Experimental demonstration of longitudinal wiggler free-electron laser

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The lowbitron—a longitudinal wiggler beam interaction device—as a novel source of submillimeter wave radiation was proposed and analyzed theoretically by McMullin and Bekefi [Phys. Rev. A **25**, 1826 (1982)]. This letter reports the first experimental measurements of lowbitron radiation obtained with a 740-kV, 400-A electron beam. The measured power spectra in the W-band range for different wiggler field periods agree with the lowbitron interaction theory. They match the intersection points of the shifted fast cyclotron wave dispersion relation $\omega - (k + k_w)v_\parallel - \Omega_0/\gamma = 0$ and the TE_{01} electromagnetic waveguide mode.

A great experimental and theoretical effort has been made in recent years in developing powerful millimeter and submillimeter microwave sources using relativistic electron beams. Mostly, two classes of electron beam instabilities have been investigated. The first is the cyclotron resonance maser (CRM) instability[1-4] characterized by tranverse bunching of gyrating electrons traveling along a uniform axial magnetic field. Its radiation frequency is associated with the gyrofrequency or one of its harmonics. The other class is the free-electron laser (FEL) instability[5] characterized by axial bunching of electrons traveling in a transversely polarized periodic magnetic wiggler. Its radiation is determined by the double Doppler shift formula resulting in an emission wavelength of $\lambda = l_w/2\gamma_\parallel^2$, where l_w is the spatial period of the wiggler and $\gamma_\parallel = [1 - (v_\parallel/c)^2]^{-1/2}$ is the relativistic factor, $v_\parallel$ being the axial beam velocity.

The lowbitron—a longitudinal wiggler beam interaction device—is a hybrid system of the above mechanisms and was proposed and analyzed *theoretically* by McMullin, Bekfi, and Davidson.[6-8] The proposed scheme assumes a thin pencil beam of relativistic electrons with a large transverse velocity $v_\perp$ (acquired before entering the interaction region) traveling on axis in a combined uniform axial magnetic field and *longitudinally* polarized periodic wiggler magnetic field. A different scheme which also utilizes the CRM and FEL mechanisms has been proposed and demonstrated by Grossman *et al.*[9] This scheme is based on a hollow relativistic electron beam traveling in a *transversely* polarized helical undulator magnetic field. The interaction mechanism in this system is essentially the magnetic bremsstrahlung FEL mechanism with a parametric (Raman) cyclotron frequency shift. Contrary to the longitudinal wiggler mechanism it is based on a transverse wiggler field, which is maximized away off the axis, upon the annular beam shape. The analysis in Ref. 9 only describes a specific waveguide mode interaction mechanism which vanishes for a transverse electromagnetic radiation wave.

The total imposed magnetic field on axis in the lowbitron is of the form

$$\mathbf{B} = \hat{z}[B_0 + \delta B \sin(k_w z)], \qquad (1)$$

where B_0 is the uniform axial magnetic field, $k_w = 2\pi/l_w$ is the wave number, l_w the period, and δB the amplitude of the wiggler magnetic field. Assuming interaction between a tenuous electron beam and a transverse electromagnetic wave, and neglecting the perpendicular spatial variations ($\partial/\partial \mathbf{x}_\perp = 0$), the dispersion relation near the intersection of the electromagnetic dispersion curve

$$\omega - kc = 0 \qquad (2)$$

and the dispersion curve of the wiggler shifted cyclotron wave

$$\omega - (k + nk_w)v_\parallel - \Omega_0/\gamma = 0 \qquad (3)$$

is

$$(\omega - kc)\{[\omega - (k + nk_w)v_\parallel - \Omega_0/\gamma]^2 + 1/2\beta_\perp^2\,\omega_p^2\,J_n^2(x)\} = 1/4\beta_\perp^2\,\omega_p^2\,J_n^2(x)nk_w c. \qquad (4)$$

In Eqs. (2)–(4) ω,k are the frequency and the wave number of the radiated wave, $\beta_\perp = v_\perp/c$, $v_\perp$ being the perpendicular velocity of the beam, $\Omega_0 = eB_0/mc$ is the electron cyclotron frequency, $\omega_p^2 = 4\pi ne^2/m\gamma$ is the plasma frequency, $\gamma = [1 - (v/c)^2]^{-1/2}$, v being the total beam velocity, and $J_n(x)$ is the Bessel function of order n with argument $x = \Omega_0\delta B/(\gamma v_\parallel k_w B_0)$. For $n \geqslant 1$ and experimental quantities for $\omega_p^2, \beta_\perp$, and $J_n(x)$ the $\omega_p^2 J_n^2$ term in the left-hand side of Eq. (4) can be neglected. In this case, Eq. (4) reduces to a simple cubic equation which can be solved for $k = \hat{k}_n + \delta k$ resulting in the growth rate for this instability

$$\mathrm{Im}(\delta k) = \sqrt{3}/2\{1/4[\beta_\perp^2\,\omega_p^2\,J_n^2(x)nk_w]/v_\parallel^2\}^{1/3}. \qquad (5)$$

