The Nonlinear Interaction Between an Electron and Multimode Fields in an Electromagnetically Pumped Free Electron Laser

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Abstract—We consider the motion of an electron under the influence of two counterpropagating multimode electromagnetic waves and a longitudinal electrostatic field. General equations of motion are presented from which a simple wavelength scaling condition is deduced for minimizing detrapping effects. An interaction coherence time or a maximum useful interaction length parameter is derived. The full equations of motion are numerically solved for a variety of external and initial conditions.

INTRODUCTION

THE idea of the utilization of an electromagnetic wave as a pump source in a free electron laser was proposed years ago by Pantell [1]. Among the attractive features of this configuration are the availability of high intensity sources (lasers) transverse uniformity of the wiggler field [2], short pump wavelengths and consequently, short operating wavelength. The main disadvantages of the utilization of short wavelength pump sources are lower efficiency, and the stringent beam quality requirements on the electron beam in the linear regime [2].

The above mentioned limitations can however be surpassed once efficiency enhancing mechanisms are considered in the nonlinear regime [3]-[5]. In these schemes an electron, trapped in the ponderomotive potential, is made to radiate efficiently into the signal field by applying an axial electrostatic field. This field performs work on a trapped electron without accelerating it, transferring the invested energy to the signal mode. Such schemes are of special relevance for experimental proposals where the pump is a pure electromagnetic wave and a strong signal field is already present to create a strong trapping potential [7], [16], and in particular, in two stage FEL configurations [6], [11], [12].

The scope of the model treated here, however, goes beyond the specific case of an electromagnetically pumped FEL. As is well known, the problem of a static magnetic or electric wiggler can also be treated in the extreme relativistic limit in terms of an equivalent electromagnetic field [8], [9].

In any frame of reference which moves at such relativistic velocities the magnetic (electric) field appears almost as an electromagnetic wave. In the frame of the reference which moves at the velocity of the ponderomotive force, the ponderomotive force becomes static, allowing the definition of a ponderomotive potential. On the same basis the equivalence between two proposed efficiency enhancement mechanisms can be shown: namely the tapering of the wiggler period and amplitude [13], [14] and the introduction of a longitudinal accelerating field. In the tapered period case a trapped electron slows down as the wiggler period is shortened; its frame of reference is no longer inertial and a forward directed D'Alambertian force is felt by the electron. In the case of an axial electric field the force is real while the ponderomotive frame of reference remains inertial. A demonstration of this equivalence is presented in the Appendix.

The presence of many modes of different frequencies in the wiggler or pump fields changes some fundamental aspects of the interaction. A ponderomotive potential cannot be straightforwardly defined any more since there exists no frame of reference where the longitudinal force can be made static. If such a potential is defined for one pair of counterpropagating modes, the other modes will give rise to slowly time varying forces whose net effect can be the detrapping or retrapping of electrons. If a longitudinal force is present, the detrapping mechanism will be dominant, since any detrapped electron will be accelerated by that force, away from resonance conditions. The overall efficiency of the FEL is therefore expected to diminish as the number of participating modes increase. The presence of several modes may be an inherent feature of high intensity sources (lasers) or may arise spontaneously in the signal wave in an oscillator configuration. This effect, which may be of practical concern in some free electron laser experiments has received so far only little attention [15].

In the present paper we consider the motion of an electron under the influence of counterpropagating multimode fields and an accelerating longitudinal electric field. General equations of motion are presented and a simple "wavelength scaling" condition is found for minimizing the detrapping effect. Under these conditions an interaction coherence time of a maximum useful coherence length parameter is derived. The nonlinear equations of motion are numerically solved showing the gradual detrapping of electrons with time for multimode interaction.

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Equations of Motion

In this section we derive the equations of motion for an electron interacting with two counterpropagating multimode electromagnetic waves and an additional electrostatic longitudinal field. This basic interaction scheme is shown in Fig. 1. No highly relativistic approximations will be made since we wish the results to be relevant to experimental situations where moderately energized electron beams are used [6], [7].

