Suppression of Current Shot Noise and of Spontaneous Radiation Emission in Electron Beams by Collective Coulomb Interaction

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***Abstract*—The effect of current-shot-noise suppression in an electron beam (e-beam) and the corresponding process of charged-particle self-ordering are analyzed using an analytic 1-D model and verified by 3-D numerical simulations. The suppression of current shot noise can be utilized to enhance the coherence of seeded free-electron lasers (FELs) and any other radiation devices using an e-beam. It is shown that this can be attained at optical frequencies with state-of-the-art high-quality e-beams. Our analysis of spontaneous emission suppression results in fundamental theoretical limit expressions for the coherence of FELs and other coherent radiation devices, which is analogous to the Schawlow–Townes limit. After exceeding the shot-noise limit, the coherence of FEL radiation is limited in the IR by velocity noise due to the e-beam energy spread. At UV and shorter wavelengths, it is fundamentally limited by quantum noise.**

***Index Terms*—Bunched particle beams, electron beams (e-beams), free-electron lasers (FELs), particle beam dynamics, shot noise.**

I. Introduction

NOISE MECHANISMS of differing natures set limits on the coherence and the spectral characteristics of radiation sources. Gordon *et al.* [1] and Schawlow and Townes [2] showed that the emission linewidths of masers and lasers are limited by incoherent thermal and spontaneous emission noise at the inputs, respectively. Their limits are used until present days as scales for appreciating linewidth measurements of coherent radiation generators [3], [4].

In the case of vacuum tubes, the noises that affect the coherence of the emitted radiation are the electron-beam (e-beam) noise and the ambient temperature blackbody radiation. The e-beam noise originates from the random oscillations associated with the charge fluctuations in a randomly distributed discrete charged-particle beam (current shot noise) and from velocity fluctuations associated with the energy spread of electron emission from the cathode (velocity noise). Noise in microwave frequency range devices has been intensively investigated in the past [5], [6]. For microwave tubes based on nonrelativistic e-beams, it was shown that the minimal noise achievable level is limited by the cathode temperature [7]–[9].

The progress in the technology of particle accelerators and accelerator-based radiation emission devices, particularly free-electron lasers (FELs), opens the way for the development of high-brightness radiation sources operating with different wavelengths from millimeter wavelengths to X-ray. These capabilities have been demonstrated recently at a wavelength of 1.5 Å in LCLS with brightness higher by eight orders of magnitude relative to any other X-ray source [10].

This result was achieved using the self-amplified spontaneous emission (SASE) scheme. However, SASE sources are characterized by temporal incoherency since they are essentially amplifiers of the current shot noise, which is a source of the spontaneous radiation [11], [12].

In recent years, a number of schemes have been developed to overcome the coherence limitation of FELs due to current shot noise. These include schemes of seed radiation injection, which have been demonstrated at UV wavelengths [13] using high harmonic generation of an intense femtosecond laser beam in a gas. Another seeding scheme is based on prebunching the e-beam by consecutive harmonic generation and high gain amplification (HGHG) in wiggler structures, which has been demonstrated at visible wavelengths [14]. In these schemes, coherence is expected to be achieved if the coherent harmonic signal (of radiation or current modulation) is strong enough to significantly exceed the current-shot-noise power.

In this paper, we present a linear response formulation for the calculation of the evolution of noise parameters in a relativistic e-beam used in the beam transport sections (dispersive section, free drift section, and accelerator transport line) preceding the wiggler. The linear model is also employed to describe the subsequent generation of coherent and incoherent radiations in the wiggler [15]–[18]. Using the combined analysis of the accelerator transport line and the wiggler, we present a scheme for the suppression of SASE radiation based on controlling the current noise of the e-beam at the entrance to the wiggler. The scheme is shown schematically in Fig. 1 and is composed of a wiggler and a preceding e-beam drift section. We present a formulation for calculating the development of the e-beam noise and the spontaneous radiation emission in the combined system. The formulation results in the conditions for suppression of the incoherent radiation power and the expressions for the ultimate coherence limits achievable in FEL.

Manuscript received September 21, 2010; revised January 4, 2011 and June 13, 2011; accepted July 14, 2011. Date of current version October 12, 2011. This work was supported in part by the Israel Science Foundation under Grant 353/09.