The lowbitron instability radiation mechanism seems to be an advantageous radiation scheme since it combines the high growth rate of the CRM obtained at large $v_\perp/v_\parallel$ parameter values, and the possibility of achieving tunable radiation at high frequency (as in FEL) by changing the wiggler period l_w or the relativistic beam factor γ.

The radiation frequency expression is

$$\omega = \frac{(1 + \beta_\parallel)\gamma^2(nk_w v_\parallel + \Omega_0/\gamma)}{1 + \gamma^2\beta_\perp^2}. \qquad (6)$$

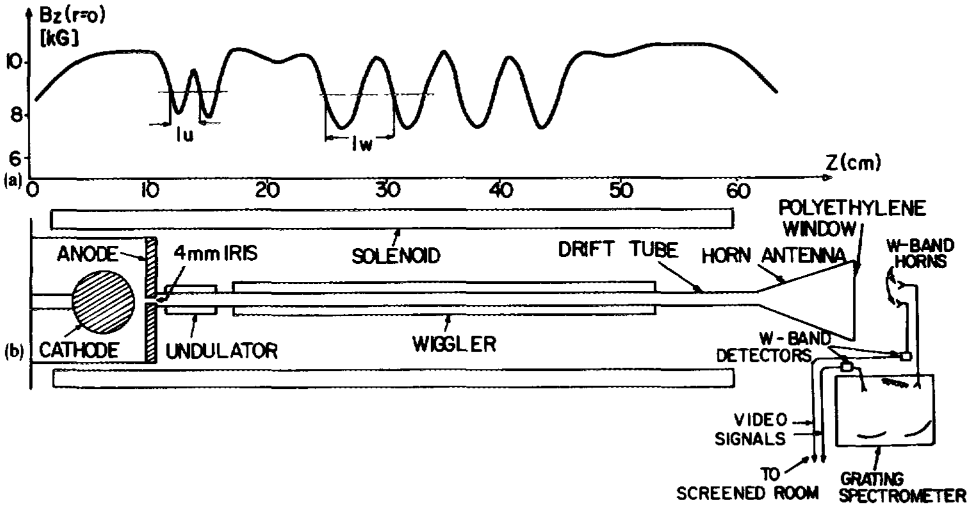

FIG. 1. (a) Total imposed magnetic field on the z axis; (b) experimental setup.

The operation at higher spatial harmonics is possible since the growth rate as given in Eq. (5) can be maximized for $n > 1$ by appropriate choice of $\delta B/B_0$ which maximizes $J_n(x)$.

The experimental setup is illustrated in Fig. 1. It consists of a 740-kV, 10-kA, 10-ns electron beam accelerator (P.I. 105) energizing a foilless diode composed of a 50-mm-diam graphite spherical cathode and a plane graphite anode with a central iris of 4 mm diameter. A 4-mm-diam pencil electron beam carrying about 400 A is extracted through the iris and launched into an evacuated stainless-steel drift tube with 11 mm i.d. and 550 mm length. The drift tube, which is also the interaction waveguide, is terminated by a conical horn antenna with 80 mm aperture diameter. A polyethylene window, attached to the horn output, provides the vacuum sealing. The diode and the drift tube are immersed in a uniform axial pulsed magnetic field produced by discharging a 150 μF/20 kV capacitor bank into a 300 μH solenoid. A 200 μs rise time magnetic pulse is obtained with 30 kG maximum amplitudes capability.

The diode configuration used was analyzed by a computer code. This analysis shows a monoenergetic axial velocity beam with $\langle \Delta E/E \rangle < 0.5\%$ and uniform radial density, emerging from the anode iris. In order to produce the transverse velocity $v_\perp$ required, the beam is passed through a short (2 periods) undulator whose period consists of a 10-mm-wide aluminum ring and a 15-mm-wide lucite ring. This undulator utilizes the magnetic diffusion technique[10] to produce a multiple mirror periodic magnetic field which can be approximated by

$$\mathbf{B} = B_0[1 + (\delta B/B_0) I_0(k_u r)\sin(k_u z)]\hat{z} - \delta B I_1(k_u r)\cos(k_u z)\hat{r}, \tag{7}$$

