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## Collective microdynamics and noise suppression in dispersive electron beam transport

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A general formulation is presented for deep collective interaction micro-dynamics in dispersive e-beam transport. In the regime of transversely coherent interaction, the formulation is applicable to both coherent and random temporal modulation of the electron beam. We demonstrate its use for determining the conditions for suppressing beam current noise below the classical shot-noise level by means of transport through a dispersive section with a small momentum compaction parameter.

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### INTRODUCTION

Recent developments in electron beam accelerator technology, and particularly the development of RF photocathode gun technology, make it possible to have high current high quality (low emittance and energy-spread) e-beams. Such beams are needed for various applications and particularly for driving free electron lasers (FELs).<sup>1</sup> As a result of these developments, new effects of collective Coulomb interaction microdynamics have been observed in accelerator and beam-transport systems. The description of these collective effects requires some revisions in the conventional single-electron accelerator physics formulations used in analyzing RF-LINAC beams.

Unexpected collective microdynamics effects were found to be responsible for electron beam microbunching instability in dispersive elements.<sup>2</sup> Such an effect of energy modulation microbunching due to longitudinal space charge (LSC) and consequent growth of current microbunching in dispersive sections was analyzed in the limit of small collective interactions (linear current-voltage response) in terms of an LSC impedance model.<sup>3,4</sup>

Unexpected optical frequency phase correlation effects in electron beams have been observed by means of optical transition radiation (OTR) diagnostics.<sup>5,6</sup> In particular, an effect of coherent optical transition radiation (COTR) referred to as “unexplained physics” was observed in the injector of SLAC’s LCLS x-ray FEL accelerator.<sup>6</sup> This effect is now understood to be the result of a collective microdynamics process of random e-beam energy bunching in the waist of a high current electron beam, which turned into enhanced coherent current (density) bunching after passage through a dispersive magnetic bend.<sup>7,8</sup>

Most recently, Musumeci *et al.*<sup>9</sup> have demonstrated that significant collective interaction effects (over half plasma wave oscillations) can be attained by coherent THz frequency modulation of RF-LINAC accelerated beams and even be driven into the nonlinear interaction regime.<sup>10</sup>

The conventional theory of charged particle beam transport is based on a linear transformation of the kinetic parameters of the individual particles in electron-optical systems.

The particle parameters are described in a 6 dimensional phase space<sup>11</sup> ( $\delta x_j$ ,  $\delta x'_j$ ,  $\delta y_j$ ,  $\delta y'_j$ ,  $c\delta\tau_j$ , and  $\delta p_j/p_0$ ), and the transformation of each particle  $j$ , independent of others, is characterized by a matrix  $R_{mn}$   $x_m^{(j)} = \sum_{n=1}^6 R_{mn} x_n^{(j)}$ . However, this formulation cannot describe collective interaction. To describe such effects, one needs to solve the 6D kinetic plasma equations, including Poisson equation,<sup>12</sup> or, if the beam quality parameters (emittance -  $\varepsilon$  and energy spread -  $\delta\gamma_j/\gamma$ ) are small, the 3D fluid plasma equations suffice.

In this paper, we show that the transverse coherence feature of collective microdynamic processes in dispersive e-beam transport, such as were observed in Ref. 6, can be described in terms of two small-signal parameters of the beam—the current modulation ( $I_1(z, t)$ ) and the energy modulation ( $\delta\gamma(z, t)$ )

$$I(z, t) = I_0 + I_1(z, t), \quad (1)$$

$$V(z, t) = V_0(z) + V_1(z, t) = -\frac{mc^2}{e} [\gamma_0(z) - 1 + \delta\gamma(z, t)], \quad (2)$$

or in terms of their Fourier transforms  $\tilde{I}_1(z, \omega) = \int_{-\infty}^{\infty} I_1(z, t) e^{-i\omega t} dt$ ,  $\tilde{V}_1(z, \omega) = \int_{-\infty}^{\infty} V_1(z, t) e^{-i\omega t} dt$ , where the parameter  $V$ , Chu’s kinetic voltage<sup>13</sup> that we extended to have a relativistic definition,<sup>8</sup> is the time dependent beam energy normalized to voltage dimensions. This formulation may be employed to both temporally coherent and random modulations of the e-beam current and energy. Furthermore, it is valid for beam transport in both free drift and dispersive transport sections.

