Appl. Phys. 23, 295–298 (1980)

Applied Physics

An Analysis of Stimulated Longitudinal Electrostatic Bremsstrahlung in a Free-Electron Laser Structure*

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Received 19 June 1980/Accepted 15 July 1980

Abstract. An analysis of stimulated emission of radiation by an electron beam passing through a periodic longitudinal electrostatic field is presented, and its use in a free-electron laser structure is discussed. The analysis is based on a coupled-mode calculation of the interaction between a waveguided electromagnetic mode and an electron-beam plasma with finite cross section. The dispersion equation derived applies to the collective as well as the single electron regime.

PACS: 41, 42.55

In the magnetic bremsstrahlung free electron laser [1, 2] (FEL) a periodic magnetic force operates transversely on the electrons of an electron beam and produces radiation by a stimulated bremsstrahlung process. The interaction in this FEL involves coupling of a transverse electromagnetic wave and the electron-beam plasma waves by means of the ponderomotive force [3]. FELs based on direct longitudinal interaction of an electron beam with a TM electromagnetic wave in a slow wave structure (Cerenkov-Smith-Purcell FELs) were also suggested and analyzed [4–6].

The use of either transverse or longitudinal electrostatic bremsstrahlung mechanisms for FELs by applying a periodic external electrostatic potential or by passing a high density beam in a rippled wall waveguide were suggested by a number of authors [7–9]. The operation of a transverse electrostatic FEL can be described quite accurately ny a simple adaptation of the magnetic bremsstrahlung FEL theory [9]. However, no detailed theory has been presented yet to characterize the longitudinal electrostatic FEL. The purpose of the present letter is to analyze the longitudinal electrostatic bremsstrahlung mechanism and its utilization in a FEL structure.

* This research is supported in part by AFOSR Grant, 77–3445.

Theoretical Description

The interaction structure is assumed to consist of an electromagnetic waveguide which supports TM modes, an electron beam which propagates along the waveguide (in the $+z$ direction), and periodic electrodes which produce a longitudinal electrostatic field along the beam which is uniform across its cross section. The beam is assumed to be narrow enough so that transverse variation of the field across the beam is negligible.

The electrostatic potential which is applied along the beam is

$$\Phi_0(z) = \Phi_0 \cos k_0 z \tag{1}$$

and the corresponding electrostatic field is

$$E_0(z) = E_0 \sin k_0 z\,, \tag{2}$$

where $E_0 = \Phi_0 k_0$ and $k_0 = 2\pi/L$, L is the period of the periodic field.

The electron beam distribution is governed by the linearized one-dimensional Vlasov equation

$$\frac{\partial f^{(1)}}{\partial t} + w_z(z)\frac{\partial f^{(1)}}{\partial z} - eE_0(z)\frac{\partial f^{(1)}}{\partial p_z} = eE_z(z,t)\frac{\partial f^{(0)}}{\partial p_z}\,, \tag{3}$$

where $f^{(0)}(\mathbf{p}, z)$ is the electron momentum distribution of the beam in the static field $E_0(z)$, and $f^{(1)}(\mathbf{p}, z, t)$ is the first-order perturbation to the electron distribution

0340-3793/80/0023/0295/$01.00

due to the total longitudinal alternating electric field $E_z(z,t)$ (including both electromagnetic and space charge wave fields). $w_z \equiv p_z/[\gamma(\mathbf{p})m]$ is the z dependent longitudinal velocity.

As in the analysis of the magnetic bremsstrahlung FEL [3], it would prove beneficial to transform the momentum variables of (3) to variables which are constant of the motion in the periodic field. The transverse momenta p_x, p_y are two constants of the motion since we assumed a vanishing transverse electrostatic field. Another constant of the motion is the total electron energy (kinetic plus potential). Thus, we may define a new momentum variable u_z, which is a constant of the motion, by

$$\{\gamma[p(z)]-1\}mc^2 - e\Phi_0(z) = [\gamma(\mathbf{u})-1]mc^2\,, \tag{4}$$

where $\gamma(\mathbf{u}) \equiv [1+(p_x^2+p_y^2+u_z^2)/(m^2c^2)]^{1/2}$.

The transformation from p_z to u_z is defined implicitly by (4). After substituting in (3): $f^{(1)}(\mathbf{p},z,t) = g^{(1)}(\mathbf{u},z,t)$; $f^{(0)}(\mathbf{p},z) = g^{(0)}(\mathbf{u})$ and rewriting the partial derivatives in terms of the new variables $(\mathbf{u},z,t)$, we get

$$\frac{\partial g^{(1)}}{\partial t} + w_z(z)\frac{\partial g^{(1)}}{\partial z} = eE_z\frac{w_z(z)}{v_z}\frac{\partial g^{(0)}}{\partial u_z}. \tag{5}$$