We start with the Lorentz force equation as follows:

$$\frac{dP}{dt} = -e(\vec{E} + \vec{v} \times \vec{B}), \tag{1}$$

that can be expressed by means of the vector and scalar potentials A and ϕ, in the form

$$\frac{d\vec{P}}{dt} = e\left[\frac{\partial \vec{A}}{\partial t} + \vec{\nabla}\phi - \frac{1}{\gamma m}(\vec{P}_c + e\vec{A}) \times (\vec{\nabla} \times \vec{A})\right]. \tag{2}$$

The velocity has been expressed here by means of the canonical momentum p_c. After expanding the vector product of the vector A and its curl we obtain

$$\frac{d\vec{p}}{dt} = e\left[\frac{\partial \vec{A}}{\partial t} + \nabla\phi - \frac{1}{\gamma m}\vec{P}_c \times (\vec{\nabla} \times \vec{A}) - \frac{e}{\gamma m}\left[\frac{1}{2}\vec{\nabla}\vec{A}^2 - (\vec{A}\vec{\nabla})\vec{A}\right]\right\}. \tag{3}$$

Since we aim towards collinear plane waves, we can choose z as the direction of propagation, explicitly

$$\vec{A} = \vec{A}(z, t),\ \phi = \phi(z). \tag{4}$$

Here, A means there is no $\hat{z}$ component to the vector potential. The conservation of the transverse canonical momentum p_c follows straightforwardly from (4) and a simplified equation for P_z is obtained.

$$\frac{dP_z}{dt} = \frac{\partial\phi}{\partial z} - \frac{e^2}{2\gamma m}\frac{\partial}{\partial z}(\vec{A}^2) \tag{5}$$

where we made use of the initial condition $p_c(0) = 0$. We introduce now the explicit form of the field as the sum of two counterpropagating waves

$$\vec{A} = \mathrm{Re}\,(\vec{A}_w + \vec{A}_s)$$

$$\vec{A}_w = \sum_{m=-M}^{M} \vec{A}_{wm} e^{i\psi_{wm}},\quad \vec{A}_s = \sum_{n=-N}^{M} \vec{A}_{sn} e^{i\psi_{sn}} \tag{6}$$

where w and s stand for wiggler and signal fields, and the total number of modes is $2M + 1$ and $2N + 1$, respectively. The phases of the forward going signal wave and backward going wiggler wave are given by

$$\psi_{sn} = -\omega_{sn}t + k_{sn}z + \psi_{osn}\ (+z \text{ direction})$$

$$\psi_{wm} = -\omega_{wm}t - k_{wm}z + \psi_{owm}\ (-z \text{ direction}). \tag{7}$$

We now introduce our first explicit approximation by assuming that an electron will interact significantly only with forcing terms which move close to its velocity and thus neglect all the terms which propagate with phase velocity larger than the speed of light, and thus disregard all contributions to (5) having phase velocity equal or greater than c. Now, inserting (6) and (7) into (5), we obtain

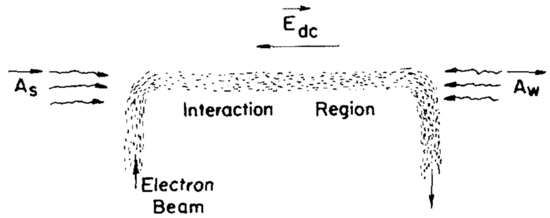


Fig. 1. Interaction scheme of two counterpropagating electromagnetic waves and an electron beam, in the presence of a longitudinal electrostatic field.

$$\frac{dP_z}{dt} = -e\sum_{n=-M}^{M}\sum_{m=-M}^{M} E^p_{nm} \cdot \sin(k_{nm}z - \omega_{nm}t + \psi_{onm}) - eE_{\mathrm{dc}} \tag{8}$$

where E^p_{nm} are the ponderomotive field amplitudes

$$E^p_{nm} = (2\gamma m)^{-1}\, ek_{nm}\vec{A}_{sn} \cdot \vec{A}_{wm} \quad \text{and} \quad E_{\mathrm{dc}} = \frac{\partial\phi}{\partial z} \tag{9}$$