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Digital Object Identifier 10.1109/TPS.2011.2162853

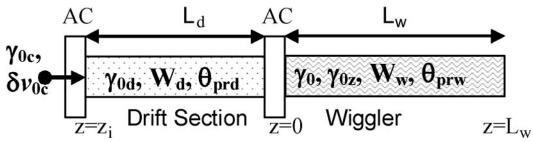

Fig. 1. FEL system composed of a free drift e-beam section, (AC) acceleration sections, and a wiggler section.

II. Single Transverse Mode Linear Response Formulation

The schemes for reduction of e-beam noise below the current-shot-noise level are based on "smoothing" the e-beam current fluctuations by means of space-charge force. This occurs because an e-beam plasma oscillation starts taking place during the transit time along the e-beam transport line [18].

Using small-signal formalism, we can express all parameters as a sum of two terms: a time-averaged part and a time-varying part whose amplitude is much smaller than the time-averaged one. The linear analysis may be now specified to the single-frequency ω case by using a phasor relation. Each variable X is presented as a function of space and time

$$X(\mathbf{r},t) = X_0(\mathbf{r}) + \mathrm{Re}\left(\widetilde{X}(\mathbf{r},\omega)\exp(-i\omega t)\right). \quad (1)$$

For the analysis of the e-beam modulation and noise dynamics, we introduce here a relativistic extension of Chu's kinetic voltage parameter [6], [19], [20]

$$\widetilde{V} = -\widetilde{v}_z \frac{m}{e}\frac{d\gamma_0(z)}{dv_{z0}} = -\frac{m}{e}\gamma_0\gamma_{z0}^2 v_{z0}\widetilde{v}_z = -\left(\frac{mc^2}{e}\right)\widetilde{\gamma}(z) \quad (2)$$

where e and m are the electron charge and its mass, respectively, c is the speed of light, v_{z0} is the axial e-beam velocity, γ_0 is the beam relativistic Lorentz factor, and γ_{z0} is the axial (average) Lorentz factor. This expansion near the average beam parameters is valid also for sections of axial acceleration and sections with transverse magnetic field $(B_\perp)$

$$\gamma_{z0}^2(z) = \gamma_0^2(z)/\left(1+a_\perp^2(z)\right),\ a_\perp(z) = -(e/mc)\int_0^z B_\perp(z')\,dz'.$$

The radiation mode amplitude, the beam current, and the kinetic voltage ($\widetilde{C}_q(z)$, $\widetilde{i}(z)$, $\widetilde{V}(z)$, respectively) as a function of the axial coordinate in the wiggler (z) can be expressed in terms of a general frequency transfer matrix

$$\begin{pmatrix}\widetilde{C}_q(z)\\ \widetilde{i}(z)\\ \widetilde{V}(z)\end{pmatrix} = \underline{\underline{\widetilde{\mathbf{H}}}}_{\mathrm{FEL}}\begin{pmatrix}\widetilde{C}_q(0)\\ \widetilde{i}(0)\\ \widetilde{V}(0)\end{pmatrix} = \begin{pmatrix}H^{EE} & H^{Ei} & H^{EV}\\ H^{iE} & H^{ii} & H^{iV}\\ H^{VE} & H^{Vi} & H^{VV}\end{pmatrix}\begin{pmatrix}\widetilde{C}_q(0)\\ \widetilde{i}(0)\\ \widetilde{V}(0)\end{pmatrix}. \quad (3)$$

The explicit expressions for the components of the transfer matrix of a uniform wiggler section are defined in all linear gain regimes in [16] and [18].

