Visible and Ultraviolet Radiation Generation Using a Gas-Loaded Free-Electron Laser

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Abstract—Gas loading of a free-electron laser modifies the phase-matching condition while the small-signal gain expression remains the same when written in appropriate form. This permits a wider parameter space than the vacuum FEL, which is particularly advantageous at shorter wavelength operation. Scattering of the electrons by the gas limits the interaction length, but available gains are still high enough to allow oscillation build-up. For example, a 0.5- \( \mu \)m wavelength helical wiggler FEL utilizing a 41 MeV electron beam is restricted to a length of 14 cm, has a small signal gain of 21 percent, and builds up to saturation in less than 1 ps. Tunability of this device over several microns is easily obtained by changing the gas pressure.

INTRODUCTION

THE free-electron laser (FEL) [1] is a coherent radiation source in which the photons are generated by an electron beam wiggled by a periodic, transverse, static magnetic field called a wiggler, or an undulator inside an optical cavity as shown in Fig. 1. Since the active medium in this device is a beam of free particles, and since the electron beam and wiggler characteristics are left to be chosen by the designer, it holds the promise of a very easily and widely tunable radiation source compared to the most common lasers in which the active medium is an atomic or molecular medium whose energy level system cannot be tailored to the user’s needs. Furthermore, electron beams used in a variety of devices, some of which are analogous to the free-electron laser [2] are known to be very powerful and efficient sources of microwaves. It was therefore hoped, when the free-electron laser principle was conceived [1], that it would be applicable to designing powerful radiation sources tunable over wide portions of a spectrum spanning from millimeter to X-rays.

The wavelength of oscillation of a free-electron laser is given by a velocity synchronism condition between the electromagnetic wave and the electrons, which can be approximately written

\[
\lambda \approx \lambda_w |B_z| \text{rel}
\]  

(1)
where $\lambda = \text{wavelength of operation}$, $\lambda_w = \text{wiggler period}$, and $B_{z, \text{rel}} = \text{z-component of the relative velocity between the wave and the electron divided by } c$, the velocity of light in vacuum. As will be shown later (1) can be more exactly rewritten in terms of the electron beam energy and the wiggler characteristics according to

$$\frac{\lambda}{\lambda_w} = 1 + \frac{a_w^2}{2\gamma^2}$$

(2)

with $\gamma = \text{total electron energy divided by } m_e c^2$, where $m_e = \text{electron rest mass}$;

$$a_w = \frac{eB_0}{2\pi nm_e^2}$$

(3)

e = \text{electron charge}$, and $B_0 = \text{wiggler magnetic field}$. Equation (2) is valid for a helical wiggler generating a circularly polarized static magnetic field. It is valid for a linear wiggler, generating a linearly polarized field if $B_0$ is taken to be the rms value of the magnetic field. Such a laser has been operated at Stanford University [3] with 43.5 MeV electrons, a wiggler period of 3.2 cm, and a magnetic field of 2.4 kG, generating 3.4 $\mu$m radiation. Equation (1) shows that to generate wavelengths shorter than the 3.5 $\mu$m value of the Stanford experiment, either the wiggler period has to be decreased, or the electron beam energy increased very drastically if one wants to design an ultraviolet or X-ray radiation source. Present magnetic technology does not provide us with wigglers of a period significantly shorter than 2 cm; on the other hand increasing electron rest mass: the electron beam energy increased very drastically if one wants to obtain the same effect. It can be seen from (1) that the relative z-velocity of the wave and the electrons can then be made arbitrarily small by slowing down the optical wave instead of accelerating the electrons. Another picture of such a device would be that the pitch angle of the helix described by the electrons in a helical wiggler is the Cherenkov angle, so that the stimulated Cherenkov interaction is in fact occurring, but without any walk-off. It will be shown later under which circumstances the latter interpretation of the interaction holds.

The purpose of this paper is to present the design of two gas-loaded free-electron lasers which would generate pressure-tunable radiation of approximately 0.2 $\mu$m and 0.5 $\mu$m, respectively, using a 40 MeV accelerator presently under construction at Stanford. This paper will take the vacuum FEL viewpoint and show how it is modified when a gaseous medium, which slows down the optical wave, is added. Section I will discuss the changes in the expressions for the quantities of interest due to the presence of the gas. Section II will illustrate how the added degree of freedom given by the presence of the gas allows it to reach a more attractive parameter space than in the vacuum case. Section III will discuss the problems introduced by the gas and how they can be overcome. Section IV will present the parameters of the two gas-loaded FEL's designed to operate on a 40 MeV accelerator at 0.5 $\mu$m and 0.2 $\mu$m. All quantities are in MKS units except when otherwise specified.

**SECTION I: ANALYSIS**

As already mentioned, the optical wave is slowed down by a gas of index $n$ inside the FEL wiggler. The analysis of the gas-loaded FEL therefore proceeds as for the vacuum FEL except that the k-vector of the optical wave now has amplitude $k_{\text{opt}} = \omega n/e$. Any of the methods used for the vacuum FEL are applicable. The gas-loaded FEL has been partially analyzed in [10] and [11].

