap shots of the filtered beam $x'-z'$ plane sections at different transport distances (top to bottom): $z=0$, $z=\pi c/4\omega_{pl}$, $z=\pi c/2\omega_{pl}$.

where (9) and (10) are satisfied. In the beam frame, this corresponds to filtering a spatial frequency range $(k_2' < k' < k_1')$, where $k'=2\pi/\beta\gamma\lambda$.

We have employed a spatial filtering procedure on the beam charge distribution $\rho'(\bar{r}',t')=-e\Sigma_j\delta[\bar{r}'-\bar{r}_j'(t')]$ where $\bar{r}_j'(t')$ is the data obtained using the GPT code using 60 000 macroparticles for the parameters of Table I. A rectangular step-function band-pass filter in k' space was employed for both positive and negative spatial frequencies in the range $|k_1'| < |k'| < |k_2'|$. It was assumed, that an equivalent convolution in z' of the particulate charge function with the point spread function of the filter was employed. In the transverse dimensions the pointlike sample-electrons were smeared by a transverse point spread function of width $n_0'^{-1/3}$ (corresponding to low pass filtering $k_x', k_y' < k_\perp' = 2\pi/n_0'^{-1/3}$). The average density was subtracted:

$$\begin{aligned} n_{\rm fil}'(r',z) = 2\sum_j \{&[k_2' \,\mathrm{sinc}\, k_2'(z'-z_j') - k_1' \,\mathrm{sinc}\, k_1'(z'-z_j')] \\ &\times (k_\perp' \,\mathrm{sinc}\, k_\perp'(x'-x_j')k_\perp' \,\mathrm{sinc}\, k_\perp'(y'-y_j')\ \} \\ &- n_0'(z). \end{aligned} \tag{12}$$

This $x'-z'$ charge distribution was computed for a simulation run of 60 000 macroparticles starting with an initial random distribution of particles with initial axial velocity spread $\Delta\beta'=0.002$. The density distribution was filtered through a band pass filter $5\mu < \lambda < 10\mu$ which corresponds to 1 mm $< \lambda' < 2$ mm in the beam rest frame. This is shown in Fig. 5 at three propagation distances using the same color scale for the three beam "snap-shots." The predicted homogenization effect is clearly depicted.

IV. CONCLUSIONS

We have shown by 3D numerical simulations that it is possible to control and suppress the optical frequency current shot noise in a drift section by choice of proper beam parameters. This was also predicted by the extended 1D analytical model.[11] For the case of a uniform drift section, in a drift length of a quarter plasma oscillation $L_d=\pi/2\theta_{pr}$ the minimal current shot noise is obtained. The plasma reduction factor effect was studied, and the results are in good agreement with the theoretical prediction: a longer drift length is required to obtain minimal shot noise for longer wavelengths. These results provide strong evidence for the validity of the single mode analytical model for current shot noise reduction for e-beam parameter range which we defined.

Shot noise suppression can be utilized to suppress spontaneous emission in any e-beam coherent radiation source. It should be pointed out, that the minimum current shot-noise point is the optimal place to position the radiating component for minimizing radiation noise if the radiation process takes place in a short interaction length (e.g., in OTR from a foil). If the radiative interaction takes place in an extended length (e.g., the wiggler of a high-gain FEL), then the beam plasma oscillation process may continue in this beam transport section and the optimal beam injection point into the wiggler deviates slightly from the quarter plasma oscillation point. This will be analyzed in a separate publication.

We suggest that the beam current shot-noise control and suppression scheme can be used to enhance the coherence of seed injected FELs and relax the requirements on the seed intensity. The simulation results indicate the possibility of coherence enhancement of FELs below the classical shot-noise limit level at optical frequencies.[12,18] Further studies and developments will be required to extend the process to X-UV frequencies, where suppression of the shot-noise is highly desirable for attaining coherent radiation emission in seeded FELs.

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