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A KINETIC SMALL SIGNAL GAIN ANALYSIS OF A PLANAR WIGGLER FEL, OPERATING IN THE HIGH HARMONIC (STRONG WIGGLER) REGIME

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A kinetic linear analysis of a strong planar wiggler FEL operating at the fundamental or odd harmonic frequencies is presented in this paper. The gain dispersion equation in the high harmonic regime is shown to be an extension of the known general FEL equation, and converges to it in the weak pump limit. In the low gain, cold, tenuous beam regime we get the known gain expression with the Bessel function correction term. A similar term also appears in the gain expression of the high gain strong pump limit.

When space charge effects are significant (Raman regime), the strong pump gain dispersion relation is more complicated than the corresponding weak pump relation. In the cold beam limit it leads to a quintic polynomial dispersion relation instead of the known cubic equation. This brings about gain curve dependences on the detuning parameter which are double humped and with a relatively high bandwidth.

The higher the order of the harmonic frequency, the stronger is the effect of the axial velocity spread on the gain and the tighter are the beam acceptance parameters. The analytical model permits an arbitrary axial velocity distribution. The gain damping effect is computed, both with a simple model of a Gaussian electron velocity distribution and with a more accurate model, taking into account an analytical expression for the asymmetrical skewed velocity distribution produced by the combined finite emittance and energy spread effects.

1. Introduction

The fundamental operating wavelength of a free electron laser (FEL) is determined by the energy of the electron beam according to the scaling law $\lambda = \lambda_w/(2\gamma_{0z}^2)$ (λ_w is the wiggler period, and γ_{0z} is the longitudinal relativistic Lorentz factor of the e-beam). One finds that a conventional wiggler FEL ($\lambda_w \sim 0.01$–0.1 m) operating in the visible range requires an electron beam energy of at least 100 MeV. Consequently, accelerators such as the Van de Graaff that are known to operate with high beam quality but relatively low energy can be used in FELs operating in the far infrared region only. High harmonic operation is one way by which such FELs can be made to operate at optical frequencies, using conventional period wigglers. Alternatively, this method can be used to make RF linac and storage ring based FELs to operate at ever decreasing shorter wavelengths down to the VUV–X-ray regime [3].

FEL operation in high harmonics has been considered by various workers as a method of extending the FEL range of operation towards shorter wavelengths. Madey and Taber [1] pointed out the feasibility of FEL interaction at high odd harmonic frequency as a unique property of the planar wiggler FEL, due to the periodic variation (at half the wiggler period) in the axial velocity of the electrons propagating along the wiggler. Colson [2] investigated nonlinear aspects of the higher harmonic operation of planar wiggler FELs in the cold, tenuous beam regime. Dattoli et al. [3] evaluated the dependence of the small signal gain on the electron beam quality factors in the low gain, tenuous beam regime. An experimental observation of gain in the third harmonic was reported by Barbini and Vignola [4]. A self-consistent kinetic description of the FEL instability in a planar magnetic wiggler was developed by Davidson and Wurtele [5], and a linear model including some collective and high gain effects was suggested by Murphy and Pellegrini [6].

The present paper presents a kinetic, linear analysis of a strong (relativistic wiggle velocity), planar wiggler FEL operating at the fundamental or odd harmonic frequencies. Space charge and axial velocity spread effects are included in this model. The resulting gain dispersion relation at fundamental and high harmonic frequencies is an extension of the known fundamental FEL equation [7], and is reduced to it in the limit of a small magnetic field wiggler.

Analytical and numerical solutions of the gain dispersion relation are shown, and some unique aspects of FEL operation at higher harmonics in the space charge dominated regime are discussed.

2. The theoretical model

The development of the gain dispersion relation of the planar wiggler FEL operating at a high harmonic frequency is based on the solution of Maxwell equations simultaneously with the linearized Vlasov equation in one dimension. Assuming small-signal operation, the perturbation method is applied. Vlasov equations in the zero and first orders are solved by the method of the characteristic lines in the p_z–z plane (the axial momentum and coordinate, respectively). After performing a Laplace transform in the z dimension, the gain dispersion relation is obtained by the assumption of a near-resonance operation at one of the harmonics.