To apply this model, one needs to assume propagation in an ideal dispersive transport section, namely one in which the predominant effect of the transport is the differential time delay of particles of different energies, while their other phase space parameters remain the same ( $R_{ij} \cong \delta_{ij}$  except  $R_{56} \neq 0$ ). Examples for such dispersive sections are a chicane<sup>11</sup> (see Fig. 1), a balanced chromate bend,<sup>6</sup> and simple free space transport<sup>8</sup> (in this latter case, the dispersion is small).

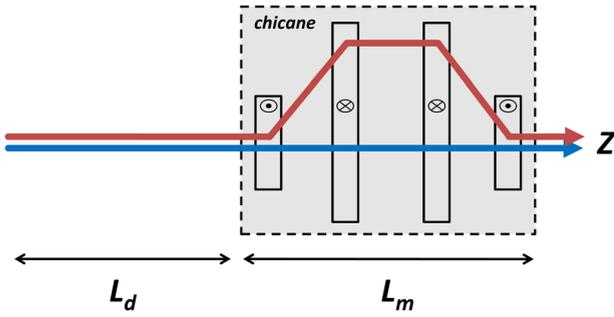


FIG. 1. (Color online) Schematics of beam transport through a drift section, followed by an ideal dispersive section (chicane).

The predominant effect of dispersive sections is assumed to be conversion of energy modulation into current (density) modulation. Any inverse process due to microdynamic collective (plasma oscillation) interaction is ignored in conventional theory. Consequently, when electron beam noise dynamics is considered in general e-beam transport, it is widely believed that the beam current noise (random current modulation) is limited by the classical shot-noise  $|\tilde{i}(\omega)|^2 = 2eI_0$  and that this shot-noise level, determined upon emission at the cathode, remains constant in general transport and may only be enhanced in dispersive transport due to transformation of energy noise into current noise. The possibility that optical frequency beam current noise can be suppressed below the shot-noise level in such transport systems, as proposed in Refs. 8 and 14, is considered controversial. The main reason for doubt is lack of experimental observation of such a noise suppression effect.

In this paper, we present a general formulation for linear regime analysis of collective microdynamics in dispersive transport. The formulation is valid in the deep collective regime (longitudinal plasma oscillation of coherent or random microbunching of the beam) as long as the beam quality parameters are good enough (small emittance and energy spread), and the beam transverse dimensions are small enough to enable transversely coherent longitudinal collective interaction throughout the total length. With these assumptions we then employ the formulation to obtain explicit expressions for shot-noise suppression in a particular case of a short section of free-drift beam transport followed by a dispersive section.

## COLLECTIVE MICRODYNAMICS IN GENERAL BEAM TRANSPORT

The beam parameter range, in which transversely coherent optical frequency microdynamic effects can take place, is<sup>8</sup>

$$\beta_0 \gamma \lambda > 2r_b, \quad (3)$$

where  $r_b$  is the beam radius. Only in this range, the transverse extent of the longitudinal Coulomb field of frequency  $\omega = 2\pi c/\lambda$  of each electron is wider than the beam diameter, and thus all electrons in the beam cross-section can apply in-phase axial force on all neighboring electrons in the beam. In this case, 3D collective effects can be neglected,<sup>15</sup> and the two parameters formulations of Eqs. (1) and (2) can be employed. Additional conditions for the validity of this

model (and occurrence of transversely coherent microdynamics) need to be satisfied by the beam quality parameters (energy spread and emittance).<sup>8</sup>