The small signal gain can be rederived using Madey's theorem [13], which is applicable here, as can be seen from [14]. This yields for the small signal gain in the case of a helical wiggler

$$G_{g, \text{FEL}} = \frac{\pi}{4} \frac{[\mu_0 / \varepsilon_0]^{1/2}}{nm_e c^2/e} \frac{L^3 a_w^2 (1 + a_w^2)}{\lambda^2 \gamma^2} \frac{I_0 / d}{A_1} \left[ \frac{\sin^2 \theta}{\theta^2} \right]$$

(5)

where $I_0 = \text{electron beam peak current}$, $L = \text{interaction length}$, and $A_1 = \text{optical mode area}$; $\theta$ is given by

$$\theta = \left( \frac{\omega / \beta_c c}{\lambda} - \frac{2\pi}{\lambda_{\text{wiggler}}} \right) L/2$$

(6)

where $\omega = \text{optical frequency}$ and $z = \text{direction of propagation}$. The index $n$ differs from unity by only a few parts in $10^4$ for
the gases and pressure range considered, so that the \( n \) in (5) can be ignored. Setting \( \theta \) equal to \(-1.3\) to maximize the gain and replacing the fundamental constants by their numerical values, (5) becomes (in MKS units)

\[
G_{\text{FEL}} \approx 3.12 \times 10^{-4} \frac{L^3 a_w^2(1 + a_w^2)}{\lambda^2} \frac{I_0}{A_1} .
\]

(7)

The phase-matching condition (corresponding to \( \theta = 9 \)) between the optical wave and a given electron becomes

\[
\left[n + \frac{\lambda}{\lambda_w}\right] \beta \cos \theta_p = 1
\]

(8)

where \( \theta_p \) is the pitch angle of the helical trajectory described by the electron in the wiggler field, and is given by

\[
\sin \theta_p = \frac{a_w}{\beta \gamma}.
\]

(9)

For relativistic electrons \( \theta_p \) is a small angle, so that (8) can be rewritten

\[
n - 1 + \frac{\lambda}{\lambda_w} \approx \frac{1 + a_w^2}{2\gamma^2}.
\]

(10)

In summary, the addition of a gas to the FEL does not alter the small signal gain but does change the synchronism condition from (2) to (10). If we put a lower value on \( \lambda_w \), of about 1 cm for conventional wigglers we see that for \( \lambda \) smaller than 1 \( \mu \text{m} \), \( \lambda / \lambda_w \) will be less than \( 10^{-4} \). A gas such as \( \text{H}_2 \) at 1.0 atm and a temperature of 273 K has an index \( n - 1 \) of 1.368 \times 10^{-4} at 1.0 \( \mu \text{m} \), increasing with decreasing \( \lambda \). Thus, at wavelengths shorter than 1 \( \mu \text{m} \), \( n - 1 \) is at least comparable to \( \lambda / \lambda_w \), and dominates as \( \lambda \) decreases. We see that when progressing towards the UV, the phase-matching condition tends towards the Cherenkov condition given by (4). The wiggler field then serves mainly to provide a geometry where there is an angle \( \theta_c \approx \theta_p \) between the electron instantaneous velocity vector and the optical mode \( \mathbf{k} \) vector while maintaining a unity filling factor over the interaction distance. The degree of freedom thereby introduced into the choice of wiggler parameters and \( \gamma \) allows higher gain through transit bunching as evidenced in (7). Thus, the “gas-loaded FEL” is really a magnetic bremsstrahlung, free-electron laser in the far IR where \( \lambda / \lambda_w \gg n - 1 \), a synergistic magnetic bremsstrahlung-Cherenkov free-electron laser in the near IR to longer wavelengths part of the spectrum where \( n - 1 \approx \lambda / \lambda_w \), and a purely Cherenkov free-electron laser in the visible to UV part of the spectrum.

The efficiency of the device can be estimated easily in the small signal limit from the argument showing that efficiency and energy acceptance are almost equal [15]. This argument applies equally well to the gas-loaded FEL. Let us now redefine the energy acceptance. The phase-slippage due to the energy spread of the electron beam is

\[
\Delta \Phi = \Delta \left[ \frac{1}{\beta_z} \right] \approx \frac{1 + a_w^2}{2\gamma^2} \left[ \frac{\Delta E}{E} \right]_{\text{FWHM}}
\]

(12)

where \( \Delta E \) is the full width at half-maximum of the distribution. Equation (12) is readily obtained by using the energy relationship \( \gamma = [1 - \beta_z^2 - \beta_e^2]^{1/2} \) and by noting that the normalized transverse velocity is \( \beta_{z} = a_w / \gamma \). By limiting the phase-slippage to \( \pi \) we finally obtain for the efficiency

\[
\eta \approx \frac{\Delta E}{E} = \frac{\lambda}{2L} \left[ \frac{2\gamma^2}{1 + a_w^2} \right].
\]