The transverse components of the e.m. wave under the 1-D assumption ($\partial/\partial x, \partial/\partial y = 0$) are described by the reduced Maxwell equations:

$$\frac{\partial^2}{\partial z^2}\boldsymbol{E}_t(z,t) - \mu_0\varepsilon\ddot{\boldsymbol{E}}_t(z,t) = \mu_0\dot{\boldsymbol{J}}_t(z,t) \tag{1}$$

and

$$\dot{\boldsymbol{B}}_t(z,t) = -\hat{z}\times\frac{\partial}{\partial z}\boldsymbol{E}_t(z,t). \tag{2}$$

The driving current $\boldsymbol{J}_t(z,t)$ is given by:

$$\boldsymbol{J}_t(z,t) = -e\int_p \boldsymbol{v} f(\boldsymbol{p},z,t)\ \mathrm{d}^3p. \tag{3}$$

The momentum distribution of the electrons $f(\boldsymbol{p},z,t)$ is governed by the Vlasov equation:

$$\dot{f}(\boldsymbol{p},z,t) + v_z\frac{\partial}{\partial z}f(\boldsymbol{p},z,t) + \boldsymbol{F}(\boldsymbol{p},z,t)\cdot\frac{\partial}{\partial \boldsymbol{p}}f(\boldsymbol{p},z,t) = 0. \tag{4}$$

The driving force $\boldsymbol{F}(\boldsymbol{p},z,t)$ is defined by the Lorentz equation:

$$\boldsymbol{F}(\boldsymbol{p},z,t) = -e\left[\boldsymbol{E}_t(z,t) + \hat{z}E_z(z,t) + \boldsymbol{v}\times\boldsymbol{B}_t(z,t) + \boldsymbol{v}\times\boldsymbol{B}_{\mathrm{w}}(z,t)\right]. \tag{5}$$

The longitudinal electric field $E_z(z,t)$ is caused by collective effects and is the solution of the Poisson equation:

$$\frac{\partial}{\partial z}E_z(z,t) = -\frac{e}{\varepsilon}\int_p f(\boldsymbol{p},z,t)\,\mathrm{d}^3p. \tag{6}$$

The "pump" force in eq. (5) ($\boldsymbol{F}_{\mathrm{w}} = e\boldsymbol{v}\times\boldsymbol{B}_{\mathrm{w}}$) is induced by the planar wiggler static magnetic field, which is approximated by:

$$\boldsymbol{B}_{\mathrm{w}}(z,t) = \hat{x}B_{\mathrm{w}}\cos k_{\mathrm{w}}z. \tag{7}$$

Eqs. (1)–(7) are a self-consistent complete set of equations. To solve them, we apply the standard perturbation method that is applicable with the small signal assumption. The electron momentum distribution is expressed in the form:

$$f(\boldsymbol{p},z,t) = f_0(\boldsymbol{p},z) + f_1(\boldsymbol{p},z,t), \tag{8}$$

where $f_0(\boldsymbol{p},z)$ is the steady state e-beam momentum distribution in the wiggler field, and $f_1(\boldsymbol{p},z,t)$ is a perturbation due to the interaction with the e.m. wave, assuming that $f_0 \gg |f_1|$ exists. In a similar manner, the driving force $\boldsymbol{F}(\boldsymbol{p},z,t)$ is written as:

$$\boldsymbol{F}(\boldsymbol{p},z,t) = \boldsymbol{F}_0(\boldsymbol{p},z) + \boldsymbol{F}_1(\boldsymbol{p},z,t), \tag{9}$$

where $|\boldsymbol{F}_0| \gg |\boldsymbol{F}_1|$ exists.