where $k_u = 2\pi/l_u, l_u$ being the undulator period and I_0, I_1 are the modified Bessel functions. By choosing the parameters so that $k_u \cong \Omega_0/\gamma v_\parallel$, we take advantage of the magnetoresonance effect to produce appreciable perpendicular velocity.[11,12] Now the beam enters the wiggler magnetic field created by six to seven periods of alternating aluminum and lucite rings. The period of the wiggler field is far from magnetoresonance and the first and last conducting rings are made thinner in order to produce an adiabatic transition region. The total magnetic field inside the drift tube is given by Eq. (7) where k_u is replaced by k_w. If we wish to maintain the transversely uniform longitudinal wiggler field assumed in the theoretical model and given by Eq. (1), a small beam radius R_b is required so that $k_w R_b < 1$. In this case $I_0 \cong 1$, $I_1 \cong 0$, and Eq. (7) reduces to Eq. (1). In our experiments, $k_w R_b$ varies around 0.2 so that this condition is satisfied. The diagnostic of the microwave radiation is carried out by means of a grating spectrometer which measures the frequency spectrum in the range of 60–140 GHz, and by a broadband crystal detector for measuring the total power in the W band.

In our experiment, the finite radial dimension of the waveguide must be considered, so that lowbitron interaction of the wiggler-upshifted cyclotron wave with a waveguide mode can be accounted for. In this case, Eq. (2) is replaced by

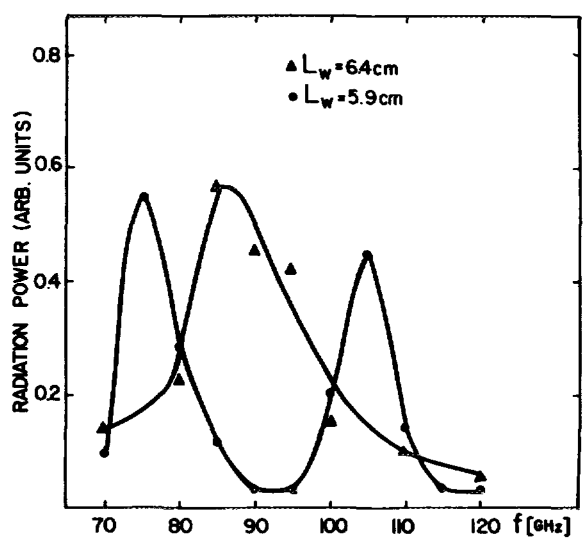

FIG. 2. Typical measured power spectra for $l_w = 6.4$ cm and $l_w = 5.9$ cm.

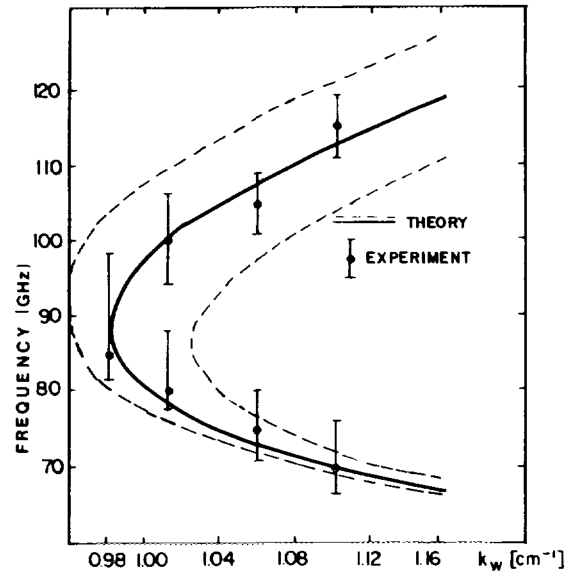

FIG. 3. Comparison of experimental data with calculations using the theoretical model. The experimental parameters assumed in the drawn theoretical curves are: $B_0 = 9.5 \pm 0.1$ kG, $\gamma = 2.45 \pm 0.08$, $\beta_\perp = 0.06 \pm 0.02$. The solid line corresponds to the nominal values and the broken lines correspond to the extreme deviations from the nominal values.

$$\omega^2 - k^2c^2 - \omega_{CO}^2 = 0\,, \qquad (8)$$

where ω_{CO} is the beam-loaded cut-off frequency of the mode. The experimental parameters were chosen so that the coupled waveguide mode is the transverse electric TE_{01} and the expected radiation frequencies are in the *W*-band range. Since the accelerator pulse is very short, the operation of the laser is in the super-radiant mode. The frequency spectra were measured for $l_w = 6.4$, 6.2, 5.9, and 5.7 cm. Typical results of the spectrum for $l_w = 6.4$ and 5.9 cm are given in Fig. 2. For $l_w = 6.4$ cm the beam wave [Eq. (3)] and the waveguide mode [Eq. (8)] dispersion curves are expected to be tangent with each other, and the spectrum is expected to have a single peak at $f = 87$ GHz. For $l_w = 5.9$ cm the dispersion curves intersect and radiation peaks at $f = 73$ GHz and $f = 108$ GHz are expected. The peak power detected around 85 GHz for $l_w = 6.4$ cm and the two peaks around 75 and 105 GHz detected for $l_w = 5.9$ cm are in good agreement with calculations.

