Feasibility of dc to visible high-power conversion employing a stimulated Compton free electron laser with a waveguided CO_2 laser pump wave and an axial electric field

A. Gover
Tel Aviv University, School of Engineering, Israel and JAYCOR, Alexandria, Virginia 22304

C. M. Tang and P. Sprangle
Naval Research Laboratory, Plasma Physics Division Washington, D.C. 20375

(Received 29 December 1980; accepted for publication 9 September 1981)

We examine the feasibility of high power generation of visible radiation by a process of applying an axial accelerating electric field on electrons trapped in the ponderomotive potential of a Compton scattering free electron laser. We consider a scheme where the pump (wiggler) field is produced by a high-power pulsed CO_2 laser and the signal wave is the radiation of a high-power pulsed dye laser. We propose to use a hollow dielectric waveguide in order to overcome the pump wave diffraction and obtain a long interaction length.

PACS numbers: 42.55. – f, 42.65.Cq, 52.35.Mw, 52.35.Py

Free electrons which propagate in a transverse electric or magnetic periodic wiggler field can give rise to amplication of an electromagnetic wave by the process of stimulated magnetic bremsstrahlung[1–9] or stimulated Compton scattering[10–12] (free electron lasers). This radiative emission process saturates by a trapping mechanism in which the electrons are trapped in the troughs of the traveling longitudinal "ponderomotive potential" wave.[13,14] Once an electron is fully trapped by the trapping potential, which propagates with constant phase velocity, it cannot further reduce its kinetic energy and stops emitting radiation. However, further energy can be extracted if at this point an axial accelerating electric field is applied along the electron beam propagation direction. Assuming this field is not strong enough to detrap the electrons from their potential wells, then the electrons, still trapped, will keep traversing with the constant axial phase velocity of the trapping potential wave, and will do work on the wave. The energy which is supplied by the dc accelerating electric field source is then directly transformed into radiation energy.[15–17,28] The purpose of the present paper is to examine the feasibility of realizing this scheme for producing high-power visible radiation from trapped electrons, utilizing high-power laser beam sources for the wiggler and signal fields.

The interaction scheme is shown schematically in Fig. 1. In this scheme the pump wave propagates in a hollow dielectric or metallic waveguide[18–20] in which there is no diffraction along the interaction region. Therefore a high-power density of the pump wave can be sustained along a long interaction length. The higher frequency signal radiation wave ω_s propagates from left to right in free space, and its diffraction is assumed to be negligible within the interaction length ℓ. A high intensity relativistic electron beam is propagated along the interaction region through the overlapping cross-sectional area of the pump and signal waves. An axial accelerating electric field is applied inductively to the trapped electrons in a way similar to the accelerating fields in induction Linacs.

The basic structure of the scheme in Fig. 1 is similar to that of a conventional single turn voltage transformer, where the secondary coil has been replaced by the electron beam. When the current intensity I is varied in the primary coil, an axial electric field (potential drop) is induced on the "secondary coil," proportional to dI/dt. In the conventional resistively loaded voltage transformer, the work done by the axial field on the electrons in the secondary conductor is transformed eventually to incoherent phonon energy (heat). In the present arrangement, the work performed on the trapped electrons is converted into coherent radiation.

In order to illustrate the basic principles of the device operation and its design considerations, let us examine in a simple model the axial interaction of a trapped electron with the ponderomotive field and the dc accelerating field.

Assume that the pump and signal radiation fields are given by

$$\mathbf{E}_w(\mathbf{r},t) = \mathrm{Re}\widetilde{\mathbf{E}}_w(x,y)e^{-i[\omega_w t + k_w z + \varphi_w(z)]}, \quad (1)$$

$$\mathbf{E}_s(\mathbf{r},t) = \mathrm{Re}\widetilde{\mathbf{E}}_s(x,y)e^{-i[\omega_s t + k_s z + \varphi_s(z)]}, \quad (2)$$

where $\mathbf{E}_w$ and $\mathbf{E}_s$ are transverse fields. The axial force equation of an electron in these fields with zero transverse canoni-

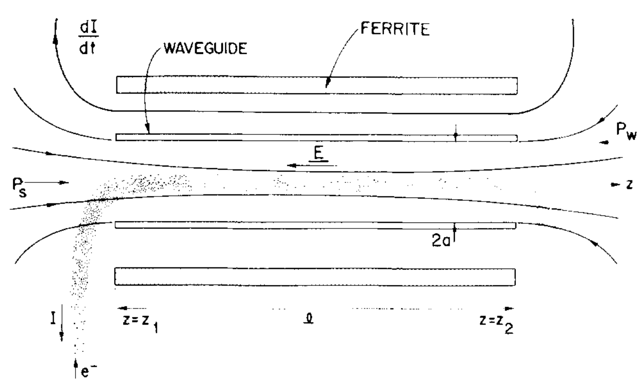

FIG. 1. Schematic of FEL experiment utilizing high-power laser sources for the wiggler and signal fields. The wiggler field propagates in a hollow dielectric or metallic waveguide in which there is no diffraction along the interaction region.