(13)

Using the resonance condition and letting \( N_w \) be the number of wiggler periods, (13) can be rewritten as

\[
\eta \approx \frac{\Delta E}{E} = \frac{1}{2N_w} \left[ 1 + \frac{1}{(n-1)\lambda_w/\lambda} \right].
\]

(14)

It is to be noted that (13) is valid for both the vacuum free-electron laser and the gas-loaded FEL. However, the often quoted formula, \( \eta \approx \Delta E / E = 1/2N_w \) is valid only in vacuum. One can also see from (14) that for a given number of wiggler periods the efficiency of the gas-loaded free-electron laser is smaller than in vacuum. For example, for \( \text{H}_2 \) gas at 1.0 atm, 293 K, for a wavelength of 0.5 \( \mu \text{m} \) and for \( \lambda_w = 3 \text{ cm} \), \( n - 1 \) is 1.5 \times 10^{-4}; the energy acceptance is thus reduced by a factor of 10 compared to a vacuum FEL that would have the same number of wiggler periods. However, (13) does not impose a stringent monochromaticity requirement on the electron beam because the gas-loaded FEL can have a very small number of wiggler periods, as will be seen later. In both cases \( L / \lambda \) is the quantity of physical significance in determining the allowed phase slip and therefore the energy acceptance.

The output power \( \Delta P_{\text{sat}} \) of the laser when it reaches saturation is obtained from the definition of efficiency

\[
\Delta P_{\text{sat}} = \eta \times \text{power in the electron beam}
\]

(15)

since the energy lost by the electrons in steady-state or at saturation should be equal to the output energy of the laser.

Another quantity of interest for a free-electron laser amplifier is its angular acceptance. Angular spread is present in the electron beam at entrance as a result of transverse thermal velocities, space charge effects, and lens aberrations. As will be seen later, in the gas-loaded FEL there is also a component due to the scattering of the electrons by the gas molecules. Here we only compute the phase-slippage due to the intrinsic angular spread of the electron beam: electrons entering the wiggler at an angle to its axis will slip out of phase with respect to the optical wave. Equation (11) applies here, but \( \Delta (1/\beta_z) \) is now

\[
\frac{1}{\cos \phi_e} - 1 \approx \frac{1}{2} \phi_e^2.
\]

(16)

where \( \phi_e \) is the half-single angular divergence. The resulting phase-slippage is thus

\[
\Delta \Phi \approx \pi \frac{L}{\lambda} \phi_e^2.
\]

(17)
Angular acceptance will be computed when the other contributions to the angular spread have been discussed.

Among the characteristics of an FEL oscillator an important one is the oscillation build-up time. The output of a conventional linac typically consists of electron microbunches of a few picoseconds occurring at the microwave frequency for a few microseconds. The microsecond-long macrobunch is repeated at tens or hundreds of hertz. The oscillation build-up time should therefore be short compared to the macrobunch length to permit steady-state saturated power level to be reached. We follow the method used in [16] to estimate this quantity. If \( N \) is the number of round-trips necessary to reach saturation, the bandwidth of the spontaneous emission is narrowed down to

\[
\Delta \omega_N = \frac{1.23 \times 10^{9}}{N \omega_1 \lambda} \left[ \frac{G'_{\text{FEL}} + 1}{G_{\text{FEL}}} \right]^{1/2} \frac{1}{N^{1/2}} \tag{18}
\]

after \( N \) round-trips, where \( G'_{\text{FEL}} \) is the excess gain per pass (gain minus output and losses). Equation (18) therefore gives the laser bandwidth. The solid angle of emission of a laser is

\[
\Delta \Omega = \frac{1}{2\pi} \frac{\lambda^2}{w_1^2} \tag{19}
\]

where \( w_1 \) is the spot size of Gaussian optical mode. If \( d^2P/d\omega d\Omega \) is the spontaneous emission per unit solid angle and unit bandwidth, the amount of spontaneous power that starts up oscillation is

\[
P_m = \frac{d^2P}{d\omega d\Omega} \Delta \Omega \Delta \omega_N = 1.5 \times 10^{-9} \frac{I_0 L^3 a_w^2}{w_1^2 N \omega_1 \lambda \gamma^2} \left[ \frac{G'_{\text{FEL}} + 1}{G_{\text{FEL}}} \right]^{1/2} \frac{1}{N^{1/2}}. \tag{20}
\]

\( N \) is calculated by noting that \( \Delta P_{\text{sat}} \) is the output power at saturation and therefore should satisfy

\[
\frac{\Delta P_{\text{sat}}}{P_{\text{in}}} \approx (1 + G_{\text{FEL}})^2 \tag{21}
\]

The oscillation build-up time is defined here as the time necessary to reach saturation. Thus it is given by

\[
\tau B = NL_c \tag{22}
\]

where \( L_c \) is the total cavity length.