III. Accelerator and Beam Transport Section

If the transport section is composed only of fast-acceleration (in which collective microdynamics is neglected) and drift sections of length L_d, the 3×3 transfer matrix from the cathode to the FEL wiggler may be modeled by [17]

$$\underline{\underline{\widetilde{\mathbf{H}}}}_T = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos\phi_p & -i\sin\phi_p/W_d\\ 0 & -iW_d\sin\phi_p & \cos\phi_p\end{pmatrix}\exp(i\varphi_b) \quad (4)$$

where $\phi_p = \theta_{pr}L_d$ is the phase shift of the plasma wave on the e-beam, $W_d = r_p^2/\varepsilon_0\omega\theta_{pr}A_e$ is the beam wave impedance, $\varphi_b = \omega L_d/v_{0z}$ is the optical phase, $\theta_{pr}^2 = r_p^2 eI_b/\varepsilon_0 m\gamma_0\gamma_{z0}^2 v_{z0}^3 A_e$ is the longitudinal plasma wavenumber [15], $r_p \leq 1$ is the finite width beam plasma reduction factor, I_b is the beam current, ε_0 is the free-space permittivity, and A_e is the beam cross-sectional area.

In previous analysis of noise dynamics in a drifting e-beam [17], we showed, based on (4), that at a drift distance

$$L_d = \pi/2\theta_{pr} \quad (5)$$

the initial beam kinetic voltage noise turns into current noise and vice versa, and then

$$\overline{\left|\breve{i}(L_d,\omega)\right|^2} = \overline{\left|\breve{V}(z_i,\omega)\right|^2}/W_d^2 = N^2\overline{\left|\breve{i}(z_i,\omega)\right|^2} \quad (6)$$

where $\breve{i}$ and $\breve{V}$ are the spectra (Fourier transform) of the current noise and kinetic voltage noise, respectively, z_i is the initial drift start point, and N is the kinetic-voltage-noise-to-current-noise initial ratio factor

$$N^2 \equiv \overline{\left|\breve{V}(z_i,\omega)\right|^2}\Big/\overline{\left|\breve{i}(z_i,\omega)\right|^2}W_d^2. \quad (7)$$

Usually, the noise of high-quality relativistic e-beams is dominated by current shot noise, namely, $N^2 \ll 1$.

Under this condition, transporting the beam through a quarter plasma oscillation length (5) reduces the current shot noise by a factor N^2 (7), and since the SASE radiation power in FEL is believed to be dominated by the input current shot noise, this would enable suppression of SASE radiation power. However, since there is continued noise evolution dynamics also within the wiggler, it is necessary to solve for the noise dynamics evolution in the combined system of the beam transport section and the wiggler in order to get more accurate expressions for the radiation-noise suppression and the optimal drift length for SASE output power minimization.

IV. 3-D Simulation of Current Noise Suppression

To verify the noise suppression predicted in the 1-D analytical theory, we performed numerical simulations. These simulations are based on full 3-D particle-to-particle Coulomb interactions in the collective interaction region. Particle locations were evaluated in both the laboratory frame of reference and the moving frame, which moves relatively to the laboratory frame with velocity v_{0z}, by solving the motion equations of all sample particles under the Coulomb field forces applied

TABLE I
LIST OF PARAMETERS FOR 3-D PARTICLE MICRODYNAMICS SIMULATION IN A DRIFTING BEAM

Energy	75[MeV]
Pulse Charge, duration	0.5[nC], 5[ps]
Beam Waist	0.5 [mm]
Normalized Emittance	2[μm]
Δγ/γ	0.05[%]
L_d	9.5[m]

on them by all other particles in a finite-dimension bunch of electrons (long enough to regard the bunch as a caustic beam and ignore coherent edge effects).

The simulations were carried out using the General Particle Tracer (GPT) code starting from a random spatial distribution of sample particles (shot noise) in a pencil-shaped charge bunch. The different cases of uniform and Gaussian transverse densities were separately examined. The positions and velocities in the moving frame $(\boldsymbol{r}', \boldsymbol{v}')$ were calculated for each particle (j) as a function of the time (t'). These variables were then transformed to the laboratory frame $(t, \boldsymbol{r}, \boldsymbol{v})$ using Lorentz transformation and calculated as a function of the position of the center of the bunch $z = v_0 t = v_0 \gamma_0 t'$. These were used to calculate the laboratory-frame current noise as a function of z

$$\left|\breve{i}(\omega, z)\right|^2 = \frac{q_e^2}{T}\left|\sum_{j=1}^{N} \exp\left[i\omega t_j(z)\right]\right|^2 \tag{8}$$

where q_e is the charge of one sample particle and T is the bunch duration. The summation is performed on all the macroparticles within the pulse. These 3-D simulation results (8) were compared to the 1-D analytical model (6) to see if the minima of the current noise corresponded to the estimated quarter plasma oscillation longitudinal distance (5).