 0021-8979/82/010124-06$02.40

cal momentum is derived in Appendix I and given by (M. K. S. units)

$$\frac{dp_z}{dt} = -\frac{e^2}{2\gamma m}\frac{\partial}{\partial z}(\mathbf{A}_w + \mathbf{A}_s)^2 + e\frac{\partial}{\partial z}\phi, \tag{3}$$

where

$$\gamma = \left[1 + \frac{p_z^2}{m^2c^2} + \frac{e^2}{m^2c^2}(\mathbf{A}_w + \mathbf{A}_s)^2\right]^{1/2}.$$

$\mathbf{A}_w(\mathbf{r},t) = \mathrm{Re}(\mathrm{A}_w e^{-i\omega}w^t)$ and $\mathbf{A}_s(\mathbf{r},t) = \mathrm{Re}(\widetilde{\mathbf{A}}_s e^{-i\omega_s t})$ are the vector potentials of the pump and signal waves. The associated electric field intensities are $\mathbf{E}_w(\mathbf{r},t) = \mathrm{Re}(\widetilde{\mathbf{E}}_w e^{-i\omega_w t})$ and $\mathbf{E}_s(\mathbf{r},t) = \mathrm{Re}(\widetilde{\mathbf{E}}_s e^{-i\omega_s t})$, where $\widetilde{\mathbf{E}}_w = i\omega_w \widetilde{\mathbf{A}}_w$ and $\widetilde{\mathbf{E}}_s = i\omega_s \widetilde{\mathbf{A}}_s$. If we neglect space charge effects, the only contribution to the scalar potential will be that of the axial accelerating field: $\phi = \phi_{\mathrm{ac}} = -\int_{z_1}^{z} E_{\mathrm{ac}}(z')\,dz'$.

When the fields (1, 2) are substituted in (3) and the quadratic term is expanded, we find that out of all the resulting terms, only the mixed "beating" term produces a wave which has a phase velocity less than the speed of light. This wave can interact with an electron and trap it. Keeping only this term (the ponderomotive force wave) and the external accelerating field E_{ac}, Eq. 3 reduces to the simple form

$$\frac{dp_z}{dt} = -eE_{\mathrm{pond}}\sin(kz - \omega t) - eE_{\mathrm{ac}}, \tag{4}$$

where $\omega = \omega_s - \omega_w$ and $k = k_s + k_w = (\omega_s + \omega_w)/c$. The ponderomotive field amplitude is

$$E_{\mathrm{pond}} = \frac{e}{2\gamma m}(k_w + k_s)|\widetilde{\mathbf{A}}_w^* \cdot \widetilde{\mathbf{A}}_s|. \tag{5}$$

It is most instructive to view the interaction in a frame which is moving with the ponderomotive wave phase velocity $v_{\mathrm{ph}} = \omega/k < c$. In this frame the ponderomotive wave is stationary and (4) reduces to a simple pendulum equation with a constant force term:

$$\frac{d}{dt'}p_z' = -eE'_{\mathrm{pond}}\sin(k'z') - eE'_{\mathrm{ac}}. \tag{6}$$

The primes denote parameter values in the moving frame.

Assume an electron is traversing with an axial velocity close to the phase velocity of the ponderomotive potential wave $v_z \simeq v_{\mathrm{ph}}$. The velocity of such an electron when viewed in the moving frame (v'_z) will be then very small and assumed to be nonrelativistic. We can then multiply Eq. (6) by $p_z' = \gamma' m dz'/dt'$ and directly integrate it:

$$\frac{p_z'^2}{2\gamma' m} - \frac{eE'_{\mathrm{pond}}}{k'}\cos k'z' + eE'_{\mathrm{ac}}(z' - z_1') = \mathscr{E}' = \mathrm{const}. \tag{7}$$

This relation simply states that the sum of the axial kinetic and potential energy of the electron is constant in the wave frame. The potential energy of the electron is composed of the ponderomotive potential energy, and the axial potential energy drop due to the accelerating dc field (see Fig. 2).