As one can see from the above discussion all the quantities of interest in designing a free-electron laser can be written in a form where the index of refraction does not appear, leading to the same expressions for both the gas-loaded FEL and the vacuum FEL. However, the numerical values for these quantities change vastly due to the different range allowed for \( a_w \), \( \gamma \), \( \lambda \), and \( \lambda_w \) by the new phase-matching condition. This will be illustrated in the next paragraph.

**Section II: Advantages of Adding a Gas**

There are a number of ways to see the effect of the new phase-matching condition on the performance of the FEL by writing the small signal gain in three different forms, as follows.

Case A:

\[
G_{\text{FEL}} = 17.6 \times 10^{-4} \frac{I_0 L^3}{A_1} \frac{a_w^2}{(1 + a_w^2)^{3/2}} \left[ n - 1 + \frac{\lambda}{\lambda_w} \right]^{1/2} \tag{23}
\]

Case B:

\[
G_{\text{FEL}} = 6.24 \times 10^{-4} \frac{I_0 L^3}{A_1} \frac{a_B^2}{(1 + a_B^2)^{3/2}} \left[ n - 1 + \frac{\lambda}{\lambda_w} \right]^{1/2} \tag{24}
\]

Case C:

\[
G_{\text{FEL}} = 3.12 \times 10^{-4} \frac{I_0 L^3 a_w^2 (1 + a_w^2)}{A_1} \frac{1}{\lambda y^5} \tag{25}
\]

For Case A it will be assumed that \( \lambda, B_0, \) and \( \lambda \) are given, i.e., the wiggler characteristics and the wavelength of operation are specified. Then, according to (23), the gain for a fixed length is a constant times the factor \([n - 1 + (\lambda/\lambda_w)]^{1/2}\). As already pointed out for short \( \lambda, n - 1 \) can be at least one order of magnitude greater than \( \lambda/\lambda_w \). Table I shows an example using \( H_2 \) at STP which gives \( n - 1 = 1.400 \times 10^{-4} \) for \( \lambda = 0.5 \mu m \). Without the gas, the required \( \gamma \) is approximately three times as large and the gain for a fixed length is reduced by over two orders of magnitude.

For Case B, \( \lambda, B_0, \) and \( \gamma \) are given, i.e., the wiggler characteristics and the electron beam energy, are fixed. Equation (10) shows that \( \lambda/\lambda_w \) will be smaller than in the \( n = 1 \) case, thus allowing for operation at a shorter wavelength. In addition, (24) also indicates that the gain per unit length at the shorter wavelength is higher due to the factor \((1 - (n - 1) \cdot (2\gamma^2)/(1 + a_B^2)))^{-1}\) in the gain expression. Table I shows an example for this case and it is seen that for the specified parameter values the addition of the gas reduces the wavelength and increases the gain per unit length by a factor of 12. However, there will be a reduction in usable length in the gFEL as a result of scattering induced emittance.

Finally for Case C, \( a_w, \lambda, \gamma \) are given, and (25) shows that the gain per unit length is the same for both devices. However, (10) yields a value for \( \lambda_w \) which is larger in the gas-loaded case, which, in turn, implies a smaller wiggler field as seen from

\[
a_w = 0.0933 B_0 (kG) \lambda_w (cm). \tag{26}
\]

An example of this case is also given in Table I, where it is shown that to achieve the same gain without the gas there is approximately a factor of 10 decrease in \( \lambda_w \), and a corresponding increase in \( B \). In addition, the B-field transverse variation for a periodic array of magnets is \( \exp(-2\pi x/\lambda_w) \) where \( x \) is the transverse dimension, so that for magnets positioned outside a 5 mm vacuum chamber the fields needed to produce 33 kG and 3.5 kG on axis as called for in Table I are 4500 kG and 5.8 kG, respectively. Clearly the first of these values is unrealizable.

Thus, in Cases A and B the gains per unit length are higher in the gas-loaded case and appreciably so when \( \lambda/\lambda_w \ll n - 1, \)
in Case C; for a given gain a more practical wiggler can be used.

Another way to see the advantage of the gas-loaded FEL is to note from the synchronism condition that if we choose a wiggler of characteristics \( \lambda_w = 2.5 \, \text{cm}; B_0 = 5 \, \text{kG}; \lambda = 0.5 \, \mu \text{m} \), the vacuum FEL would generate 4 \( \mu \text{m} \) radiation whereas if loaded with 0.9 atm of hydrogen at 293 K it would generate 0.25 \( \mu \text{m} \) radiation. To generate 0.25 \( \mu \text{m} \) radiation in vacuum with the same wiggler parameters one would have to raise the electron energy to 175 MeV. In this section the advantages of a gas have been considered, and in Section III the disadvantages are presented.

