Predicted effect of mode cooperation and white-light lasing in warm-beam free-electron lasers

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Received July 22, 1980

The saturation process of free-electron-laser oscillators (FEL's) in the warm-beam regime is examined, considering radiation-extraction efficiency and spectral width. The spectral width and conversion efficiency can be estimated in both high- and low-gain regimes when the laser-oscillator mode population is sparse enough. Under certain conditions, which can be achieved in a relativistic FEL with a Fabry–Perot resonator, a superinhomogeneous-broadening saturation process is predicted in which a hill-heaping effect will accompany the hole burning in the gain curve. In an oscillator, the resonator modes will interact and tend to spread both the emission spectrum and the electron-beam distribution uniformly and over a wide region.

In conventional atomic lasers, there is a known distinction between two kinds of saturation behavior, homogeneous broadening and inhomogeneous broadening.[1] In homogeneously broadened lasers (such as collision-dominated gas lasers), the linewidth of electronic transitions in any atom in the assembly of population-inverted atoms is wide enough so that it can amplify a number of cavity modes. The different modes in this laser must compete with one another for the same electrons to make the transition at one mode frequency or the other. In inhomogeneously broadened lasers (such as Doppler-broadened gas lasers), the width of the emission line is due to the presence of many species of atoms, each having radiative transition at a different frequency. Thus, in such a laser, the different cavity modes are amplified by stimulated radiative transitions of different noninteracting atoms, and therefore there is no mode competition or mode interaction of any sort.

In free-electron lasers in the warm-beam regime, we predict a new kind of saturation characteristic (different from homogeneous and inhomogeneous saturation) in which neighboring modes may interact with one another in a cooperative way so that the saturation of one mode will increase the gain of other modes. In such a superinhomogeneous laser, there will be a tendency for multimode operation, and the laser radiation will spread evenly over a wide spectrum.

The single-pass gain of a free-electron laser in the high-gain warm-beam regime is given by[2]

$$G(\omega) \equiv \ln \frac{P(\omega,l)}{P(\omega,0)} = 2\delta k_i l = \pi\kappa l \frac{k_D'^2}{(k_{z0}+k_0)^2} \int_{-\infty}^{\infty} g'\left(\frac{v_z - v_{0z}}{v_{zth}'}\right) \frac{1}{\pi} \times \frac{v_{0z}\dfrac{\delta k_i}{k_{z0}+k_0}}{(v_z - v_{ph})^2 + \left(v_{0z}\dfrac{\delta k_i}{k_{z0}+k_0}\right)^2} \mathrm{d}v_z, \quad (1)$$

where

$$v_{ph} = \frac{\omega}{k_{z0}+k_0} \quad (2)$$

is the phase velocity of the longitudinal space-charge electrostatic potential wave that is propagating in the electron beam of the free-electron laser (ponderomotive potential in bremsstrahlung FEL's).

This expression applies to various kinds of FEL's, which differ only with the value of the coupling coefficient κ, which is listed for various FEL's in Ref. 2. l is the laser-interaction length, k_{z0} is the wave number of the electromagnetic mode, $k_0 = 2\pi/L$ is the wave number of the periodic structure of the FEL (periodic magnetic field in magnetic bremsstrahlung FEL), k_D' is the Debye wave number of the electron beam, $g(v_z - v_{0z}/v_{zth}')$ is the normalized longitudinal velocity-distribution function of the electron beam at the entrance to the interaction region, v_{0z} is the beam-average longitudinal-beam velocity, and v_{zth}' is the longitudinal velocity spread. δk_i is the imaginary part of the wave number of the growing wave.

In the low-gain regime ($\delta k_i l \ll 1$), the single-pass gain at the warm-beam limit is given by

$$G(\omega) = \frac{\Delta P(\omega)}{P(\omega)} = \pi\kappa l \frac{k_D'^2}{(k_{z0}+k_0)^2} \int_{-\infty}^{\infty} g'\left(\frac{v_g - v_{0z}}{v_{zth}'}\right) \frac{\sin^2\left[\dfrac{\omega l}{2v_{0z}^2}(v_z - v_{ph})\right]}{2\pi\left[\dfrac{\omega l}{2v_{0z}^2}(v_z - v_{ph})\right]^2} \mathrm{d}v_z. \quad (3)$$

The gain is thus given in both cases by the convolution of the warm-beam gain curve $g'(v_z - v_{0z}/v_{zth}')$ with a spectral line-shape function attributed to the wave-number uncertainty that is due to wave growth (in the first case) or to the finite interaction length (in the second case). When the spectral line-shape functions are narrow enough, both equations give