If the accelerating field is not too high:

$$E'_{\mathrm{ac}} << E'_{\mathrm{pond}}, \tag{8}$$

then the energy diagram of Fig. 2 will depict a slightly tilted array of potential wells (traps, buckets), which can trap electrons. A necessary condition for trapping is that the electron in the wave frame will have a maximum axial kinetic energy which is smaller than the height of the potential well:

$$\frac{p_z'^2}{2\gamma' m} << 2\frac{eE'_{\mathrm{pond}}}{k'}. \tag{9}$$

The electron will then oscillate back and forth inside the potential well. In the laboratory frame the electron will be closely in phase with the ponderomotive wave despite the accelerating axial field. A fully trapped electron will have zero kinetic energy in the moving frame, and will sit at the

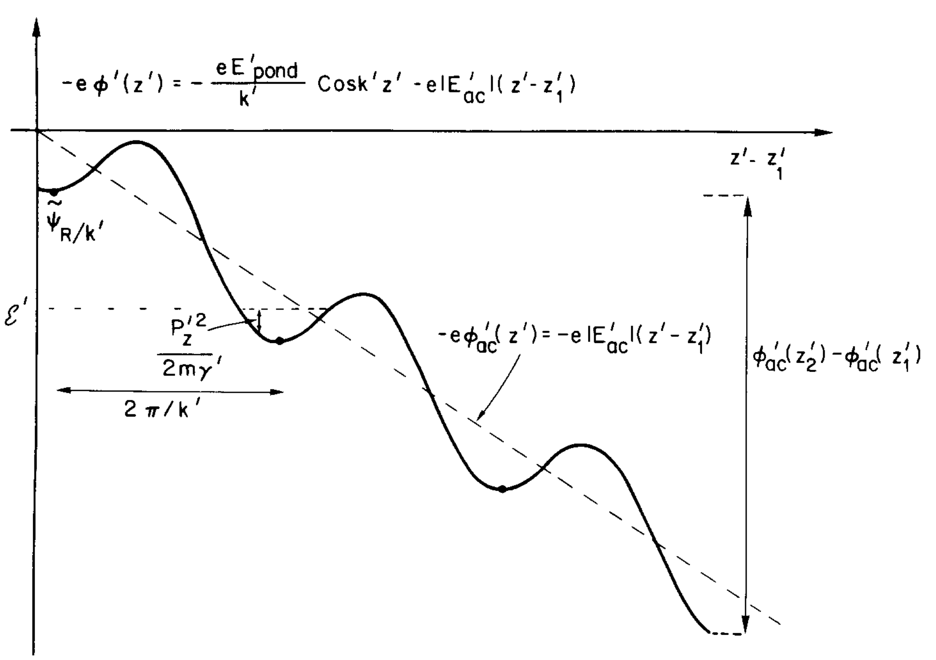

FIG. 2. Plot of the ponderomotive potential with an applied accelerating dc electric field.

bottom of the potential well. It will remain at constant phase with respect to the ponderomotive wave at a value ψ_R which corresponds to the location of the potential well minimum.

A third inequality which is required [in addition to (Eqs. 8, 9)] in order to keep the electrons trapped, and in order to validate our simple single electron interaction model is:

$$E'_{\rm sc} = \frac{e\, n'_{\rm trap}}{\epsilon_0 k'} << E'_{\rm pond}. \tag{10}$$

This simply states that the space charge field associated with the spatial bunching of the trapped electrons should be negligible in comparison to the ponderomotive field. The quantity $n'_{\rm trap}$ is the average density of trapped electrons in the moving frame.

The trapped electrons are seen in the laboratory frame to be traversing at a constant phase velocity across the potential drop $\phi_{\rm ac}(z_2) - \phi_{\rm ac}(z_1) = -\int_{z_1}^{z_2} E_{\rm ac}\,dz'$ applied along the interaction length, where $E_{\rm ac}$ is a constant or slowly varying (as a function of z) "accelerating" field. Since no kinetic energy has been added to the electrons, the full potential energy which was released, $e[\phi_{\rm ac}(z_2) - \phi_{\rm ac}(z_1)]$ is transformed into radiation field energy. The radiation process is, of course, governed by the Maxwell equations, which provide the formal justification for this conservation of energy argument.[17]