### Section III: Drawbacks

In the same region of space simultaneously present are an electron beam, a strong static magnetic field, a gas, and a potentially intense electromagnetic wave. The gaseous medium cannot be modeled by a single quantity, its index of refraction. Various other physical phenomena might occur which must be taken into account. The electron beam suffers collisions with the molecules of the medium and this results in angular scattering of the electron beam, energy spread, and energy loss. Ionization of the gas, both by the electron beam and the optical beam, creates free charges in the gas that might modify the focusing properties of the electron beam. The index of refraction and therefore the synchronism condition might be modified during oscillation build-up due to gas ionization, density fluctuations, and nonlinear optics phenomena such as self-focusing. The optical wave might be depleted by scattering phenomena such as stimulated Raman and Brillouin scattering, self-focusing, and gas breakdown. In this section those phenomena will be evaluated for 50 MeV electrons and visible radiation.

The main side-effect is angular scattering of the electrons by the collisions in the gas. It results in an angular spread that adds to the intrinsic spread of the beam, and thus causes phase-slippage of the electrons with respect to the wave. To limit such a phase-slippage, the interaction length has to be limited.

The phase-slippage due to scattering has a quadratic dependence upon the scattering angle, similar to the dependence upon divergence from beam emittance, plus a linear dependence that results from the product of the scattering angle and the pitch angle for the trajectory, but which averages to zero over integral half-wavelengths of the wiggler. Depending upon the number of scattering events per wiggler period, the residual linear term has been estimated to be in the range of 0.1-0.5 of the quadratic term for typical parameter values. Allowing for a pessimistic estimate, the phase-slippage from scattering has been taken to be twice the value of the quadratic term. A reasonable approximation to the Highland scattering formula [17] gives the scattering angle \( \theta_s(z) \) as

\[
\theta_s(z) \approx \frac{12}{E} \left[ \frac{z}{X_0} \right]^{1/2}
\]

where \( E \) is the particle energy in meaelectronvolts and \( X_0 \) is the radiation length in the medium. Hydrogen provides the least scattering for a specified index \( n \), and thus we consider hydrogen for the gas. For \( E = 50 \, \text{MeV} \) electrons, for example, \( \theta_s \) after traversing 30 cm of \( \text{H}_2 \) gas at STP is 1.4 mrad. The corresponding phase-slippage is

\[
\Delta \Phi = \Delta \left[ \omega \int_0^L \frac{dz}{v_e(z)} - k_{\text{opt}}L - k_wL \right]
\]

\[
\Delta \Phi \approx 900 \frac{L^2}{\lambda X_0 T^2}.
\]

Allowing for a phase-slippage due to scattering during the interaction of \( \pi/2 \) leads to an approximate formula for the allowed length

\[
L \approx 2.8 \frac{2\lambda^{1/2}}{p^{1/2}}
\]

where \( T \), the gas temperature, is taken as 293 K. Examples of values of \( L \) calculated from (29) for 1.0 atm of gas are given in Table II.

Thus, the total length \( L \) is limited. However it has been shown [6], [18], in the case of the vacuum FEL, that by using smaller beam areas and by the increase in current made possible by an increase in emittance, the small signal gain could be made independent of the laser length while the efficiency and power output scaled, respectively, as the inverse and inverse cube of the length within limits. Similar considerations will be used here to obtain high gains and

### Table I

**Parameters for the FEL and Gas-Loaded FEL**

<table>
<thead>
<tr>
<th>( n = 1 )</th>
<th>( n &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>123</td>
</tr>
<tr>
<td>( G(x)/L^2 )</td>
<td>0.74</td>
</tr>
</tbody>
</table>

**Case A:** \( \lambda_w = 2.5 \, \text{cm}; B_0 = 5 \, \text{kG}; \gamma = 0.5 \, \mu \text{m} \); \( 1/\alpha_1 = 100 \, \mu \text{A/cm}^2 \); \( n = 1 \approx 1.400 \times 10^{-1} \)

| \( \lambda(\mu \text{m}) \) | 2.3 | 86 |
| \( G(x)/L^2 \) | 0.2 | 43 |

**Case B:** \( \lambda_w = 1.25 \, \text{cm}; B_0 = 10 \, \text{kG}; \gamma = 80; \) \( 1/\alpha_1 = 100 \, \mu \text{A/cm}^2 \); \( n = 1 \approx 1.691 \times 10^{-1} \)

| \( \lambda(\mu \text{m}) \) | 1.3 | 153 |

**Case C:** \( \lambda_w = 1; \gamma = 86; \lambda = 0.5 \, \mu \text{m} \); \( 1/\alpha_1 = 100 \, \mu \text{A/cm}^2 \); thus \( 1/\alpha_1 = 38.1 \, \mu \text{A/cm}^2 \); \( n = 1 \approx 1.400 \times 10^{-1} \)

| \( \lambda(\mu \text{m}) \) | 0.32 | 3.1 |
| \( E_0(kG) \) | 33 | 3.5 |
output power from the gas-loaded FEL. In addition, the gas-loaded FEL benefits from the other advantages of the short FEL’s: absence of pulse slippage effects, relaxed mechanical alignment and stability constraints, short build-up time, and lower wiggler cost. The latter is reduced both by the shortness of the device and less demanding values for the magnetic field strength and the wiggler period.