The simulations of beam drift presented in this paper are based on the parameters of the ATF (Table I) of Brookhaven National Laboratory. The simulations were carried out using 250 000 sample particles in order to allow sufficient number of particles per modulation wavelength. In these simulations, we assumed a Gaussian current density distribution beam.

The current noise variation as a function of drift distance is shown in Fig. 2. Noise was evaluated in the wavelength region of 1–2 μm. It is clear that the noise minima occur at a distance slightly greater than but very close to the calculated quarter plasma oscillation length $L_d = 8.5$ m.

Fig. 3 shows the sample particle noise (8) as a function of wavelength in the range of 3–15 μm. The classical expression level of the sample particle current shot noise $(q_e I_b)$ is marked as a black line for comparison. An appreciable reduction of the noise in all wavelengths is observable.

V. COHERENT AND INCOHERENT RADIATIONS IN THE COMBINED E-BEAM TRANSPORT AND FEL SECTIONS

Consider now a general FEL structure that consists of a nonradiative section (acceleration and drift sections) and a wiggler (Fig. 1). Based on the solution of the FEL linear response problem (3) [16], the expressions for the FEL emission output

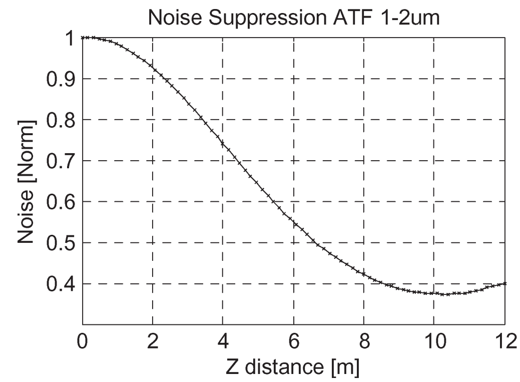

Fig. 2. ATF simulation results: Current noise reduction for 1–2 μm for a Gaussian beam.

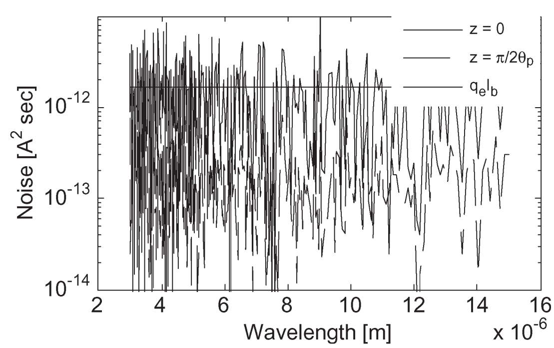

Fig. 3. Noise as a function of wavelength in a logarithmic scale according to the ATF beam parameters. The constant line represents the classical current shot noise for comparison.

powers in the respective schemes of *coherent radiation* seed injection and *coherent beam prebunching* are

$$[P_s(L_w)]_{\mathrm{coh}} = P_q\left|\widetilde{C}_q(L_w)\right|^2 = P_q\left|\widetilde{H}^{EE}_{\mathrm{FEL}}\right|^2\left|\widetilde{C}_q(0)\right|^2 \tag{9}$$

$$[P_s(L_w)]_{\mathrm{prebunch}} = P_q\left|\widetilde{H}^{Ei}_{\mathrm{FEL}}(\omega)\widetilde{i}(0) + \widetilde{H}^{Ev}_{\mathrm{FEL}}(\omega)\widetilde{V}(0)\right|^2 \tag{10}$$

where $z = 0$ is the wiggler entrance point, L_w is its length, and P_q is the mode normalization power.