Equation (5) can be Fourier transformed in time $\left(\frac{\partial}{\partial t} \to -\mathrm{i}\omega\right)$ and straightforwardly integrated

$$g^{(1)}(\mathbf{u},z) = \int_0^z dz' \exp\left[\mathrm{i}\omega\int_{z'}^{z}\frac{dz''}{w_z(z'')}\right] \cdot E_z(z')\frac{e}{v_z}\frac{\partial g^{(0)}(\mathbf{u})}{\partial u_z}, \tag{6}$$

where $v_z \equiv u_z/[\gamma(\mathbf{u})m]$ and we assumed $g^{(1)}(\mathbf{u},0)=0$ (no prebunching of the electron beam). This gives an expression for the current density induced in the beam

$$J_z^{(1)}(z) = -e^2\int_{-\infty}^{\infty} dp_x dp_y du_z \frac{\partial g^{(0)}(\mathbf{u})}{\partial u_z} \cdot \int_0^z dz' \exp\left[\mathrm{i}\omega\int_{z'}^{z}\frac{dz''}{w_z(z'')}\right]E_z(z'). \tag{7}$$

To complete the coupled mode analysis of the interaction between the electron-beam plasma and the TM waveguide mode, we need now to use Maxwell equations which describe the excitation of the EM mode in the waveguide by the current (7). The Maxwell equations can be simplified to a large degree if the transverse electric field in the structure is expanded in terms of the waveguide modes: $\mathbf{E}_t(x,y,z) = \sum_m a_m(z)\mathcal{E}_{mt}(x,y)$. This procedure leads to simple one-dimensional equations for the mode complex amplitude $a_m(z)$ [10, 6]. Assuming that only a single mode is excited in the waveguide (and therefore, for simplicity, dropping from now on the subscript m) these equations can be written as

$$\frac{da(z)}{dz} - \mathrm{i}k_{z0}a(z) = \frac{1}{4P}\iint_{A_e} dxdy\mathcal{E}_z(x,y)J_z^{(1)}(x,y,z)\,, \tag{9}$$

$$E_z(x,y,z) = a(z)\mathcal{E}_z(x,y) + \frac{1}{\mathrm{i}\omega\varepsilon}J_z^{(1)}(x,y,z)\,, \tag{10}$$

where A_e is the electron beam cross section, k_{z0} the wave number of the uncoupled waveguide mode and P the mode power. Since $E_0(z)$ (2) and consequently $w_z(z) \equiv p_z(z)/[\gamma(p)m]$ are periodic functions of z, we will expect that the solution of the coupled equations (7, 9, 10) will satisfy the Floquet theorem

$$J_z^{(1)}(z) = \sum_{n=-\infty}^{\infty} J_{zn}^{(1)}\exp\{\mathrm{i}[k_z+(n+1)k_0]z\} \tag{11}$$

$$E_z(z) = \sum_{n=-\infty}^{\infty} E_{zn}\exp\{\mathrm{i}[k_z+(n+1)k_0]z\}\,. \tag{12}$$

Also $a(z)$ should satisfy the Floquet theorem, but if the coupling is weak enough, we would expect the fundamental space harmonic with wave number $k_z \simeq k_{z0}$ to be dominant.

$$a(z) \simeq a_0\exp(\mathrm{i}k_z z)\,. \tag{13}$$

To allow an approximate closed-form solution of the coupled system, we will not keep in (11, 12) the infinite number of terms. The space harmonic $n=-1$ in (11) is evidently a significant non-negligible term since in (9) it will resonantly induce the electromagnetic mode (13). The $n=0$ space harmonic is also significant if we assume operation near the synchronizm condition

$$\omega/(k_{z0}+k_0) \simeq v_0\,, \tag{14}$$

where v_0 is the electron beam average velocity. In this case, the alternating field (12) will be synchronous with the electron beam plasma through the $n=0$ space harmonic. Thus, the significant terms in (11, 12) are

$$J_z^{(1)}(z) \simeq J_{z-1}^{(1)}\exp(\mathrm{i}k_z z) + J_{z0}^{(1)}\exp[\mathrm{i}(k_z+k_0)z]\,, \tag{15}$$

$$E_z(z) \simeq E_{z-1}\exp(\mathrm{i}k_z z) + E_{z0}\exp[\mathrm{i}(k_z+k_0)z]\,. \tag{16}$$

When (16) is substituted in (7), the integration over z' can be carried out, if an explicit functional form for $w_z(z)$ is available. In the case of a small amplitude of periodic potential (1), we may expand

$$w_z(z) = p_z(z)/[\gamma(\mathbf{p})m]$$

and Eq. (4) to first order in Φ_0

$$w_z(z) \simeq v_z + v_{z1}\cos k_0 z\,, \tag{17}$$

where $v_{z1} \simeq -e\Phi_0/(\gamma_z^2 u_z)$ and $\gamma_z \equiv (1-v^2/c^2)^{-1/2}$. Using the mathematical identity

$$\exp(\mathrm{i}M\sin k_0 z) \equiv \sum_{n=-\infty}^{\infty} J_n(M)exp(\mathrm{i}nk_0 z)$$