If we consider electron energies below the critical energy \( E_c = (400/Z) \text{ keV} \) ionization is the main cause of mean-energy loss. From Evans [19] the mean-energy loss can be estimated to be

\[
\frac{dE}{dl} (\text{MeV/cm}) \approx 0.3 \rho \frac{Z}{A} \left[ \ln \left( \frac{0.511}{l} \right) \gamma^{3/2} \right] + \frac{1}{2}
\]

where \( \rho \) is the density of the gas in g/cm\(^3\), \( Z \) is its atomic number, \( A \) is its atomic weight, and \( l \) is the geometric-mean ionization and excitation potential in megaelectronvolts. For 1.0 atm of hydrogen at 273 K, 50 MeV electrons, and \( l \approx 15 \text{ eV} \) we find \( dE/dl \approx 48 \text{ keV/m} \). For 1 m of gas the mean-energy would therefore amount to a reduction of 0.1 percent of the initial energy. This number is to be compared to the intrinsic energy spread on the electron beam and the energy acceptance of the FEL, those two quantities usually being equal by design. For the examples given in Table III, \( \Delta E/E \) is around 0.5 percent; so that the mean-energy loss due to scattering is thus negligible. For any design where 0.1 percent would be non-negligible compared to the energy acceptance of the laser, the mean-energy loss could be compensated by tapering the wiggler or the index of refraction of the gas. The latter technique will be discussed at the end of this paper.

Energy spread (or straggling) is also introduced on the beam both by ionization and radiative losses, the former being once again dominant below the critical energy. From Landau [21] it can be estimated to be

\[
\frac{d\Delta E}{dl} (\text{MeV/m}) = 15.4 \rho \frac{\Sigma Z}{\Sigma A}
\]

where \( \Sigma Z \) is the sum of the atomic numbers in a molecule of the medium and \( \Sigma A \) is the sum of the atomic weights. For an atmosphere of hydrogen at 273 K, \( \Delta E \) is 1.4 keV/m. This is negligible compared to the intrinsic energy spread on the entering beam, so that the energy spread introduced by the gas can be ignored.

To estimate the effect of the gas ionization on the electron beam [22] one needs to consider its time structure. The microbunches ionize the gas along their path, at a rate approximatively given by the energy loss divided by the ionization energy, i.e., 50 keV/m/15 ev/ionization event \( \approx 3000 \) ionization events/m/electron. For a peak current of 2500 A/cm\(^2\) in a 5 ps pulse at 350 ps intervals (S-band accelerator) this yields a rate \( \frac{dn_e}{dt} = 7 \times 10^{21}/\text{cm}^2/\text{s} \). The equilibrium plasma density will be determined by three body recombination processes. Taking the recombination coefficient to be \( \alpha = 1.6 \times 10^{-7} \text{ cm}^3/\text{s} \) for \( \mathrm{H}_2 \) at STP and setting \( \frac{dn_e}{dt} = \alpha n_e^2 \) yields an equilibrium density \( n_e = 2 \times 10^{14} \text{ cm}^{-3} \). This is almost three orders of magnitude greater than the electron density in the beam. However, its effect upon refractive index at the optical frequencies of interest is only of order one part in ten million. Potential instability mechanisms associated with a relativistic beam traversing a plasma have been considered but are not significant because of the short pulse length (1.5 mm for 5 ps).

We next consider stability requirements on the gas density. Index of refraction deviations from the value given by (10) will cause phase-slippage of the electrons with respect to the optical wave. Allowing for a phase-slippage of \( \pi \) gives a limitation of \( 2 \times 10^{-6} \) on \( \Delta n/n \) or about 1 percent on the fractional change in index of refraction \( \Delta(n - 1)/n - 1 \) for 0.5 \( \mu \text{m} \) radiation and 14 cm of interaction length in \( \mathrm{H}_2 \) at 1.0 atm and 293 K. This implies a pressure stability and a temperature stability of 1 percent over the about 10 \( \mu \text{m} \) of the macrobunch, both easily achieved. At higher duty cycles the thermal effects of the small energy loss can cause temperature gradients across the gas. The effect of the refractive index profile in the gas on the optical mode will be equivalent to a lens and may be compensated by a change in mirror curvature. The effect upon the electron beam will be averaged out by the wiggler motion. Finally the nonlinear index of refraction, due to the presence of the electromagnetic wave in the gas, is much less than \( 6 \times 10^{-6} \) for hydrogen at STP.