The power generated by the "accelerated" trapped electrons is therefore:

$$\Delta P = P(z_2) - P(z_1) = I_{\rm trap}\,[\phi_{\rm ac}(z_2) - \phi_{\rm ac}(z_1)], \tag{11}$$

where $I_{\rm trap}$ is the current intensity of trapped electrons. This radiative power generation scheme is quite different from the normal operation of free electron lasers. In this scheme the power for generating the radiation is supplied solely by the source of the accelerating potential and not by the kinetic energy of the electron beam. It is also instructive to point out the analogy between this electric to optical power conversion scheme and the electric to acoustic power conversion in the acoustoelectric effect when operating at the trapped electron mode.[21]

The amount of power which can be transformed from the dc accelerating potential source to the radiation field is limited essentially only by the trapping conditions (Eqs. 8–10) and the practical interaction length ℓ. The trapping conditions (Eqs. 8–10) can be expressed in terms of the laboratory frame parameter values by making the standard relativistic transformations. This results in the following inequalities:

$$E_{\rm ac} << E_{\rm pond} = \frac{e}{2\gamma_0 m}(k_w + k_s)|\tilde{\mathbf{A}}_w^* \cdot \tilde{\mathbf{A}}_s| \tag{12}$$

$$E_{\rm sc} = \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \frac{J_{\rm trap}}{\beta_{\rm oz} k} << E_{\rm pond} \tag{13}$$

$$\Delta\mathscr{E} \ll \mathscr{E}_{\rm trap} \equiv 2\sqrt{2\beta_{\rm oz}}\,\gamma_{\rm oz}\, c\, e(\tilde{\mathbf{A}}_w^* \cdot \tilde{\mathbf{A}}_s)^{1/2}, \tag{14}$$

where $\Delta\mathscr{E}$ is the beam axial energy spread in the laboratory frame, and $J_{\rm trap}$ is the current density of the trapped electrons. See Appendix II for derivation of Eq. (14).

The basic experimental design considerations are derivable from Eqs. 11–14. In order to obtain appreciable power generation, the trapped electron current $I_{\rm trap}$ and the "accelerating" potential drop $\phi_{\rm ac}(z_2) - \phi_{\rm ac}(z_1)$ should be maximized. The amount of trapped current which can be used is limited mainly by the total current which can be produced by the electron beam and propagated through the interaction region, as well as by the beam axial energy spread. If the beam energy spread is small compared to the trapping energy (Eq. 14), most of the beam electrons can be trapped. In the opposite case, only a portion of the electrons (roughly $\mathscr{E}_{\rm trap}/\Delta\mathscr{E}$) will be trapped. The untrapped electrons will be freely accelerated within the interaction region and will not contribute to increasing the radiative power. The magnitude of the accelerating potential drop that can be applied to the trapped electrons is limited by the interaction length $\ell = z_2 - z_1$ and the maximum field which can be applied without tilting the trapping potential wells to the point where the potential barriers diminish and the trapped electrons "spill out" (Eq. 12).

Bearing these considerations in mind, we note from (Eqs. 12, 14) that high initial field intensity of both the pump and signal radiations are essential for obtaining appreciable output radiation power in our scheme. This consideration requires strong focusing of the high power pump and signal laser beams to the smallest cross-section area through which the electron beam can be focused and propagated. However, unfortunately, when the longer-wavelength pump wave is focused in free space, it also diffracts and its field intensity falls off axially. This then permits only a short interaction length ℓ through which condition (Eq. 12) is satisfied and restricts the total potential drop $\phi_{\rm ac}(z_2) - \phi_{\rm ac}(z_1)$ along the interaction length. If the pump wave diffracts freely in free space, an optimal focusing condition can be found for obtaining maximum laser gain.[27] Another approach, which is proposed here, is to guide the pump radiation in a hollow dielectric (metallic) waveguide (Fig. 1), which has relatively small propagation losses when the waveguide cross-section size is large compared to a wavelength.[18–20] This method permits the pump field intensity to be kept high and unattenuated for a long interaction length.