$$\omega_{nm} = \omega_{sn} - \omega_{wm} \qquad k_{nm} = k_{sn} + k_{wm}$$

$$\psi_{onm} = \psi_{osn} - \psi_{owm}.$$

We observe here that the ponderomotive waves created by the multimode wiggler and signal fields propagate at velocities $v_{nm} = (\omega_n - \omega_m)/(k_n + k_m)$, which can be significantly smaller than the speed of light. These velocities, related to different mode pairs are in general different from each other. An electron launched at a velocity v_{nm} will see a static force due to the corresponding ponderomotive field. Other terms in (8), however, will still be time dependent in the electron frame of reference and will slowly affect its trajectory. It should be remembered that when a dc longitudinal field is applied as an efficiency enhancing mechanism, the fact that the electrons are continuously trapped is crucial: only then can the dc field perform work on them which it totally transferred into the radiation fields [5]. The multimode perturbation may act then as a detrapping agent which lowers the general efficiency. In the next section we show that this perturbation can be minimized by fulfilling a proper "scaling condition" on the wavelengths of the fields involved.

Further Considerations Regarding Multimode Interaction

A. Frequency Scaling Condition

Assume now that an electron is drifting with a velocity close to resonance with the ponderomotive field E^p_{11}, i.e.,

$$v \simeq v_{ph_{11}} \equiv \frac{\omega_{1s} - \omega_{1w}}{\omega_{1s} + \omega_{1w}}\, c. \tag{10}$$

As stated before this electron will feel in a frame of reference moving with velocity v_{ph11} a static force as a consequence of the modes of frequency ω_{1s} and ω_{1w}. Assume now a second pair of modes is present with frequencies $\omega_{2s} = \omega_{1s} + \Delta\omega_s$ and $\omega_{2w} = \omega_{2w} + \Delta\omega_w$. The frequency difference between the two additional waves in the electron inertial frame will be

$$\Delta\omega' = \gamma(\omega_{1s} + \Delta\omega_s)(1-\beta) - \gamma(\omega_{1w} + \Delta\omega_w)(1+\beta)$$
$$= \gamma(\Delta\omega_s - \Delta\omega_w) - \gamma\beta(\Delta\omega_s + \Delta\omega_w). \quad (11)$$

It is apparent that the frequency difference in the moving frame is smaller than that measured in the laboratory frame. If we control the frequency difference $\Delta\omega_s$ and $\Delta\omega_w$, we may be able to satisfy $\Delta\omega' = 0$, implying

$$\frac{\Delta\omega_s}{\Delta\omega_w} = \frac{1+\beta}{1-\beta} = \frac{\omega_s}{\omega_w}. \quad (12)$$

Fulfillment of this frequency scaling condition means that the electron will feel a static force from the ponderomotive field arising from the modes at frequencies ω_{2w} and ω_{2s} also. For multimode laser electromagnetic sources, $\Delta\omega_s$ and $\Delta\omega_w$ are constant so that the scaling conditions can be fulfilled simultaneously for all pairs of modes with equal mode indexes ($n = m$). The total number of pairs that can be made to correspond is of course not greater than min($2N$, $2M$). The frequency difference $\Delta\omega$ between adjacent modes depends for a laser source mainly on its length L:

$$\Delta\omega = \frac{\pi c}{n^r L} \quad (13)$$

where n^r is the refraction index of the laser gain medium. The scaling condition (12) can be then simply reformulated:

$$\frac{\lambda_w}{\lambda_s} = \frac{n_w^r L_w}{n_w^r L_s}. \quad (14)$$

In other words "the lasers cavity lengths should be made proportional to the respective wavelengths." This condition can be implemented in a variety of existing laser sources.

B. The Detrapping Time

Our purpose now will be to find an expression for the detrapping time τ^D due to the presence of several modes in the wiggler and signal fields. It is intuitively apparent that this detrapping time should be related to the coherence time of the laser souces involved. For this purpose it is convenient to write the force equation (8) in the frame of reference moving with a velocity given by (10). After performing the fully relativistic transformation and inserting scaling conditions (12) we find the following expression:

$$\frac{dP_z'}{dt} = -\sum_{n=-N}^{M}\sum_{m=-M}^{M} E_{nm}^p \cdot \sin\{\gamma(k_{oo}(1-\beta^2) + \Delta k_s(1-\beta)(n+m)z' - \gamma(\Delta\omega_s(1-\beta)(n-m)t' + \psi_{onm}'\} - eE_{dc} \quad (15)$$

where we used the fact that the modes frequencies are equally spaced, i.e.,

$$k_{mn} = k_{oo} + n\,\Delta k_s + m\,\Delta k_w$$
$$\omega_{mn} = \omega_{oo} + n\,\Delta\omega_s - m\,\Delta\omega_w.$$