For the *incoherent* radiation power calculation, we need to keep a transfer matrix that includes both the FEL and drift ($\underline{\underline{\widetilde{\mathbf{H}}}}_{\mathrm{FEL}}$ and $\underline{\underline{\widetilde{\mathbf{H}}}}_{T}$, respectively) sections (starting from the "cathode" position $z = z_c$ or, more correctly, from the drift section entrance point): $\underline{\underline{\widetilde{\mathbf{H}}}}_{\mathrm{TOT}} = \underline{\underline{\widetilde{\mathbf{H}}}}_{\mathrm{FEL}}\underline{\underline{\widetilde{\mathbf{H}}}}_{T}$. The total incoherent spectral power at the FEL output is then

$$\begin{aligned}
&\left(\frac{dP(L_w, \omega)}{d\omega}\right)_{\mathrm{incoh}} \\
&\quad = \frac{2P_q}{\pi}\left\{\left|\widetilde{H}^{EE}_{\mathrm{FEL}}\right|^2 \overline{\left|\breve{C}(0,\omega)\right|^2} + \left|\widetilde{H}^{Ei}_{\mathrm{TOT}}(\omega)\right|^2 \overline{\left|\breve{i}(z_c,\omega)\right|^2}\right. \\
&\qquad + \left|\widetilde{H}^{EV}_{\mathrm{TOT}}(\omega)\right|^2 \overline{\left|\breve{V}(z_c,\omega)\right|^2} \\
&\qquad + 2\mathrm{Re}\left(\widetilde{H}^{Ei}_{\mathrm{TOT}}\widetilde{H}^{EV*}_{\mathrm{TOT}}\right)\mathrm{Re}\overline{\left(\breve{i}\breve{V}^*_c\right)} \\
&\qquad \left. -2\mathrm{Im}\left(\widetilde{H}^{Ei}_{\mathrm{TOT}}\widetilde{H}^{EV*}_{\mathrm{TOT}}\right)\mathrm{Im}\overline{\left(\breve{i}\breve{V}^*_c\right)}\right\}.
\end{aligned} \tag{11}$$

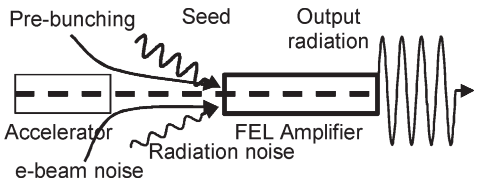

Fig. 4. Coherent and incoherent radiations and beam modulation input signals at the FEL amplifier input. The amplified signal output power is partially coherent.

At the entrance to the drift section, we assume that various dissipative processes that increase the e-beam energy spread and emittance render the kinetic voltage noise (namely, the velocity noise) to be *uncorrelated* with the current shot noise [15]. At this no-correlation point, single particle analysis produces the following expressions for the beam noise parameters (the current noise, the kinetic voltage noise, and the kinetic power noise, respectively):

$$\overline{|\breve{i}|^2} = eI_b \tag{12}$$

$$\overline{|\breve{V}|^2} = \left(mc^2\delta\gamma\right)^2/eI_b \tag{13}$$

$$\overline{\breve{i}\breve{V}^*} = mc^2\delta\gamma\delta\beta_{zc}/\beta_{zc}. \tag{14}$$

Here, $\delta_\gamma = \gamma_{z0c}^2\gamma_{0c}\beta_{z0c}\delta\beta_{zc}$, where $\beta_{z0c} = \langle\beta_{zj}\rangle_j$ and $(\delta\beta_{zc})^2 = \langle(\beta_{zj} - \beta_{z0c})^2\rangle_j$ are the averages over the e-beam axial velocity β_{z0j} distribution at the start point ($\delta\beta_{zc} < \beta_{z0c}$). The beam energy spread is fundamentally limited by the cathode temperature (for thermionic cathode): $\delta\gamma = \delta\gamma_c = k_BT_c/mc^2$, where T_c is the cathode temperature and k_B is the Boltzmann constant. However, in practice, after transport through the e-gun and accelerator sections, the energy spread is increased or even intently heated [21]. The effective energy spread $\delta\gamma$ as measured within a slice (a slice is defined as a narrow axial section along the e-beam) after acceleration is hard to measure but is at least three orders of magnitude larger than the cathode temperature energy spread.