Consider a case where the pump radiation is coupled to the EH_{11} mode of a circular waveguide[18]:

$$\tilde{\mathbf{E}}_w = \hat{e}_x E_w(0) J_0\left(u\frac{r}{a}\right), \tag{15}$$

where $2a$ is the waveguide diameter, and $u = 2.405$ is the first root of Bessel function J_0. The power attenuation of this mode is

$$2\alpha_{11} = 2\left(\frac{u}{2\pi}\right)^2 \frac{\lambda_w^2}{a^3}\,\mathrm{Re}\,\frac{\frac{1}{2}(\nu^2+1)}{(\nu^2-1)^{1/2}}, \tag{16}$$

where ν is the complex index of refraction of the waveguide material. We can see immediately that very low attenuation can be obtained with an oversized waveguide $(a/\lambda_w >> 1)$. Furthermore, it has been shown[23] that the EH_{11} mode can be excited in the waveguide end by an appropriately focused Gaussian wave with very high efficiencies (98%).

Assuming that the signal wave diffraction is negligible, the ponderomotive field and trapping potential will remain constant along the interaction length. For simplicity we also assume that the ratio between the applied "accelerating" field E_{ac} and the ponderomotive field E_{pond} along the interaction length is constant.

$$\sin \tilde{\psi}_R = \frac{E_{ac}}{E_{pond}} = \text{const} < 1. \tag{17}$$

In this case the phase $\tilde{\psi}_R$ of an electron fully trapped in the trough of the potential well remains constant (see Fig. 2). The maximum potential drop along the interaction length ℓ is then

$$\phi_{ac}(z_2) - \phi_{ac}(z_1) = E_{pond}(0)\ell \sin \tilde{\psi}_R. \tag{18}$$

The ponderomotive field amplitude $E_{pond}(0)$ can be calculated from (Eq. 5) using the values of the pump and signal field intensities on the waveguide axis. The pump electric field intensity of the EH_{11} mode $E_w(0)$ is related to the mode power P_w by the relation,

$$P_w = J_1^2(u)\frac{\pi a^2}{2}\left(\frac{\epsilon_0}{\mu_0}\right)^{1/2}|E_w(0)|^2 = 0.135\pi a^2\left(\frac{\epsilon_0}{\mu_0}\right)^{1/2}|E_w(0)|^2. \tag{19}$$

In considering specific examples we should, of course, consider extremely powerful lasers for producing both the pump and signal radiation. For producing high-power tubeable visible radiation an obvious choice for the pump wave would be an intense pulsed CO_2 laser ($\lambda_w = 10.6 \mu$m) and for the signal wave an intense flash lamp pumped pulsed dye laser.

To calculate the electron beam energy required for the interaction we use the synchronism (resonance) condition $(\omega_s - \omega_w)/(k_s + k_w) \simeq v_{oz}$, or

$$\frac{\lambda_w}{\lambda_s} = (1 + \beta_{oz})^2 \gamma_{oz}{}^2. \tag{20}$$

For $\lambda_w = 10.6\,\mu$m and $\lambda_s = 0.5\,\mu$m we get $\gamma_{oz} \simeq \gamma_0 = 2.3$ ($\mathscr{E}_k = (\gamma_0 - 1)\,mc^2 = 665$ Kev). We assume for the pump power $P_w = 10$ GW and for the signal power $P_s = 10$ MW. These waves are launched into a $2a = 8$-mm-diam copper tube. The pump wave is focused appropriately to exicte the EH_{11} mode (Eq. 15). The signal wave is assumed to be a freely diffracting Gaussian wave,[22] which is focused to a $2w_{os} = 4$-mm beam waist size. This corresponds to a Rayleigh length $2z_{os} = 2\pi w_{os}^2/\lambda_s = 50$ m which justifies the neglect of the signal wave diffraction along the interaction length. The Gaussian beam field on axis is related to the beam power through the relation

$$P_s = \frac{\pi w_{os}^2}{4}\left(\frac{\epsilon_0}{\mu_0}\right)^{1/2}|\tilde{\mathbf{E}}_s(0)|^2 \tag{21}$$

This equation and Eq. 19 give for the signal and wiggler fields on axis $|\tilde{\mathbf{E}}_s(0)| = 3.46 \times 10^7$ v/m and $|\mathbf{E}_w(0)| = 7.46 \times 10^8$ v/m. Using (Eq. 5) we find $E_{pond}(0) = 9.26 \times 10^3$ v/m and assuming an interaction length $\ell = 10\,m$ and $\sin \tilde{\psi}_R = 1/2$ we obtain from (Eq. 18) $\phi_{ac}(z_2) - \phi_{ac}(z_1) = 46.3$ KV.