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FEASIBILITY OF HIGH GAIN X-RAY FREE ELECTRON LASERS

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We investigate the feasibility of a tunable VUV/soft X-ray FEL operating in a dedicated storage ring. Because of poor mirror reflectivity available at these wavelengths, the FEL must operate in the intermediate high gain regime (gain = 3–5). In order to obtain this high gain, a long interaction length (27 m), a high current (270 A) and strong magnetic fields ($\bar{a}_w > 1$) are necessary. Consequently some velocity spread effects and possibly even some space charge effects must be considered. We use a one-dimensional, comprehensive small signal gain analytical model to conduct a parametric study of the FEL operating parameters and their scaling laws. The results of the analytic model are compared with results obtained with the 2-D FEL simulation code FRED, and the effect of optical guiding on the gain is estimated using an enhanced filling factor based on the observed guided mode size. We conclude that oscillation below 300 Å is feasible with existing mirror technology.

1. Introduction

Since the first demonstration of a free electron laser (FEL) amplifier [1] and oscillator [2], at 10.6 and 3.4 μm wavelengths, respectively, much research effort has been directed towards short wavelength operation of FELs. Whereas the first experiments were carried out with a superconducting linac, shorter visible wavelength lasing (6400 Å) was obtained recently with an optical klystron operating in a storage ring [3]. Visible wavelength radiation at the third harmonic (and possibly lasing) was also measured in another recent experiment at 5300 Å, using again a superconducting linac [4].

A number of proposals have been made for operating FELs in the short wavelength UV and X-ray regime [5–7]. There is, of course, considerable interest in the development of presently nonexistent high-power bright coherent sources in the X-ray regime. Considering only scientific aspects, such development will bear a significant impact on the field of microbiology, biochemistry, material sciences, radiology, medical diagnostics and treatment. Recent technological progress makes it now possible for us to predict, on the basis of reliable basic theory, the feasibility of laser operation in the soft X-ray wavelength regime. We use in this study the electron beam parameters expected to be realizable with present storage ring accelerator technology [8]. A study carried out recently at Los Alamos National Laboratory also suggested the feasibility of VUV FELs operating at wavelengths down to 500 Å [9] utilizing a pulsed rf linac. Our study, based on the design parameters of a FEL dedicated storage ring [10], suggests the feasibility of X-ray FELs operating at wavelengths down to 300 Å.

One of the main obstacles thus far in realizing X-ray oscillators is the present poor state-of-the-art of laser mirrors at wavelengths below 100 Å [11]. The low mirror reflectivity presently available, or feasible in the near future, directs us to consider FEL operation in the high gain regime. If we consider $R = 60\%$ as an attainable mirror reflectivity value, and would like to have, say, $T = 10\%$ transmission through one end, then a minimum single-pass gain of $G \equiv P_{out}/P_{in} = 3.2$ is necessary in order to reach the oscillation threshold. Basic FEL small signal gain theory [12] indicates that long interaction length and high peak current are necessary to obtain such a high gain. However, in most accelerators, high peak current requirements lead to pour beam quality parameters (energy spread and emittance) [13]. This may cause the laser to operate in the warm beam regime, where gain is substantially reduced relative to a cold beam [14]. The longer interaction length, the better the beam quality parameters should be in order to maintain operation in the cold beam regime. This results in a trade-off between the long interaction length requirement and the high current requirement. This contradictory requirement dilemma is somewhat relieved if operation in the high gain regime is feasible because then the beam quality acceptance parameters of the FEL are relaxed [14].

The discussion above gives the rationale for choosing a storage ring for soft X-ray FEL operation. In a storage ring, high instantaneous current (hundreds of amperes) is quite readily available, while the beam quality parameters (emittance, energy spread) stay very small. This suggests that in order to study the limits of X-ray wavelength operation of FELs, a comprehensive small signal analytical model should be used, including high gain, exact axial velocity spread, and space-charge (longitudinal plasma waves) effects [15]. Using the basic design parameters of the Stanford FEL dedicated storage ring, we calculate the gain at the first and third harmonics of various wiggler schemes. The results show considerable gain at 300 Å wavelength.