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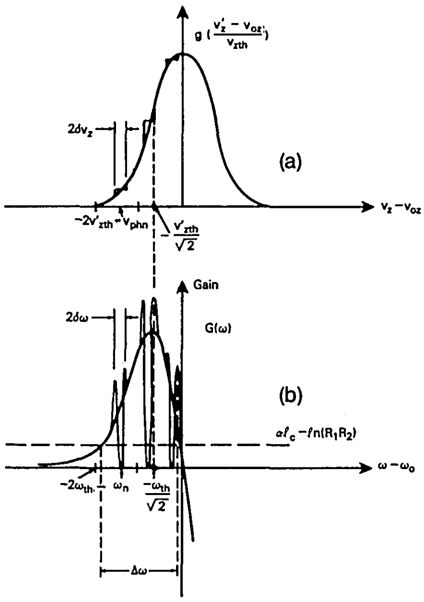

Fig. 1. (a) The electron distribution function and (b) the gain curve before interaction (heavy line) and after saturation (light line) in a cavity with sparse mode density.

$$G(\omega) = \pi\kappa l \frac{{k'_D}^2}{(k_{z0}+k_0)^2} g'\left(\frac{v_{ph}-v_{0z}}{v'_{zth}}\right) = \pi\kappa l \frac{{k'_D}^2}{(k_{z0}+k_0)^2} g'\left(\frac{\omega-\omega_0}{\omega_{th}}\right), \qquad (4)$$

where, for the case of a TEM wave ($k_{z0} = \omega/c$),

$$\omega_0 = \frac{\beta_{0z}}{1-\beta_{0z}} k_0, \qquad (5)$$

$$\omega_{th} = \frac{1}{1-\beta_{0z}} \frac{v'_{zth}}{v_{0z}} \omega_0. \qquad (6)$$

This gain curve is shown in Fig. 1, assuming a Maxwellian velocity distribution:

$$g(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}. \qquad (7)$$

Neglecting the effect of saturation on the electron-distribution function, the laser-oscillation frequency domain is determined by requiring a positive round-trip net gain

$$G(\omega) - 2\alpha l_c + \ln(R_1R_2) > 0, \qquad (8)$$

where α is the electromagnetic waveguide loss per unit length, l_c is the laser-cavity length, and R_1 and R_2 are the power reflectivities of the back and front mirrors, respectively. From Fig. 1(b) one can see that for gain considerably larger than the losses, the oscillation bandwidth is

$$\Delta\omega \simeq 2\omega_{th}. \qquad (9)$$

The laser cavity is usually a Fabry–Perot resonator, but it can be also a distributed-feedback or a distributed Bragg-reflector resonator or another filtering (dispersive) resonator, which sustains fewer numbers of modes. All the modes with frequencies within the spectral width $\Delta\omega$ will be excited and will oscillate.

The saturation mechanism of each mode is electron trapping, which is familiar in plasma physics, for instance, in Langmuir wave saturation.[3] During this process, an electron at velocity $\delta v_z > 0$ relative to the wave-phase velocity v_{ph} will reverse its velocity at full trapping, and then it cannot contribute more energy to the wave. Thus, around the phase velocity of each mode, there will be a plateau formation in the electron-distribution function, and $g'_{\mathrm{sat}}(v_{ph} - v_{0z}/v'_{zth}) = 0$ [Fig. 1(a)], which can be interpreted as a depletion of population inversion.

The class of electrons that interacts with each mode is different in the high- and low-gain regimes, as seen from Eqs. (1)–(3). In the first case,

$$\delta v_z = \frac{\delta k_i}{v_{z0}+k_0} v_{0z}. \qquad (10)$$

In the second case,

$$\delta v_z = \frac{\pi/l}{k_{z0}+k_0} v_{0z}. \qquad (11)$$

The radiation-extraction efficiency of a single mode can thus be estimated from the equation

$$\eta_0 = \frac{\Delta E_{KE}}{E_{KE}} = \frac{1}{\gamma_0-1}\int_{-\infty}^{\infty}(\gamma-1)\left[g\left(\frac{v_z-v_{0z}}{v'_{zth}}\right) - g_{\mathrm{sat}}\left(\frac{v_z-v_{0z}}{v'_{zth}}\right)\right]\frac{dv_z}{v'_{zth}} = \frac{1}{\gamma_0-1}\int_{v_{ph}-\delta v_z}^{v_{ph}+\delta v_z} \times \left[g\left(\frac{v_z-v_{0z}}{v'_{zth}}\right) - g\left(\frac{v_{ph}-v_{0z}}{v'_{zth}}\right)\right]\frac{dv_z}{v'_{zth}}. \qquad (12)$$

This gives, for the nth-order mode in the high-gain limit,

$$\eta_{0n} = \frac{\pi^3}{12}\frac{{\gamma_{0z}}^2}{{\beta_0}^2{\beta_{0z}}^5}(1+\gamma_0^{-1})\left[g'\left(\frac{v_{phn}-v_{0z}}{v'_{zth}}\right)\right]^4 \times \frac{1}{(v'_{zth}/v_{0z})^2}\frac{{k'_D}^6\kappa^3}{(\omega/c)^9}, \qquad (13)$$

and in the low-gain limit,

$$\eta_{0n} = \frac{2\pi^3}{3}\frac{\beta_{0z}}{{\beta_0}^2}{\gamma_{0z}}^2(1+\gamma_0^{-1}) \times g'\left(\frac{v_{phn}-v_{0z}}{v'_{zth}}\right)\frac{1}{(v'_{zth}/v_{0z})^2}\frac{1}{(\omega c/l)^3}. \qquad (14)$$