The trapping potential energy for this case (Eq. 14) is $\mathscr{E}_{trap}(0)/(\gamma_0 - 1)\,mc^2 = 5.8 \times 10^{-4}$ ($\mathscr{E}_{trap}(0) = 383$ eV). If we assume $I_{trap} = 1$ KA, we find for this case $\Delta P \simeq 46.3$ MW, $\Delta P/P = 4.63$.

The power attenuation of the pump wave due to losses in the waveguide walls are found to be sufficiently small for the parameters of this example. Assuming $\nu = 14.2 - i64.5$ for a copper tube,[24] Eq. (16) results in a power attenuation constant $2\alpha_{11} = 3.65 \times 10^{-3}\ \text{m}^{-1}$, which for the $\ell = 10$-m length will reduce the pump power by $\Delta P_w/P_w(z_2) = 3.6\%$. The power dissipation on the waveguide walls is 1.45×10^5 W/cm^2, which for a pulse duration of say 300 nsec, corresponds to an acceptable energy dissipation level of 1.37 J/cm^2.

The example given above does not necessarily represent the optimal or the easiest configuration to realize the proposed dc to visible power conversion scheme. It merely demonstrates the potential as well as difficulties which should be expected in an experimental realization of such a scheme.

Perhaps the most difficult restriction in the above example is the low values of energy spread required of the electron beam. These values would require improvements in the state of the art beam emittance values, and the prevention of space charge potential depression inside a non-neutralized beam by means of Brillouin flow[25] or other techniques.

It is apparent from inequalities (Eq. 12–14) that very high-power densities of both the pump and signal waves are desirable in order to increase the ponderomotive field intensity E_{pond} and the trapping potential depth $\mathscr{E}_{trap}$, so that high optical power generation can be realized. Small increases in the ponderomotive field and the trapping potential depth occur along the interaction length ℓ due to the signal wave amplification. This effect, which was neglected in the example will not, however, relax the design constraints significantly. One way in which the power densities in the interaction region can be increased without increasing the power input into the pump and signal lasers, is by incorporating the interation set up (Fig. 1) inside the cavity of the pump or signal lasers, or both. If high Q cavities are employed it is possible to build up the laser power densities in the interaction region to extremely high levels prior to the injection of the electron beam. This alternative scheme will be limited by the absorption losses and power handling capabilities of existent mirrors and optical components.

High-input radiation power levels associated with both the pump and signal waves are possible with short pulse operation. However, it should be noticed that the pump radiation pulse duration τ_w should be long enough to fill up the interaction region for as long as it takes the signal radiation pulse (of τ_s duration) to traverse the interaction region ($\tau_w > 2\ell/c + \tau_s$.) A possible mode of operation is one in which the "free electron laser" is operating as an "energy amplifier" or "pulse train oscillator." In this mode of operation a very high-power (P_s) short-duration (τ_s) signal pulse is injected into the interaction region and partly reflected back (after amplification) into the interaction region by means of a Fabri-Perot cavity of length ℓ_c. The initial signal pulse will then produce a train of N pulses ($N = c\tau_w/2\ell_c$). If the pump wave pulse can be kept long enough in the cavity, appreciable energy gain $\Delta\mathscr{E}_s/(P_s\tau_s) \simeq N\Delta P_s/P_s$ can be obtained even if the single pass power gain $\Delta P_s/P_s$ is small.

Finally, we point out some of the limitations of the theoretical model assumed in the proposed scheme. The neglect of the nonsynchronous driving terms during the derivation of Eq. (4) from Eq. (3) is a very good approximation. Even though the electromagnetic fields of the pump and signal waves apply transverse forces on the electrons in the first-order solution of the force Eq. (3), the oscillations due to these forces (at frequencies ω_s, ω_w) are very fast, and consequently the amplitude of these oscillations is found to be negligible. In a Fabri-Perot resonator oscillator structure one gets in second order also axial ponderomotive forces due to the beating of the right- and left-going pump waves or right- and left-going signal waves or right-going pump wave with left-going signal wave. The first two pairs produce static periodic ponderomotive potentials (in the laboratory frame of reference) and the third pair produces a travelling potential which propagates to the left with velocity $\sim(-v_{oz})$ and frequency $\omega_s - \omega_w$. All these fields produce axial oscillation of negligible amplitude. They could in principle also be completely eliminated in a ring laser structure.

Other theoretical concerns which require further consideration and are beyond the scope of the present article are the effects of sideband instability[24] and finite monochromaticity or finite coherence of the pump and signal waves.[26]

This research is supported in part by the U. S. Air Force Office of Scientific Research under Grant No. AFOSR 80-0073.