We also applied here the Lorentz invariance of the longitudinal amplitudes E_{nm}^p. From (15) it is clear that when frequency scaling condition (14) is satisfied, a time independent force is obtained for all terms with $m = n$. There is however a difference in the wavevector values of these terms which introduce a spatial amplitude modulation of the resultant ponderomotive potential. When the length of the source lasers is larger than the interaction length L_{int}, then $2\pi/\Delta k_s >> L_{\text{int}}$ and this spatial variation is negligible. In the special case when the source lasers are mode locked, the phases of different ponderomotive potentials ψ_{onm} are identical and they sum up coherently to produce an intense and narrow resultant ponderomotive potential region. Notice though that we still have the mixed terms $m \neq n$ and they can still cause electron detrapping.

The detrapping time for a given pair of modes (m, n) will be defined as the time required to dephase the corresponding argument in (15) by $\pi/2$, namely

$$(\tau_{mn}^D) = \pi/[2\gamma\Delta\omega_s(1-\beta)(n-m)] \quad (16)$$

or, returning to the laboratory frame,

$$\tau_{mn}^D = \pi/[2\Delta\omega_s(1-\beta)(n-m)] = \pi/[2\Delta\omega_w(1+\beta)(n-m)] \quad (17)$$

which is the desired expression for detrapping time independently of whether condition (14) is satisfied or not. We observe here that if the wiggler and signal fields have the same number of modes, and we insert in the last expression the maximum value of $n - m$ (namely $2N$), the detrapping time will be larger than the signal coherence time, but shorter than that of the wiggler. This estimate of the detrapping time is basically a lower limit estimate assuming worst detrapping conditions. The true detrapping time is of course a result of the contributions of all pairs (n, m) with the corresponding weights E_{nm}^p. In the detrapping process the additional axial electric field will also play a major role. The total influence of these factors will be taken into account in the next section where the full nonlinear equations are solved numerically.

C. Trapping Criterion

In this section we derive an approximate trapping criterion in a situation where several modes are present in the electromagnetic fields. Unlike the single mode case no "trapping bucket" can be drawn in the phase space since forces are explicitly time dependent in all frames of reference. We develop here an energy spread acceptance expression which has two practical meanings. In the case of a monoenergetic electron beam it will evaluate the spread in energy of trapped electrons caused by the interaction, and in the case when the beam has a finite temperature (initial energy spread) it will reflect the admissible spread in energies for a significant trapping fraction to be expected.

Start with a single mode case. The field can then be integrated over the distance to obtain a time independent potential in the moving frame of reference:

$$V' = V_p'(1 - \cos k_{oo}' z') \quad (18)$$

where

$$V_p' = E_{oo}^p / k_{oo}' \quad k_{oo}' = \gamma^{-1} k_{oo}.$$

The maximum velocity spread that electrons initially at rest can acquire due to the interaction, assuming a nonrelativistic spread will be

$$\Delta v_s' = 4(eV_p'/m)^{1/2}. \tag{19}$$

Back in the laboratory frame, this velocity spread will imply a spread in the electron beam energy

$$eV = mc^2(\gamma - 1). \tag{20}$$

Differentiating this last expression with respect to velocity, assuming $\gamma_z \simeq \gamma$ and inserting $\Delta v_s = \gamma^{-2} \Delta v_s'$ one obtains

$$\Delta V_s = 4(\gamma + 1)^{1/2} (VV_p')^{1/2}. \tag{21}$$

We finally postulate that in a multimode case with random mode phases, these energies will sum up in the average like an incoherent process yielding

$$\Delta V_s = 4(\gamma + 1)^{1/2} V^{1/2} \sum_{n,m} (E_{mn}^p/k_{nm}')^{1/2}. \tag{22}$$

This would be the statistically averaged energy acceptance and the beam energy spread caused during trapping, assuming of course that the interaction time is shorter than the detrapping time (12).