Among the phenomena that could deplete the optical wave, gas breakdown and self-focusing have a higher threshold than stimulated scattering phenomena. In stimulated Brillouin scattering the incident optical wave scatters on the density fluctuations it creates in the medium. A fraction of the optical beam is thus reflected by the medium. However, this phenomenon is usually not observed with picosecond optical pulses, since the incident and scattered pulses are counterpropagating, which reduces the interaction length to a few millimeters. Stimulated Raman scattering remains, which has a particularly large cross-section in hydrogen. However, the single-pass Stokes power produced at saturation is at most 6.5 mW for the 0.5 \( \mu \text{m} \) laser, the parameters of which are given in Table III, whereas the saturation power is about 5 MW leading to a 10\(^{-7}\) percent depletion of the optical wave if no feedback exists at the Stokes wavelength, equal to 0.63 \( \mu \text{m} \) for a pump wavelength of 0.5 \( \mu \text{m} \). Due to the large shift there will be no difficulty in preventing feedback at the Stokes line.

We thus have shown that angular scattering of the electron beam by the gas molecules is the only significant drawback of the gas-loaded free-electron laser. It has to be taken into account in the design of a device, as will be shown in the next section.
SECTION IV: EXPERIMENTAL PARAMETERS

Our interest lies in the generation of viable and UV radiation using a medium energy (up to 50 MeV) accelerator. This upper limit on the electron beam energy is chosen so that the whole device (accelerator and FEL) can be packaged into a compact, commercial unit of the size and cost of a medical linac.

The accelerator characteristics are assumed given: the peak current \( I_0 \), the energy \( \gamma \), the emittance of the electron beam \( \epsilon \) defined as

\[ \epsilon = \pi \omega_{e} \phi_{e} \]  

(32)

where \( \omega_{e} \) is the radius of the electron beam at its focus, \( \phi_{e} \) is its intrinsic angular divergence (half-angle); the intrinsic energy spread \( \Delta E/E \); the time structure of the electron beam. The desired wavelength of oscillation \( \lambda_{0} \) is also assumed chosen.

It will be shown hereafter that gain, efficiency, output power, and build-up time can be written in terms of gas transmission formulas. The second constraint is set by the definition of emittance, given by (32). The last constraint is provided by limiting the total phase-slippage due to angular spread generating phenomena, namely gas scattering and intrinsic angular spread of the electron beam, to \( \pi \). The total phase-slippage is simply taken to be the sum of the two contributions, given by (17) and (28) as

\[ \Delta \Phi_{\text{TOT}} = 972 \frac{L^{2}}{\lambda X_{0} \gamma^{2} (n - 1)^{2}} + \pi \frac{L}{\lambda} \phi_{e}^{2} \]  

(40)

To increase the gain one would like to choose a large value of \( L \). However, one sees from (40) that the electron beam divergence \( \phi_{e} \) is then small; from the emittance constraint \( \omega_{e} \) is large, which in turn makes \( A_{1} \) large, thereby reducing the term \( L^{3}/A_{1} \). This argument shows that \( L^{3}/A_{1} \) has an optimum.

In the gain expression the remaining unknown is \( L^{3}/A_{1} \). Several constraints bear on \( L \) and \( A_{1} \). The electron beam will be chosen narrower than the optical beam here. The overlap condition we have chosen is that the electron beam area be equal to the optical mode effective area at both ends of the interaction region (neglecting the emittance growth due to the gas)

\[ A_{1}(0) = A_{e}(0), \]

i.e.,

\[ \pi \frac{w_{1}^{2}(0)}{2} = \pi \omega_{e}^{2}(0) \]  

(38)

where the electron beam radius at the beginning of the interaction region \( w_{e}(0) \) can be estimated from

\[ w_{e}(0) = \left[ w_{e}^{2} + \left( \frac{\phi_{e} L}{2} \right)^{2} \right]^{1/2} \]  

(39)

The laser spot size at the center of the interaction region \( w_{1} \) can be obtained from \( w_{1}(0) \) by using Gaussian beam propagation formulas. The constraint on the build-up time. For the designs of Table III the pressure has been chosen to have the maximum value allowed by the intrinsic energy spread on the electron beam and the cavity length as small as the mechanical design of the.
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Fig. 2. Gain, output power, and efficiency of the gas-loaded FEL versus gas pressure. The parameters of the plot are: hydrogen gas, optimization wavelength = 0.5 \( \mu \text{m} \), electron beam energy = 41 MeV, peak current = 15 A, emittance = 1.5 mm \( \cdot \) mmrad. Each point on the plots represents a different device.

![Gain versus scattering angle A. The angle A accounts for the additional angular spread introduced on the electron beam by the input foil and travel through the gas before the wiggler entrance.](image)

Laser allows. Finally, output coupling has been chosen for a small oscillation build-up time compared to the accelerator macropulse length. It should be noted that in Figs. 2 and 3 each point represents a different device; in particular, devices get shorter as the pressure increases. Depending on the application one might want to lower the gas pressure to increase the efficiency, for example, to extend the short wavelength range of the device.