[where $J_n(M)$ is the n order Bessel function], and neglecting the transient wave components which correspond to non-growing waves [11], we get near synchronizm

$$J^{(1)}_{z0} = i\omega\chi_p(\omega, k_z+k_0)(\alpha E_{z-1}+E_{z0}), \tag{18}$$

$$J^{(1)}_{z-1} = i\omega\chi_p(\omega, k_z+k_0)\alpha(\alpha E_{z-1}+E_{z0}), \tag{19}$$

$$\alpha \equiv J_1\left(\frac{\omega/v_0}{k_0}\frac{v_{z1}}{v_0}\right) \simeq \frac{1}{2}\frac{\omega/v_0}{k_0}\frac{v_{z1}}{v_0}, \tag{20}$$

$$\chi_p(\omega,k_z) \equiv -\frac{e^2}{\omega}\int_{-\infty}^{\infty} dp_x dp_y du_z \frac{\partial g^{(0)}/\partial u_z}{k_z-\omega/v_z}. \tag{21}$$

χ_p is the well-known plasma susceptibility of the electron beam.

The simultaneous solution of (9, 10, 13, 15, 16, 18, 19) gives the dispersion relation of the coupled system

$$(k_z-k_{z0})[1+(1+\alpha^2)\chi_p(\omega,k_z+k_0)/\varepsilon] - \kappa\chi_p(\omega,k_z+k_0)/\varepsilon = 0 \tag{22}$$

$$\kappa \equiv \alpha^2\frac{\pi}{\lambda}\left(\frac{1}{2}\iint_{A_e} dxdy|\mathcal{E}_z(x,y)|^2/P\right). \tag{23}$$

Equation (22) is similar to the dispersion relation obtained before for the Carenkov-Smith-Purcell FELs [6]. Indeed, it can be shown that all FELs reduce to this simple dispersion relation [12]. Hence, the gain regimes of the electrostatic bremsstrahlung FEL are the same as in all other FELs [3, 6, 12]. For example, in the low-gain regime (22) results in the conventional FEL gain formula [1, 2].

$$\frac{\Delta P}{P} = Q\frac{d}{d\theta}\left[\frac{\sin(\theta/2)}{\theta/2}\right]^2, \tag{24}$$

where $\theta \equiv (\omega/v_0 - k_0 - k_{z0})l$ is the "detuning parameter" and

$$Q \equiv \frac{\kappa}{\gamma_0^3}\frac{\omega_p^2}{v_0^2}l^3. \tag{25}$$

To investigate the dependence of the gain parameter Q on the operating parameters, we should calculate κ (23) for a particular waveguide structure. For example, in a rectangular waveguide of cross-section A_g, (23) gives for a narrow electron beam

$$\kappa = \alpha^2\frac{\pi}{\lambda}\frac{\sin^2\varphi}{\cos\varphi}\frac{A_e}{A_g}, \tag{26}$$

where $\cos\varphi \equiv k_{z0}/k$; $k \equiv 2\pi/\lambda$. We should also use the radiation condition

$$\lambda \simeq (\beta_0^{-1} - \cos\varphi)L \tag{27}$$

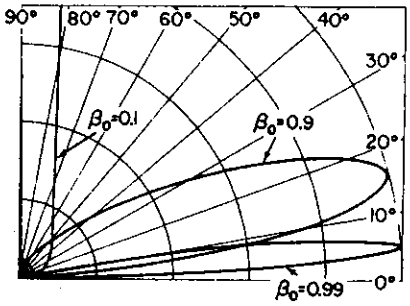

Fig. 1. The normalized angular spectrum of the gain parameter Q

which is directly derived from the synchronizm condition (14). Equations (20, 25–27) result in

$$Q = \frac{1}{16\pi}\frac{\omega_p^2}{c^2}\left(\frac{eE_0}{mc^2}\right)^2\frac{1}{\beta_0^4\gamma_0^9}\frac{1}{(1-\beta_0\cos\varphi)^4} \cdot\frac{\sin^2\varphi}{\cos\varphi}\frac{A_e}{A_g}\lambda l^3. \tag{28}$$

The normalized angular spectrum of the gain parameter Q is shown in Fig. 1. As $\beta_0 \to 1$ the gain peaks up at smaller angles. For $\gamma_0 \gg 1$ we get from (28) that the peak angle is $\varphi_m = 1/(\sqrt{3\gamma_0})$ and the corresponding peak gain parameter is

$$Q_m = \frac{27}{256\pi}\omega_p^2\left[\frac{eE_0}{mc^2}\right]^2\frac{1}{\beta_0^4\gamma_0^3}\frac{A_e}{A_g}\lambda l^3. \tag{29}$$

Equations (28, 29) are simple formulas which allow to estimate the gain available with a longitudinal electrostatic bremsstrahlung mechanism, and its dependence on the various operating parameters. In a comparison to the magnetic bremsstrahlung [12] it appears that the latter has a significant advantage in gain by approximately the factor $(B_0c/E_0)^2\gamma_0^2$ at relativistic energies. Nevertheless, it should be pointed out that in oscillator structures, the gain per pass is not always the most important parameter, if it is large enough to sustain oscillation. In this case, the device efficiency and power generation are more important parameters. These can be shown to be similar for electrostatic and magnetic bremsstrahlung FELs [12].

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