Numerical Solution of Equations of Motion

Equation (8) was solved numerically for the time varying ponderomotive field by means of a sixth order predictor-corrector routine. The calculations were performed each time for 20 particles of different initial phases (launching times) uniformly distributed. All the runs had the following data in common.

Totol pump source intensity = 10 MW

Total signal source intensity = 10 MW

$E_{dc} = -585$ V/m

$\lambda_w = 10.8\ \mu$m

$\lambda_s = 9.2\ \mu$m

Interaction length: 10 cm

Beam cross section 0.1 cm^2

$\beta = 0.08$.

These data are based on the parameters of an on going trapping investigation experiment [16]. In this experiment the radiation sources are two pulsed CO_2 lasers tuned at two different transition lines. The maximum attainable ponderomotive field is in this case 2.3 kV/m (single mode case). Results are presented for the cases of 1, 3, and 5 modes in each of the interacting radiation beams. In the multimode case the mode intensity ratios were 2:3:2 and 2:3:4:3:2, respectively. In the multimode cases the mode field phases were picked up randomly.

A. Final Electron Energy Distribution

The electron energy distribution after the interaction is shown in Fig. 2 for electrons starting the interaction with different initial phases. One observes here that the presence of many modes not only decreases the number of trapped particles, but also makes the distinction between trapped and untrapped electrons more gradual.

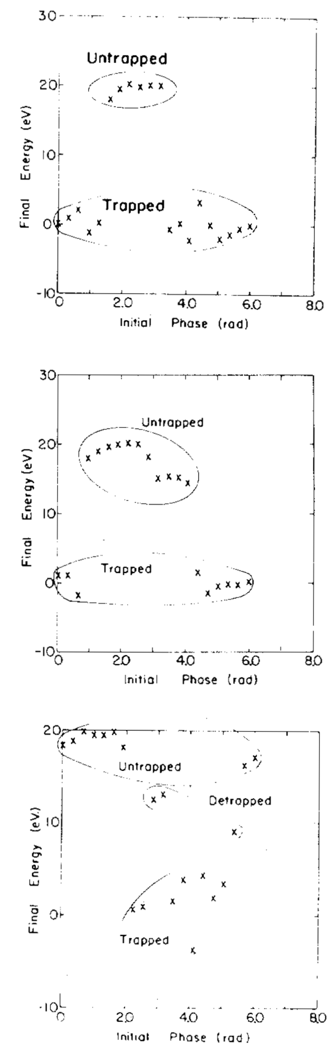


Fig. 2. Energy distribution of electrons after 3 cm of interaction as a function of the initial phases, for different number of modes in the electromagnetic fields. Upper graph: one mode. Center graph: three modes. Lower graph: five modes.

B. Exit Times (Bunching Effect)

The trapped electrons dwell mainly in the ponderomotive potential wells. Once a dc field is applied, the untrapped electrons are stripped away creating gaps in the trapped electrons axial distribution. This feature can be seen in Fig. 3, where the relative exit times are shown. Only trapped electrons times are plotted. The untrapped electron times are much shorter, out of the drawn scale.

C. Overall Trapping Efficiency

Fig. 4 shows clearly the effect that many modes have on the trapping process: the trapping fraction gradually decreases with the interaction distance. The effect of the relative phases of the different modes is also seen here. These initially random phases determine the trapping fraction at the beginning of the process. The detrapping times are significantly larger

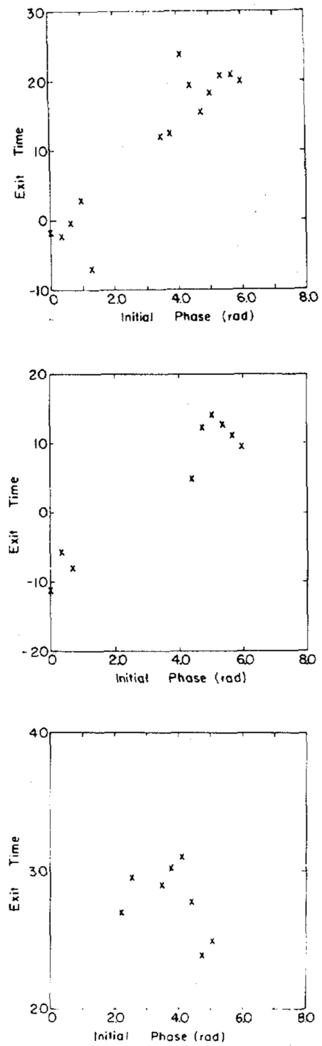


Fig. 3. Exit times of trapped electrons entering the interaction region with different phases. Time is measured relative to the exit time of the main ponderomotive wave, in units of 10^{-14} s. Upper graph: one mode in the fields. Center graph: three modes. Lower graph: five modes.