To deal further with the incoherent FEL radiation, it is convenient to define an incoherent (noise) equivalent radiation input power [noise equivalent power (NEP)], which lumps all incoherent input signal sources and is composed of the e-beam noise contributions (of current, kinetic voltage, and kinetic power) and the "radiation noise," composed of quantum spontaneous emission and blackbody radiation at the FEL entrance (see Fig. 4)

$$(dP_{\rm in}/d\omega)^{eq} = (dP(L_w)/d\omega)_{\rm incoh} \Big/ \left|\widetilde{H}_{\rm FEL}^{EE}\right|^2. \tag{15}$$

VI. Conventional FEL Theory

In conventional FEL theory, it is customary to assume $\underline{\underline{\widetilde{\mathbf{H}}}}_{\rm TOT} = \underline{\underline{\widetilde{\mathbf{H}}}}_{\rm FEL}$ (no plasma oscillation dynamics in the beam transport sections preceding the FEL). Substituting then (12)–(14) into (11) and (15), one obtains that the e-beam NEP is composed of three contributions (see Fig. 4): current shot noise ($(dP_{\rm in}/d\omega)_{\rm conv}^i$), kinetic voltage noise ($(dP_{\rm in}/d\omega)_{\rm conv}^V$), and kinetic power noise ($(dP_{\rm in}/d\omega)_{\rm conv}^{iV}$)

$$(dP_{\rm in}/d\omega)_{\rm conv}^i = \frac{eI_b}{16\varepsilon_0 c\pi A_{\rm em}}\left(\frac{a_w}{\gamma_0\beta_{z0}\Gamma}\right)^2 \tag{16}$$

$$(dP_{\rm in}/d\omega)_{\rm conv}^V = \left(\frac{mc^2\delta\gamma}{eI_b}\right)^2\left(\frac{\theta_{prw}}{W_w\Gamma}\right)^2\left(\frac{dP_{\rm in}}{d\omega}\right)_{\rm conv}^i \tag{17}$$

$$(dP_{\rm in}/d\omega)_{\rm conv}^{iV} = \frac{2}{\pi}\sqrt{3}mc^2\delta\gamma\delta\beta_{zc}/\beta_{zc} \tag{18}$$

where $A_{\rm em}$ is the electromagnetic mode area, Γ is the FEL gain coefficient, a_w is the wiggler parameter, and W_w and θ_{prw} are the beam wave impedance and the plasma reduction factor in the wiggler [19].

In addition, there may be an equivalent input radiation noise due to the radiation noise [22]

$$(dP/d\omega)_{\rm in}^E = \hbar\omega/(1 - e^{-\hbar\omega/k_BT}) \tag{19}$$

where $\hbar$ is the Planck constant.

FELs operate, in practice, only in the cold beam regime, when the axial velocity spread in the FEL is small [16] ($\delta\beta_{z0}/\beta_{z0} \ll \Gamma c/\omega$). Therefore, kinetic voltage noise (17) and kinetic power noise (18) are negligible relative to current shot noise (16). This justifies in retrospect the nonobvious common neglect of kinetic voltage noise in conventional SASE-FEL theory.

Thus, under the assumptions of conventional FEL 1-D linear theory, neglecting all noise contributions except current shot noise, the FEL coherence condition for seed radiation and prebunching schemes is simplified to

$$[P_s(0)]_{\rm coh} \gg \frac{eI_b}{16\varepsilon_0 c\pi A_{\rm em}}\left(\frac{a_w}{\gamma_0\beta_{z0}\Gamma}\right)^2\Delta\omega \tag{20}$$

$$\left|\widetilde{i}_s(0)\right|^2 \gg eI_b\Delta\omega. \tag{21}$$

Here, $\Delta\omega$ is the frequency bandwidth of the incoherent power [16]. If filtering is employed, then $\Delta\omega$ is the filter bandwidth. In a pulse of duration t_p, the bandwidth is Fourier transform limited: $\Delta\omega \approx \pi/t_p$.

VII. SASE Power Control and Suppression

We now show that, by proper control of the e-beam collective dynamics in the transport line, it may be possible to reduce the current noise below the kinetic voltage noise so that the incoherent power of the FEL would not be limited by current shot noise but by the slice energy spread of the beam.