Table III presents the parameters of two devices optimized for the same accelerator and two different wavelengths 0.5 \( \mu \text{m} \) and 0.2 \( \mu \text{m} \). The accelerator parameters are compatible with these expected from the Stanford University Mark III accelerator presently being renovated: a peak current between 15 and 30 A, an energy spread of 0.5 percent and a macropulse length of about 8 \( \mu \text{s} \). A microwave gun is planned [24] that could give an emittance better than the Lawson-Penner limit of 1.5 mm \( \cdot \) mmrad by more than two orders of magnitude. The wiggler is assumed helical; optical cavity losses (including output coupling) are taken to be 4 percent and the cavity index of refraction has a very slow wavelength variation. It is to be noted that for the given parameters the optimum of \( L^3/A_1 \) corresponds to comparable phase-slippages due to gas scattering and intrinsic angular spread. It is also to be noted that somewhat higher gains would be obtained by focusing the optical beam more tightly than has been done for the examples given in Table III, where the area used in the gain calculation is the optical beam area at the wiggler entrance.

An important practical point remains to be discussed. The electrons have to enter the gas chamber by some means: differentially pumped aperture, window, etc. This results in some additional scattering of the electrons, which has to be included in (40). Calling \( A \) the additional angular spread, the resulting phase-slippage is of the same form as the phase-slippage due to the intrinsic angular spread

\[
 \Delta \Phi = \pi \frac{L^2}{\lambda} A^2
\]  

and is simply added to the two terms of (40). The optimization of the term \( L^3/A_1 \) proceeds then as described above. The effect of the angle \( A \) on the gain is shown in Fig. 3. Table III assumes a differentially pumped aperture, for which the resulting additional scattering has a negligible influence on gain. However, this would require costly large capacity pumps and a carefully designed input nozzle to avoid turbulences in the gas in the interaction region. An experimental arrangement that would avoid those two problems is shown in Fig. 4. In the back mirror is drilled a small hole of \( \approx 200 \mu \text{m} \) diameter, small enough for the resulting optical loss to be negligible. A beryllium input foil contains the gas and lets the hydrogen in. The hole being very small, the foil can be made as thin as 2 \( \mu \text{m} \) (one stage of differential pumping might be used if

### Table III

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<th>Parameters for the Gas-Loaded FEL</th>
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<tr>
<td>( p (\text{atm}) )</td>
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the foil is not leak tight). The electron beam is focused before the hole and reexpanded before the wiggle entrance. By using strongly focusing quadrupoles with gradients on the order of 100 T/m, the distance between the input foil and the wiggle entrance could be as short as about 3 cm. Combining the scattering due to the input foil and the distance traveled through the gas, and using the plot given in Fig. 3, one obtains a 0.5 μm laser with a gain of approximately 18 percent, an efficiency of 0.6 percent, and parameters close to those given in Table III. Another advantage of the arrangement shown in Fig. 4, is the shortness of the optical cavity, which could be about 50 cm long, taking into account the fact that the electrons have to be bent out of the cavity.

A drawback of the gas-loaded FEL is its low efficiency. Efficiency improvement techniques such as tapering of the magnetic field or gain expansion are applicable to the gas-loaded FEL. However, the presence of the gas provides another means of efficiency improvement. The synchronism condition is approximately

\[
(n - 1)(z) \approx \frac{1 + aw^2}{2\gamma(z)^2}.
\]

(42)

Energy extraction is limited because the resonance condition is no longer met as the electrons lose energy. To increase efficiency, the index of refraction of the gas is increased along z so that the condition imposed by (42) is continuously met, which can be achieved by establishing a constant temperature gradient along z. Following the theory of Kroll et al. [20], we find the desired index profile to be

\[
(n - 1)(z) = (n - 1)(z=0) \frac{2\eta + 1}{(1 + 4\eta) - 4\eta z/L}.
\]

(43)

if we taper the profile over the last half of the interaction region (where \( \eta \) is the desired efficiency).

**Conclusion**

We have shown that gas-loading of the FEL introduces an additional degree of freedom which typically gives it higher gains than a vacuum FEL, and allows shorter wavelengths for a given beam energy. The high gains should permit operation on conventional accelerators with a short macropulse for visible and near UV radiation production. Furthermore, recent calculations [25] show that near electronic resonances, gain, and gas absorption have different dependencies on gas pressure. This allows obtaining of substantial net gains (gain minus absorption) near the hydrogen Lyman band (centered around 1050 Å) with electron beam energies on the order of 40 MeV, by appropriately lowering the gas pressure. Work is under way to estimate the potential of a gas-loaded FEL operating near the resonances of various light gases below 1000 Å and using medium energy electrons (40 MeV or lower). Interesting consequences of the shortness of the device include qualifying it as a good candidate for a room size integrated visible and near UV radiation source which would incorporate a compact dedicated linac. The wavelength of the source would be tuned by varying the gas pressure or the electron beam energy.

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**References**


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