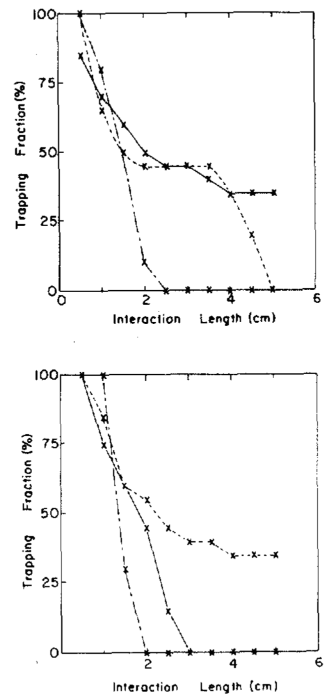


Fig. 4. Trapping fraction as a function of interaction length for three mode (upper graph) and five mode (lower graph) fields. Different curves correspond to different initial phase distributions.

than the predictions based on the lower limit expression (17). This is explained mainly because scaling condition (12) was assumed in the simulation that ensuring that the $2N + 1$ ponderomotive field terms are temporarily synchronized.

D. Detuning Effect

All the preceding results assumed a synchronous cold beam, i.e., a beam with energy spread small compared to ΔV_s and a velocity equal to the phase velocity of the main ponderomotive field. Fig. 5 shows how the trapping efficiency is affected by detuning the electron beam by ±2 eV. In the three mode case, it is seen that detuning affects mainly the initial trapping fraction, the detrapping rate afterwards remaining essentially the same.

Conclusions

It was shown that efficient trapping of electrons in the ponderomotive potential of multimode fields is possible within a definite coherence length. This length can be maximized by fulfilling a suitable wavelength scaling condition. The trapping fraction is then mainly affected by the initial phase distribution of the participating modes.

Appendix

We demonstrate here the equivalence of axial electric field and wiggler amplitude or period tapering in the trapped electron problem.

In the case when the wiggler amplitude and period are allowed to vary as a function of z, the force equation (8) can be rederived from (5) in the form

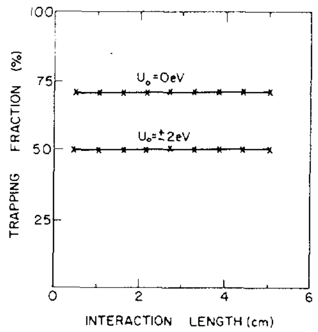
$$\frac{d}{dt}(\gamma m v) = \frac{e^2}{2m\gamma}\frac{\partial}{\partial z}\left[\vec{A}_w(z)\cdot\vec{A}_s \cos\left(\int_0^z k_{pm}(z')\,dz' - \omega_s t + \psi_0\right)\right] - \frac{e^2}{4m\gamma}\frac{\partial}{\partial z}A_w^2(z) - eE_{dc} \tag{A1}$$

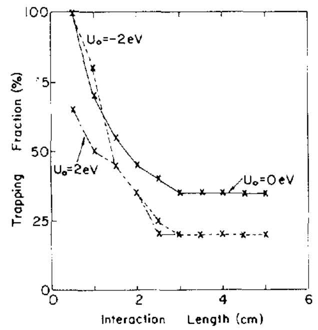
where $k_{pm}(z) = k_w(z) + k_s$, and we assumed $\omega_w = 0$ and a single mode in the wiggler and signal fields.