Assume that there is a nonradiative section between the starting point (noncorrelation) and the wiggler entrance that contains a fast-acceleration section to any middle energy level (γ_{0d}), a long enough drift section, and a second fast-acceleration stage, which accelerates the beam to the energy level (γ_0) required for the wiggler radiation (Fig. 1). In this case, the equivalent input power derived from (15) and (11) is modified due to the dynamics of energy transfer and correlation between the beam dynamic parameters ($\breve{i}$ and $\breve{V}$) in the drift section expressed by (4). After some derivation steps [19], the modified current, kinetic voltage, and kinetic power NEP expressions can be written in terms of the corresponding conventional (no drift section) equivalent radiation input noise power (16)–(18). It is found that these NEP parameters are modified by factors that depend only on a "radiation suppression parameter" S, which we define as

$$S = W_d\theta_{prw}/W_w\Gamma \approx \left(\gamma_{0d}^3\beta_{0d}^3/\gamma_0\gamma_{0z}^2\beta_{0z}^3\right)^{\frac{1}{2}}(\theta_{prw}/\Gamma) \quad (22)$$

and the phase shift of the plasma wave on the e-beam ϕ_p.

It turns out (because of the continued interaction in the wiggler) that maximum radiation power suppression does not take place exactly at the condition of maximum current noise suppression (5) but at a slightly smaller phase shift $(S \ll 1)$

$$\phi_p = \pi/2 - \sqrt{3}S/2. \quad (23)$$

At this point, one needs to examine the contribution of the other noise sources to the total NEP of the FEL. The next noise source of importance is the kinetic voltage noise, and since this noise grows in the drift section when the current noise diminishes, it turns out that the contribution to the NEP due to kinetic voltage noise is bigger relative to the conventional case expression (17) by a factor $(1/S)^2 \gg 1$. Also, in this case, the total NERP is suppressed relative to the basic case (no drift section), but the new value of the total NEP of the FEL depends on the ratio between the two parameters N and S, both $ll1$.

In the case when the noise factor is smaller than the noise suppression factor $(2N \ll S)$, the current noise contribution to the NEP is still dominant and given by

$$\left(\frac{dP}{d\omega}\right)^{eq} = \left(\frac{S}{2}\right)^2 (dP_{\rm in}/d\omega)^i_{\rm conv} = \frac{2}{\pi}\frac{\theta_{prw}}{\Gamma}\frac{W_d^2}{W_w}(eI_b). \quad (24)$$

In the opposite case $(S \ll N)$, the kinetic voltage noise contribution to the NEP becomes dominant, given by

$$\left(\frac{dP}{d\omega}\right)^{eq} = \left(\frac{1}{S}\right)^2 (dP_{\rm in}/d\omega)^V_{\rm conv} = \frac{2}{\pi}\frac{\Gamma}{\theta_{prw}}\frac{W_w}{W_d^2}\frac{(mc^2\delta\gamma)^2}{eI_b}. \quad (25)$$

In this case, the NERP of the FEL depends on the beam energy spread (note though that the FEL still operates in the cold beam gain regime).

The FEL coherence condition for the seeding power (20) is thus relaxed in either case by a factor $(S/2)^2 \ll 1$ or $(N/S)^2 \ll 1$ corresponding to (24) and (25). It remains to be seen up to what high frequencies the noise suppression scheme can be exploited. Preliminary estimates, based on presently available beam quality parameters, suggest that noise can be suppressed up to the UV spectral range with present state-of-the-art technology.

It is still of fundamental physics interest to have an expression for the ultimate NEP of FEL, after the current noise is suppressed and assuming that lower beam energy spread $\delta\gamma$ can be attained. In this limit, one needs to consider also the radiation-noise contribution (19). At long wavelengths down to FIR, this term becomes $(dP_{\rm in}/d\omega)^E = k_BT$. At high frequencies of X-UV, it becomes $\hbar\omega$ and may exceed the beam energy spread. The fundamental limit of FEL coherence is then the quantum noise limit

$$(dP_{\rm in}/d\omega)^{eq} = \hbar\omega. \quad (26)$$

This limit may be considered the equivalent of the Schawlow–Townes limit for atomic laser oscillators [1], [2]. We note, in conclusion, that, in *prebunching* schemes like HGHG, the noise suppression scheme may be still effective even for very short wavelength lasing, since the main contribution to noise in this scheme originates from HGHG of the current shot noise at the *fundamental harmonic frequency*, where current noise suppression is presently plausible.

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