Using the same formulation as in Refs. 8, 16, and 17, the cold-beam plasma fluid equations reduce into equations for the spectral current and kinetic voltage modulation parameters,  $(\tilde{I}_1, \tilde{V}_1)$ , that describe the evolution of the longitudinal plasma waves on the electron beam (Langmuir modes). We define the beam plasma phase

$$\phi_p(z) = \int_0^z \theta_{pr}(z') dz', \quad (4)$$

where  $\theta_{pr}(z) = r_p \theta_p(z)$  is the wavenumber of the Langmuir plasma-wave fundamental transverse mode,  $\theta_p$  is the longitudinal 1D plasma wavenumber

$$\theta_p^2(z) = \frac{\omega_{p0}^2}{\gamma_0 \gamma_{z0}^2(z) v_{z0}^2(z)} = \frac{eI_0 \sqrt{\mu_0/\epsilon_0}}{mc^2 A_e(z) \gamma_0 \beta_0 \gamma_{z0}^2(z) \beta_{z0}^2(z)}, \quad (5)$$

where  $v_{z0}(z) = c\beta_{z0}(z)$  is the axial velocity of the beam,  $\gamma_{z0}(z) = (1 - \beta_{z0}^2(z))^{-1/2}$ ,  $A_e(z)$  is the beam cross-section area, and  $r_p(z) \leq 1$  is the plasma reduction factor due to the finite cross section of the beam plasma.<sup>15,18,19</sup> For the fundamental Langmuir plasma-wave mode of a transverse uniform beam distribution,  $r_p^2 = 1 - (kr_b/\gamma)K_1(kr_b/\gamma)$ .

Assuming slow variation of the beam transport parameters along the propagation axis  $z$ , we define slow varying small signal amplitudes of the spectral current and kinetic voltage,  $\tilde{i}(z, \omega) = \tilde{I}_1(z, \omega) \exp[-i\omega \int_0^z dz'/v_{z0}(z')]$ ,  $\tilde{v}(z, \omega) = \tilde{V}_1(z, \omega) \exp[-i\omega \int_0^z dz'/v_{z0}(z')]$ . The equations for the slow varying parameters are

$$\frac{d}{d\phi_p} \tilde{i} = -\frac{i}{W(z)} \tilde{v}, \quad (6)$$

$$\frac{d}{d\phi_p} \tilde{v} = -iW(z) \tilde{i}, \quad (7)$$

where

$$W(z) = \sqrt{\mu_0/\epsilon_0} r_p^2 / (kA_e \theta_{pr}(z)) \quad (8)$$

is the plasma-wave beam-impedance and is related to the parameter of beam impedance per unit length  $Z_{LSC}$  used in Refs. 3 and 4 and others, by the relation  $W = -iZ_{LSC}/\theta_{pr}$ . This is the essential parameter transformation needed to extend the earlier ‘‘lumped circuit’’ model for collective interaction to the ‘‘transmission line’’ model of the present paper and to employ it to describe collective interaction in extended dispersive transport and in general, when  $W = W(z)$ .

In the case of free drift uniform beam-transport,  $\theta_{pr}(z) = \theta_{prd}$  and  $W(z) = W_d$  are constants, and the integration of Eqs. (6) and (7) is straightforward<sup>8</sup>

$$\begin{pmatrix} \tilde{i}(L_d, \omega) \\ \tilde{v}(L_d, \omega) \end{pmatrix} = \begin{pmatrix} \cos\phi_{prd} & -\frac{i}{W_d} \sin\phi_{prd} \\ -iW_d \sin\phi_{prd} & \cos\phi_{prd} \end{pmatrix} \begin{pmatrix} \tilde{i}(0, \omega) \\ \tilde{v}(0, \omega) \end{pmatrix}, \quad (9)$$

where  $\phi_{prd} = \theta_{prd} L_d$ .

In the general case when  $\theta_{pr}(z)$  and  $W(z)$  are  $z$  dependent and known, Eqs. (6) and (7) can be generally solved in principle in terms of the initial conditions  $\vec{i}(0, \omega)$ ,  $\vec{v}(0, \omega)$ . In this paper, we demonstrate the use of this general linear formulation by using Eq. (9) in a free drift section and by employing an iterative integration procedure on Eqs. (6) and (7) in the dispersive section in which  $W = W(z)$ . This is valid if the collective interaction effect is small in this section. The result of the first order iteration is