We substitute the electron coordinate $z(t)$ in terms of the perturbation coordinate $\delta_z(t)$ which is the position coordinate of the electron relative to the position of a perfectly trapped electron (which moves at the phase velocity of the ponderomotive wave).

$$z(t) = z_{ph}(t) + \delta z(t) \tag{A2}$$

$$\frac{d}{dt}z_{ph}(t) = v_{ph} = \frac{\omega}{k_{pm}(z) + k_s}. \tag{A3}$$





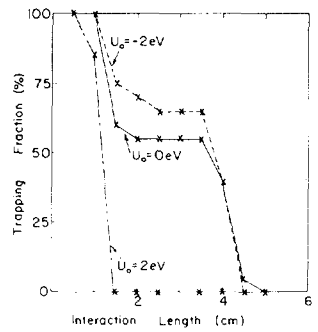


Fig. 5. Influence of detuning on the trapping fraction. Perfect tuning ($U_o = 0$) is defined with respect of the main ponderomotive wave. Upper graph: one mode fields. Center graph: three mode fields. Lower graph: five mode fields.

To a very good approximation this results in

$$\frac{d}{dt}(m\gamma_{ph}\,\delta z) = \frac{e^2}{2m\gamma}\, k_{pm}\vec{A}_w(z) \cdot \vec{A}_s \sin\psi - \frac{e^2}{4\gamma_{ph}m}\frac{d}{dz}A_w^2(z) - \frac{d}{dt}(m\gamma_{ph}v_{ph}) - eE_{\text{dc}} \tag{A4}$$

and

$$\psi = \int_{z_{ph}}^{z_{ph}+\delta z} k_{pm}(z')\,dz' + \psi_0 \tag{A5}$$

where we used the approximation

$$\gamma_{ph} \equiv (1 - \beta_{ph}^2)^{-1/2}\ \gamma \simeq \gamma \tag{A6}$$

and

$$\gamma \equiv [1 + (eA_w/mc)^2]^{1/2}.$$

Equation (A4) is basically the pendulum equation [13], [14]. The first term on the right hand side is the ponderomotive field (9). Clearly, the second and third term play the same role as the fourth term and can be regarded as equivalent axial forces which result from the wiggler amplitude and period tapering

$$-eE_{\text{dc, equiv}} = -\frac{e^2}{4\gamma m}\frac{d}{dz}A_w^2(z) - v_{ph}\frac{d}{dz}(m\gamma_{ph}v_{ph}). \tag{A7}$$

The first term can be interpreted as the force resulting from transfer of transverse kinetic energy into longitudinal energy (which is directly transformed into radiative energy). The second term can be interpreted as an imaginary D'Alambertian force as it is viewed in a noninertial reference frame moving at velocity v_{ph}. It transforms the longitudinal kinetic energy into radiative energy. Note that the real axial electric field E_{dc} generates radiating energy on account of its source and not on the account of the electron energy. Equation (A6) is in agreement with the analysis of [14], except for a factor $2\gamma_z^2$ which appears there in the second term erroneously in our opinion.

The equivalence relation (A6) can also be reversed. The amplitude tapering which is equivalent to a given dc field $E_{\text{dc}}(z)$ is found to be

$$A_w(z) = \left[\frac{4m\gamma_{ph}}{e}\int_0^z E_{\text{dc}}(z')\,dz' + A_w^2(0)\right]^{1/2}. \tag{A8}$$

The equivalent period tapering is given by

$$\gamma_{ph}(z) = \frac{e}{mc^2}\int_0^z E_{\text{dc}}(z')\,dz' + \gamma_{ph}(0) \tag{A9}$$

and (A3), (A6).

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Between 1978 and 1979 he was a Research Associate at Cornell University, Ithaca, NY, where he worked on laser bistability effects and laser spectroscopy. In 1980, he joined the Department of Electronic Devices at Tel Aviv University, Tel Aviv, Israel. His present interests are in free electron lasers, gas lasers, and waveguided optics.

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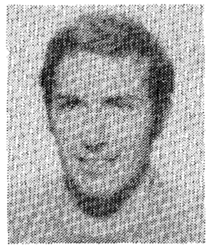

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He is now working toward the Ph.D. degree in electrical engineering at Tel Aviv University, studying the theory of free electron lasers.

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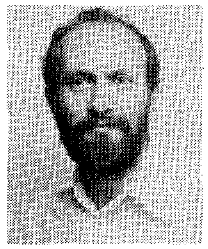

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