APPENDIX I.

The longitudinal force Eq. (3) is derived directly from the Lorenz force equation

$$d\mathbf{p}/dt = -e(\mathbf{E} + \mathbf{v}\times\mathbf{B}), \tag{I-1}$$

and the definitions of the electric scalar potential ϕ and magnetic vector potential $\mathbf{A}$

$$\mathbf{B} = \nabla\times\mathbf{A}, \tag{I-2}$$

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}. \tag{I-3}$$

The electron velocity is given in general by

$$\mathbf{v} = (\mathbf{p}_c + e\mathbf{A})/\gamma m, \tag{I-4}$$

where $\mathbf{p}_c$ is the canonical momentum and

$$\gamma = [1 + (\mathbf{p}_c + e\mathbf{A}])^2/(m^2c^2)^{1/2}. \tag{I-5}$$

Substituting (I-2) to (I-4) in (I-1) for the particular case of zero transverse canonical momentum ($\mathbf{p}_{c_\perp} = 0$) gives for the axial component

$$\frac{dp_z}{dt} = e\frac{\partial\phi}{\partial z} - e\frac{\partial A_z}{\partial t} - \frac{e^2}{\gamma m}\left[\frac{1}{2}\frac{\partial}{\partial z}\mathbf{A}^2 - (\mathbf{A}\nabla)A_z\right]. \tag{I-6}$$

For a purely transverse vector potential $A_z = 0$

$$\frac{dp_z}{dt} = e\frac{\partial\phi}{\partial z} - \frac{e^2}{2\gamma m}\frac{\partial}{\partial z}\mathbf{A}^2, \tag{I-7}$$

where $\gamma = \left[1 + \frac{1}{m^2c^2}p_z^2 + \frac{e^2}{m^2c^2}\mathbf{A}^2\right]^{1/2}$.

APPENDIX II.

In this appendix we will derive the trapping condition in the laboratory frame, Eq. (14), from the trapping condition in the frame moving at the phase velocity of the ponderomotive wave, Eq. (9). The allowable energy spread in the moving frame is the same as the full depth (trough to peak) of the ponderomotive potential well (Eq. 9)

$$\frac{p_z'^2}{2\gamma' m} = \frac{1}{2}mc^2\gamma'\beta_z'^2 << \mathscr{E}'_{\text{trap}} = 2\frac{E'_{\text{pond}}}{k'} = \frac{e^2}{m\gamma'}|\widetilde{\mathbf{A}}_s^*\cdot\widetilde{\mathbf{A}}_w|, \tag{II-1}$$

where $k' = (k_w + k_s)/\gamma_{\text{ph}}$; $\gamma_{\text{ph}} = (1-\beta_{\text{ph}}^2)^{-1/2}$; and β_{ph} is the phase velocity of the ponderomotive wave. It is assumed that this velocity is equal to the average velocity of the electron beam $\beta_{\text{ph}} = \beta_{oz}$, $\gamma_{\text{ph}} = \gamma_{oz}$.

In the moving frame the maximum allowable velocity of the trapped electrons is

$$|\beta_z'| = \left(\frac{2\mathscr{E}'_{\text{trap}}}{mc^2\gamma'}\right)^{1/2}. \tag{II-2}$$

To find the corresponding energy spread in the laboratory frame we use the Lorentz transformation for the energy

$$\frac{\mathscr{E}}{c} = \gamma mc = \gamma_{\text{ph}}(\gamma' mc + \beta_{\text{ph}}p_z'),$$
$$\gamma = \gamma_{\text{ph}}\gamma'(1 \pm \beta_{\text{ph}}|\beta_z'|). \tag{II-3}$$

For $|\beta_z'| = 0$ this equation results the identity $\gamma_0' = \gamma_0/\gamma_{oz}$.