If there are N isolated modes in the oscillation frequency domain, so that the frequency spacing between the modes $\Delta\omega_{nm}$ is much greater than $\delta\omega$,

$$\delta\omega = \frac{k_{z0}+k_0}{1-\beta_{0z}}\delta v_z = \begin{cases} \dfrac{\beta_{0z}}{1-\beta_{0z}}\delta k_i c & \delta k_i l \gg 1, \\ \dfrac{\beta_{0z}}{1-\beta_{0z}}\dfrac{\pi}{l}c & \delta k_i l \ll 1, \end{cases} \qquad (15)$$

then the total radiation-extraction efficiency is given simply by summation over the efficiencies of the oscillating modes:

$$\eta_0 = \sum_{n=1}^{N}\eta_{0n}. \qquad (16)$$

The spectral width of the oscillation in this case will be $2\omega_{th}$ [Eq. (6)].

Let us now examine the effect of one mode-saturation process on the laser gain curve. When a mode at fre-

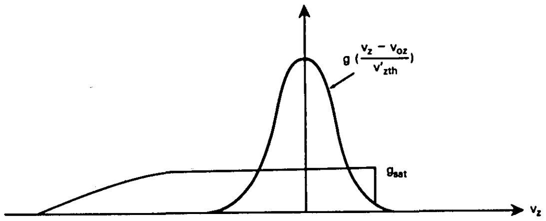

Fig. 2. The electron distribution function before interaction (heavy line) and after saturation (light line) in a cavity with close mode density.

quency ω_n saturates, there is a plateau formation in the distribution function at $v_z = v_{phn}$, and the saturated curve exhibits from the point of saturation on (within the interaction length l) an effect of hole burning as in conventional inhomogeneously broadening lasers.[1] However, in addition to the hole-burning effect, there is a change in the electron-distribution function that generates hill heaping on both sides of the holes around $v_{ph} \pm \delta v_z$. Thus, if the cavity modes are close enough, there is an effect of mode cooperation, in which the saturation of one mode increases, along the interaction length, the population inversion or the gain seen by a neighbor mode. Such an effect is unique to free-electron lasers and cannot occur in atomic lasers, in which each excited electron can make only one transition and emits only one radiation mode.

In view of this discussion, it can be predicted that in a strongly pumped free-electron laser in the warm-beam regime, whose cavity sustains oscillating modes with density

$$\Delta\omega_{mn} < 2\delta\omega, \tag{17}$$

there will be a tendency toward multimode operation. The mode-cooperation process will tend to wash away any bumps in the derivative of the distribution function and will spread its velocity distribution as shown in Fig. 2. The laser spectrum will tend to spread uniformly over a wide spectrum, which is considerably wider than $2\omega_{th}$.

In the most common laser cavity, the Fabry–Perot resonator, the spacing between modes is

$$\Delta\omega_{n,n+1} = \frac{\pi}{l_c} c. \tag{18}$$

Since the cavity length is always longer than the interaction length, $l_c > l$, we see from comparing Eqs. (15) and (18) that, at relativistic velocities $(1 - \beta_{0z} \ll 1)$, inequality (17) will hold in both gain regimes, and thus mode cooperation and wide-spectrum oscillation will take place.

We should note also that, in free-electron lasers operating in the cold-beam regime, an involved radiation spectrum broadening is likely to happen during the saturation process, involving (in the highly relativistic limit) a large number of mutually interacting modes. The saturation of a cold-beam free-electron laser always involves broadening of the beam momentum distribution,[4] which in turn would lead to radiation-spectrum broadening. In addition, other effects in the saturation process, such as side-band instability,[5] will further conspire to spread the laser spectrum.

Limitations of state-of-the-art electron-gain technology make it hard to satisfy the cold-beam condition $v'_{zth}/v_{0z} \ll \beta_{0z}\lambda/(2\pi l)$ at short-wavelength (visible) radiation. This directs our attention to warm-beam-regime operation of FEL's. The wide-spectrum lasing (white-light lasing) in this regime is not necessarily a disadvantage in many power applications of lasers and may be advantageous in some specific applications, such as blinding and jamming in electronic-countermeasures warfare. The radiation produced by such a laser is of course temporally incoherent; however, it can be highly bright and collimated and can be used for concentrating high power on a target almost as efficiently as do conventional lasers, assuming that nondispersive reflective optics is used. The superinhomogeneous broadening of such lasers is of fundamental interest as a new kind of saturation process in laser oscillators. Its full description and the calculation of conversion efficiency calls for a more involved nonlinear analysis of the coupling among a large number of cavity modes.

This research is supported by the U.S. Air Force Office of Scientific Research under grant no. AFOSR-80-0073.

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