$$\begin{pmatrix} \vec{i}(L, \omega) \\ \vec{v}(L, \omega) \end{pmatrix} = \begin{pmatrix} 1 - \int_0^{\phi_p(L)} \frac{1}{W(\phi_p)} \int_0^{\phi_p} W(\phi'_p) d\phi'_p d\phi_p & -i \int_0^{\phi_p(L)} \frac{d\phi_p}{W(\phi_p)} \\ -i \int_0^{\phi_p(L)} W(\phi_p) d\phi_p & 1 - \int_0^{\phi_p(L)} W(\phi_p) \int_0^{\phi_p} \frac{d\phi'_p}{W(\phi'_p)} \int d\phi_p \end{pmatrix} \begin{pmatrix} \vec{i}(0, \omega) \\ \vec{v}(0, \omega) \end{pmatrix}. \tag{10}$$

We now specify to the case where the  $z$  dependence of  $W$  and  $\theta_{pr}$  is due to the variation of  $\gamma_{z0}(z)$  as the beam propagates in a transverse magnetic field (e.g., Fig. 1):  $\gamma_{z0}^2 = \gamma_0^2 / (1 + a_{\perp}^2(z))$ , where  $a_{\perp}$  is the normalized transverse vector potential

$$a_{\perp}(z) = \frac{e}{mc} \int_0^z B_{\perp}(z') dz'. \tag{11}$$

The compact and general presentation of the transfer matrix (10) in terms of the beam plasma phase  $\phi_p$  becomes explicit when expressed in terms of  $z$  by substitution of  $d\phi_p = \theta_{pr}(z) dz$ . For an ultra-relativistic beam and assuming that  $r_p^2/A_e$  does not vary significantly with  $z$ , the axial variation of the beam parameters is only due to the transverse magnetic field:  $\theta_{pr}^2(z) = \theta_{prd}^2 (1 + a_{\perp}^2(z))$  and  $W(z) = W_d \theta_{prd} / \theta_{pr}(z) = W_d (1 + a_{\perp}^2(z))^{-1/2}$ . Here,  $\theta_{prd}$  and  $W_d$  are, respectively, the constant reduced plasma wave-number and the beam impedance in the absence of magnetic field ( $\gamma_{z0} = \gamma_0$ ).

For a magnetic structure of length  $L_m$ ,

$$\underline{\underline{M}}_m = \begin{pmatrix} 1 - \theta_{prd}^2 \int_0^{L_m} z (1 + a_{\perp}^2(z)) dz & -i \frac{\theta_{prd}}{W_d} \int_0^{L_m} (1 + a_{\perp}^2(z)) dz \\ -i W_d \theta_{prd} L_m & 1 - \theta_{prd}^2 \int_0^{L_m} \int_0^z (1 + a_{\perp}^2(z')) dz' dz \end{pmatrix}. \tag{12}$$

The matrix element  $M_{m12}$  describes the dispersion effect of beam current modulation due to energy modulation at the entrance. It is related to the momentum compaction parameter by<sup>8,11</sup>

$$R_{56} = -\frac{1}{\gamma_0^2} \int_0^{L_m} (1 + a_{\perp}^2(z)) dz. \tag{13}$$

The element  $M_{m21}$  describes the collective plasma effect of energy modulation in the beam due to current (density) modulation. This process keeps taking place in the dispersive section just as in the drift section and should be kept if the dispersive section is long enough.

Neglecting now the second order terms produced by the collective interaction in the matrix elements  $M_{m11}$ ,  $M_{m22}$ , we obtain

$$\underline{\underline{M}}_m = \begin{pmatrix} 1 & i \frac{\gamma_0^2 \theta_{prd}}{W_d} R_{56} \\ -i W_d \theta_{prd} L_m & 1 \end{pmatrix}. \tag{14}$$

**BEAM NOISE MICRODYNAMICS IN A DISPERSIVE SECTION**

We now draw attention to the case shown schematically in Fig. 1, where some energy modulation takes place in a drift section because of the random current modulation associated with the e-beam charge granularity (current shot-noise) in the drift section  $L_d$ . The beam is then injected into an ideal magnetic dispersive section of length  $L_m$  with the current noise and energy noise being correlated. In the dispersive section, both processes of LSC interaction and momentum compaction take place, and the beam current noise may be enhanced (gain) or suppressed.