With $|\beta_z'| << \beta_{\text{ph}}$ we find that the deviation in laboratory frame beam energy which corresponds to condition (II-2) is

$$|\Delta\gamma| = |\gamma - \gamma_0| = \gamma\beta_{\text{ph}}\left(\frac{2\mathscr{E}'_{\text{trap}}}{\gamma' mc^2}\right)^{1/2}, \tag{II-4}$$

Defining

$$\mathscr{E}_{\text{trap}} = 2|\Delta\gamma|mc^2, \tag{II-5}$$

and substituting Eqs. (II-4), (II-1) and $\beta_{\text{ph}} = \beta_{oz}, \gamma_{\text{ph}} = \gamma_{oz}$ we get the trapping condition in the laboratory frame

$$\Delta\mathscr{E} << \mathscr{E}_{\text{trap}} = 2\sqrt{2\gamma_{oz}}\beta_{oz}ce\sqrt{|\widetilde{\mathbf{A}}_w^*\cdot\widetilde{\mathbf{A}}_s|}. \tag{II-6}$$

[1]L. R. Elias, W. Fairbank, J. Madey, H. A. Schwettman, and T. Smith, Phys. Rev. Lett. **33**, 717 (1976).
[2]F. A. Hopf, P. Meystre, M. O. Sully, and W. H. Louisell, Opt. Commun. **18**, 413 (1976).
[3]N. M. Kroll and W. A. McMullin, Phys. Rev. A**17**, 300 (1978).
[4]W. B. Colson, *Physics of Quantum Electronics Vol. 5*, edited by S. Jacobs, M. Sargent, III, and M. Scully (Addison-Wesley, New York, 1978), p. 157.
[5]T. Kwan, J. M. Dawson, and A. T. Lin, Phys. Fluids **20**, 581 (1977).
[6]A. Hasegawa, Bell Syst. Tech. J. **57**, 3069 (1978).
[7]A. Bambini, A. Renieri, and S. Stenholm, Phys. Rev. A **19**, 2013 (1979).
[8]I. Bernstein and J. L. Hirshfield, Phys. Rev. A **20**, 1661 (1979).
[9]P. Sprangle and R. Smith, Phys. Rev. A **21**, 293 (1980).
[10]R. H. Pantell, G. Soncini, and H. E. Puthoff, IEEE J. Quantum Electron. **4**, 905 (1968).
[11]V. P. Sukhatme and P. W. Wolff, J. Appl. Phys. **44**, 2331 (1973).
[12]P. Sprangle and A. T. Drobot, J. Appl. Phys. **50**, 2652 (1979).
[13]P. Sprangle, C. M. Tang, and W. M. Manheimer, Phys. Rev. A **12**, 302 (1980).
[14]N. M. Kroll, P. L. Morton, and M. N. Rosenbluth, *Physics of Quantum Electronics Vol. 7* edited by S. Jacobs, H. Piloff, M. Sargent, III, and M. Shully (Addison-Wesley, New York, 1980).
[15]A. T. Lin, Phys. Fluids **24**, 316 (1981).

[16]Y. P. Ho, Y. C. Lee, and M. N. Rosenbluth, Sherwood Meeting, Tucson, Arizona, April 23–25 (1980).
[17]P. Sprangle and C. M. Tang, NRL Memo Report 4280 (1980), accepted for publication in the AIAA J (1981).
[18]E. A. Marcatili and R. A. Schmeltzer, Bell Syst. Tech. J. **43**, 1783 (1964).
[19]N. Nishihara, T. Inoue, and J. Koyama, Appl. Phys. Lett. **25**, 391 (1974).
[20]E. Garmire, T. McMahon, and M. Bass, Appl. Opt. **15**, 145 (1976).
[21]A. Rose, Series of Articles in RCA Rev **27**, 98 (1966); **27**, 600 (1966); **28**, 634 (1967); **30**, 435 (1969); **32**, 463 (1972).
[22]A. Yariv, *Introduction to Optical Electronics* (Holt, Rinehart and Winston, New York 1976).
[23]R. L. Abrams, IEEE J. Quantum Electron. **8**, 838 (1972).
[24]A. P. Lenham and D. M. Terhene, J. Opt. Soc. Am. **56**, 683 (1966).
[25]J. D. Lawson, *The Physics of Charged Particle Beams* (Carendon, Oxford, 1977).
[26]A. Gover and P. Sprangle, J. Appl. Phys. **52**, 599 (1981).
[27]A. Gover, C. M. Tang, and P. Sprangle, Design Considerations of a Compton Scattering Free Electron Laser with an Axial Electric Field. In *Physics of Quantum Electronics Vol. 9*, edited by S. Jacobs, H. Piloff, M. Sargent, III, and M. Scully, (Addison-Wesley, New York, to be published in 1982).
[28]H. R. Hiddleston and S. B. Segal, *Physics of Quantum Electronics Vol. 9* (edited by S. Jacobs, H. Piloff, M. Sargent, III, and M. Scully), Addison-Wesley, New York 1982) and in J. Quant. Elecs. **QE-17**, Aug. (1981).