All the measured frequencies for the various wiggler field periods mentioned before are compared with calculations in Fig. 3. The solid line represents calculated frequencies expected from intersection of the beam wave and the TE_{01} dispersion curves [Eqs. (3) and (8)] as a function of k_w, for nominal experimental values of B_0, γ. The value of $v_\perp$ was estimated from a computer code simulation. The broken lines represent the limits when experimental errors of the above parameters are taken into account. The frequencies detected at the power peaks are represented by dots when the error bars define the -3 dB bandwidth around each peak. A good agreement is obtained for all the measured data.

As mentioned before, the lowbitron interaction mechanism requires an electron beam having perpendicular energy as an initial condition. Therefore, one can expect possible arising of CRM instability. In our experiment, the cyclotron mode dispersion curve [obtained by setting $k_w = 0$ in Eq. (3)] does not intersect with the TE_{01} mode dispersion curve, but coupling with lower modes and especially with the fundamental TE_{11} mode is still possible. In this case, radiation around $f = 110$ GHz is expected. In experiments where the lowbitron did not radiate around this frequency, as in the case of $l_w = 6.4$ cm (see Fig. 3), we could not observe any radiation which can be associated with the CRM instability.

The interpretation that the interacting waveguide mode is the TE mode is based on the excellent fit of the theoretical curve of Fig. 3 (based on the TE_{01} mode dispersion equation) with the experimental data. The intersections of other waveguide modes dispersion curves with the beam mode give alternate ω versus k curves, way off the error bars of the experimental data. However, we cannot rule out the possibility that the transverse magnetic TM_{11} mode was also excited, since it has the same dispersion curve as the TE_{01} mode and has sufficiently large transverse electric field component to participate in a lowbitron interaction process. Symmetry considerations and the good fit between the field profile of the TE_{01} mode and the electron beam transverse energy excitation profile lead us to prefer the conjecture of the TE_{01} mode excitation. Unfortunately, mode pattern diagnostic means were not available in the present experiment to resolve the question.

In summary, we have presented an experimental demonstration of the theoretically predicted longitudinal wiggler FEL—lowbitron—operating as a microwave radiation source in the *W*-band frequency range. The measured frequency spectra for different wiggler field periods agree with radiation frequencies predicted from the theoretical model assuming interaction between the longitudinal wiggler upshifted cyclotron wave and the TE_{01} electromagnetic waveguide mode.

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[1] R. Q. Twiss, Aust. J. Phys. **11**, 564 (1958).
[2] J. Schneider, Phys. Rev. Lett. **2**, 504 (1959).
[3] J. L. Hirshfield and J. M. Wachtel, Phys. Rev. Lett. **12**, 533 (1964).
[4] J. L. Hirshfield and V. L. Granatstein, IEEE Trans. Microwave Theory Tech. **MTT-25**, 522 (1977).
[5] P. Sprangle, R. A. Smith, and V. L. Granatstein, in *Infrared and Millimeter Waves*, edited by K. J. Button (Academic, New York, 1979), Vol. 1, p. 279.
[6] W. A. McMullin and G. Bekefi, Appl. Phys. Lett. **39**, 845 (1981).
[7] W. A. McMullin and G. Bekefi, Phys. Rev. A **25**, 1826 (1982).
[8] R. C. Davidson and W. A. McMullin, Phys. Fluids **26**, 840 (1983).
[9] A. Grossman, T. C. Marshall, and S. P. Schlesinger, Phys. Fluids **26**, 337 (1983).
[10] K. D. Jacobs, R. E. Shefer, and G. Bekefi, Appl. Phys. Lett. **37**, 583 (1983).
[11] I. Shraga, C. Leibovitch, Y. Goren, S. Eckhouse, and A. Gover, in *Beams 83: Proceedings of the Fifth International Conference on High Power Particle Beams, Sept. 1983, San Francisco, CA*, edited by R. J. Briggs and A. J. Toepfer (L. L. N. L., Livermore, CA), p. 576.
[12] V. L. Granatstein, P. Sprangle, M. Herndon, R. K. Parker, and S. P. Schlesinger, J. Appl. Phys. **46**, 3800 (1975).