Taking the product  $\underline{\underline{M}} = \underline{\underline{M}}_m \underline{\underline{M}}_d$  of the drift and dispersion matrices (9) and (14) one gets

$$\underline{\underline{M}} = \begin{pmatrix} \cos\phi_{prd} + \gamma_0^2 \theta_{prd} R_{56} \sin\phi_{prd} & -\frac{i}{W_d} (\sin\phi_{prd} - \gamma_0^2 \theta_{prd} R_{56} \cos\phi_{prd}) \\ -i W_d (\theta_{prd} L_m \cos\phi_{prd} + \sin\phi_{prd}) & \cos\phi_{prd} - \theta_{prd} L_m \sin\phi_{prd} \end{pmatrix}, \tag{15}$$

where  $\phi_{prd}$  is the plasma phase accumulated in the drift section and  $\theta_{prd}$ ,  $W_d$  are the free drift constant parameters.

Applying the matrix  $\underline{\underline{M}}$  on the input vector  $(\vec{i}(0, \omega), \vec{v}(0, \omega))$  and calculating the averaged absolute value squared of the output parameters (at the point  $Z = L_d + L_m$ ), one obtains

$$\frac{|\overline{\tilde{i}(L, \omega)}|^2}{|\overline{\tilde{i}(0, \omega)}|^2} = (\cos\phi_{prd} + \gamma_0^2\theta_{prd}R_{56}\sin\phi_{prd})^2 + N^2(\sin\phi_{prd} - \gamma_0^2\theta_{prd}R_{56}\cos\phi_{prd})^2, \quad (16)$$

$$\frac{|\overline{\tilde{v}(L, \omega)}|^2}{W_d^2|\overline{\tilde{i}(0, \omega)}|^2} = (\sin\phi_{prd} + \theta_{prd}L_m\cos\phi_{prd})^2 + N^2(\cos\phi_{prd} - \theta_{prd}L_m\sin\phi_{prd})^2, \quad (17)$$

where  $N^2 = \frac{|\overline{\tilde{v}(0, \omega)}|^2}{W_d^2|\overline{\tilde{i}(0, \omega)}|^2}$  (the weighted ratio of initial energy noise to current noise), and it is assumed that the current and energy noises are not correlated at the entrance to the drift section.

In the limit of a current shot-noise dominated beam  $N^2 \ll 1$ , Eq. (16) reduces into

$$\frac{|\overline{\tilde{i}(L, \omega)}|^2}{|\overline{\tilde{i}(0, \omega)}|^2} = (\cos\phi_{prd} - K_d\phi_{prd}\sin\phi_{prd})^2, \quad (18)$$

where

$$K_d = \frac{|R_{56}|}{L_d/\gamma_0^2} \quad (19)$$

is essentially the ratio between the momentum compaction coefficients of the dispersive section and that of the free drift section.

Fig. 2 displays the current noise gain (18) as a function of  $K_d$  for different values of plasma modulation phase  $\phi_{prd}$  in the drift section. It appears from Eq. (18) that for each

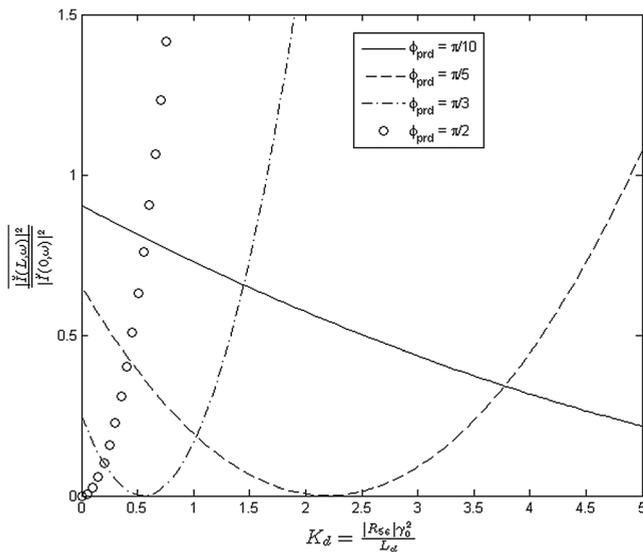


FIG. 2. Normalized current-noise as a function of the dispersive section parameter  $K_d$  for different plasma phase values  $\phi_{prd}$  accumulated in the preceding free drift section of length  $L_d$ .