2. The theoretical model

The theoretical model that is used in this paper is based on the solution of maxwell's equations, simultaneously with the linearized Vlasov equation with characteristic lines adequate to the electron trajectories in a strong planar Wiggler [15]. This results in a gain dispersion relation at the $2n+1$ harmonic wavelength, as follows:

$$\left.\frac{\bar{E}(s)}{E(0)}\right|_{2n+1} = \frac{\left[1+J_n^2(\ )\chi_{2n+1}\right]\left[1+J_{n+1}^2(\ )\chi_{2n+1}\right]}{(s-\mathrm{i}k_{z0})\left[1+J_n^2(\ )\chi_{2n+1}\right]\left[1+J_{n+1}^2(\ )\chi_{2n+1}\right]} \times \frac{1}{-\mathrm{i}\kappa_{2n+1}\left[J_n(\ )-J_{n+1}(\ )\right]^2\left[1+J_n(\ )J_{n+1}(\ )\chi_{2n+1}\right]\chi_{2n+1}}. \tag{1}$$

The argument of all Bessel functions is:

$$\alpha_{2n+1} = \frac{1}{2}\frac{\bar{a}_\mathrm{w}^2}{1+\bar{a}_\mathrm{w}^2}(2n+1), \tag{2}$$

where

$$\bar{a}_\mathrm{w} = \frac{eB_\mathrm{w}}{\sqrt{2}\,mck_\mathrm{w}} \tag{3}$$

is the normalized rms vector potential of the static planar wiggler field

$$\boldsymbol{B}_\mathrm{w} = \hat{x}B_\mathrm{w}\sin k_\mathrm{w}z.$$

The longitudinal susceptibility of the electron beam $\chi(s)$ is defined by:

$$\chi(s) = -\frac{\mathrm{i}e^2}{\omega}\int\frac{\partial f(\bar{P}_z)}{\partial P_z}\frac{\mathrm{d}P_z}{s-\mathrm{i}\omega/\bar{V}_z}, \tag{4a}$$

and

$$\chi_{2n+1} \equiv \chi(s+\mathrm{i}(2n+1)k_\mathrm{w})/\epsilon_0. \tag{4b}$$

The "exact" axial momentum distribution can be evaluated for a storage ring FEL, assuming that the electron total energy, $E=\gamma_0 mc^2$, and the electrons initial angle at the wiggler entrance, ϕ, are Gaussian distributed independent random variables [8] with standard deviations of $\Delta E/\sqrt{2}$ and $\Delta\phi/\sqrt{2}$, respectively. This results in the normalized distribution function [15,16]:

$$f(x) = R\exp[R(R+2x)]\ \mathrm{erfc}(R+x), \tag{5}$$

where we assume the optimal beam radius to be:

$$r_{\mathrm{b}0} = \frac{\Delta\phi}{k_\beta} \tag{6}$$

and use the definitions:

$$x \equiv \frac{\bar{V}_z - \bar{V}_{0z}}{\bar{V}_{0z}\delta\gamma}, \tag{7a}$$

$$\delta\gamma \equiv \frac{\Delta\gamma/\gamma_0}{\gamma_{0z}^2}, \tag{7b}$$

$$R \equiv \frac{\delta\gamma}{\Delta\phi^2}, \tag{7c}$$

$$\gamma_{0z} \equiv \frac{\gamma_0}{\left(1+\bar{a}_w^2\right)^{1/2}}, \tag{7d}$$

and

$$k_\beta = \frac{k_w \bar{a}_w}{\gamma_0} \tag{7e}$$

is the betatron oscillation wavenumber. The complementary error function erfc(x) is defined by:

$$\operatorname{erfc}(x) \equiv \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt. \tag{7f}$$

The longitudinal susceptibility of the electron beam, using the "exact" distribution of eq. (5), becomes [15]:

$$\chi(s) = \frac{\epsilon\theta_p^2}{s^2\delta\gamma} 2R \int_0^\infty e^{-2Ry} Z'(\xi + y)\, dy, \tag{8}$$

where θ_p, the space-charge parameter, is defined by

$$\theta_p = \left[\frac{eI_0}{\pi mc^2 \epsilon r_{b0}^2 \gamma_{0z}^2 \gamma_0}\right]^{1/2}. \tag{9}$$