plasma modulation phase  $\phi_{prd}$ , it is possible to attain noise suppression by setting  $R_{56}$  in the range

$$0 < |R_{56}| < \frac{L_d}{\gamma_0^2} \frac{1 + \cos\phi_{prd}}{\phi_{prd}\sin\phi_{prd}} \approx \frac{2L_d}{\gamma_0^2\phi_{prd}^2}, \quad (20)$$

and maximal suppression is attained for

$$R_{56} = \frac{L_d/\gamma_0^2}{\phi_{prd}\tan\phi_{prd}} \approx \frac{L_d/\gamma_0^2}{\phi_{prd}^2}. \quad (21)$$

The last approximate expressions correspond to the common case  $\phi_{prd} \ll 1$ . This result is consistent with the finding of Ref. 14.

### CONCLUSIONS

Long free drift sections, bends, chicanes, and other dispersive sections are very common in e-beam transport. But collective microdynamic effects in such systems have appeared to be significant only recently with the development of high current high quality e-beam technology. The 1D modeling of the coherent collective interaction processes presented in this paper can describe the main features of the interaction under conditions of dominant LSC interaction (3) and high quality beam parameters.<sup>8</sup> We presented an analytical description of these processes under simplifying assumptions of a coasting beam. The formulation is applicable to a description of the coherent evolution of random current and energy modulation (noise) in an e-beam passing through a general dispersive transport section, in which collective interaction effect may be large. The formulation suggests that the noise evolution in beam transport lines can be controlled to some degree by appropriate design.

We employed the formulation to an often encountered case of a beam drift section followed by a dispersive section. The derived formulation describes well the effect of coherent enhancement of beam current noise observed in Ref. 6, but it also predicts the possibility of current noise suppression in such dispersive transport sections—an effect that has never been observed yet at optical frequencies.

The absence of experimental observation of current shot-noise suppression in e-beam transport is hardly surprising, considering the need of satisfying special conditions for the appearance of this effect. The analysis of this paper reveals the conditions for coherent collective interaction that need to be satisfied, in a dispersive transport configuration as in Fig. 1. It delineates the range of the momentum compaction parameter values where noise suppression can take place (Eq. (20), Fig. 2). Substituting in practical parameters, it appears that usually very small values of  $R_{56}$  are required to attain noise suppression. In the case of the LCLS experiment,<sup>6,8</sup>  $-L_d = 2.5$  m,  $\gamma_0 = 265$ ,  $\phi_{prd} = 0.35$ , and consequently one needs  $|R_{56}| < 0.6$  mm for obtaining current noise suppression. This is an order of magnitude smaller than the value associated with the chromate bend used in the experiment ( $R_{56} = 6.3$  mm). In general, it seems that in order to observe the predicted noise suppression effect, a very small momentum compaction is required, much less than normally

in use. This may explain why coherent enhancement of current shot-noise has been observed and suppression has not.

Beyond the presently available beam transport theory, the general I-V parameterization model presented in this article enables consideration of coherent collective interaction, even beyond the range  $\phi_p \ll 1$ , in any transport configuration where one can solve Eqs. (6) and (7). In particular, it enables defining in all these configurations the parameter range where the yet unobserved effect of current shot-noise suppression can be found. Since the dispersion parameters of a dispersive section can be easily controlled electronically, control of this effect may be useful for avoiding beam instabilities and may be used for FEL coherence enhancement.<sup>16</sup>

It should be pointed out in conclusion that partially coherent collective interaction effects may take place in intense beam transport under conditions where the present single transverse mode plasma wave model does not apply (in particular, if Eq. (3) is violated). In this case (which is evidenced experimentally by observation of speckled OTR patterns), more elaborate multi-transverse modes or partial coherence kinetic formulation is needed to describe the collective interaction effects.

## ACKNOWLEDGMENTS

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