$Z(\xi)$ is the well known plasma dispersion function (the complex error function) and its argument ξ is given by:

$$\xi = \frac{i\omega/s - \bar{V}_{0z}}{\bar{V}_{0z}\delta\gamma}. \tag{10}$$

The contribution of the energy spread to the total detuning spread of the FEL can be estimated by the detuning spread parameter due to energy spread:

$$\bar{\theta}_{th}\left(\frac{\Delta E}{E}\right) = 4\pi(2n+1) N_w \frac{\Delta E}{E}. \tag{11}$$

The parameter R gives the ratio between the contributions of the energy spread and the emittance to the detuning spread.

The coupling parameter κ in eq. (1) is expressed in this case in the form:

$$\kappa_{2n+1} = \pi N_w \frac{\bar{a}_w^2}{1+\bar{a}_w^2}(2n+1)\mathrm{FF}, \tag{12}$$

where we used the synchronism condition of the $2n+1$ harmonic:

$$\lambda_{2n+1} = \frac{\lambda_w}{2\gamma_{0z}^2(2n+1)}. \tag{13}$$

The filling factor FF in eq. (12) is given for small free space diffraction of the e.m. radiation beam by:

$$FF_{fs} = \frac{1}{1 + \frac{\omega_0^2}{2r_{b0}^2}}, \tag{14}$$

where ω_0, the Gaussian mode waist size, is given in the case where the interaction length L is twice the Rayleigh length Z_R and the focusing is optimal, by:

$$\omega_0 = \sqrt{\frac{\lambda_{2n+1} L}{2\pi}}. \tag{15}$$

The assumption of free space diffraction may lead to underestimation of the gain in the high gain regime where the effect of optical guiding by the electron beam [17,18] may confine the e.m. beam to a smaller waist size than in free space. This effect leads to a higher effective filling factor, and therefore to a higher coupling parameter value and higher gain. For cases in which optical guiding occurs, we approximate the effect by using the filling factor of eq. (14) with ω_0 replaced by the radius of the focused e.m. radiation beam. This enhanced filling factor is denoting by FF_{og} (optically guided).

In the next section, we describe the results of the gain calculation in various cases. This was done in each case by combining the results of two computer codes. The FRED 2-D simulation code [19] was operated to find the waist size of the e.m. radiation beam in each case. In the cases considered here the amplified mode exhibited guiding once exponential power growth began. The resulting guided mode size was used to estimate an enhanced filling factor and was fed to the extended WARM program in order to solve the gain dispersion relation (1), including space-charge and "exact" electron distribution (eq. (5)) effects.

3. The numerical results

Using the model described in the previous section we calculated the gain at the first and third harmonics for various FEL schemes considered for implementation in the Stanford FEL dedicated storage ring. The aim of this investigation was to check the feasibility of building up to oscillation at 300 Å wavelength, which requires a small signal gain of at least 3.2 per pass.

The basic electron beam parameters of the Stanford storage ring are [10]:

$$\gamma_0 = 2000, \quad I_0 = 270 \text{ A}, \quad \epsilon_x = \epsilon_y = \pi \times 1.7 \times 10^{-8} \text{ (m rad)}, \quad \frac{\Delta E}{E} = 8.2 \times 10^{-4}.$$

The maximum wiggler length L is 27 m and the minimum gap between the poles is 30 mm. Three different wiggler schemes with periods of 11.4, 8.3 and 6.4 cm, and magnetic field densities (on the axis) of 7.2, 5.25 and 3.8 kG respectively, have been considered. The resulting FEL parameters of the various cases are listed in table 1.

In case 1 ($\lambda_w = 0.114$ m), the fundamental radiation wavelength is relatively long (4300 Å) and free space diffraction leads to a small filling factors: $FF_{fs} = 0.11$. The resonance detuning curves in fig. 1 for the first and third harmonic are based on this free space filling factor. The electron beam in case 1 may be considered cold as the fundamental wavelength is relatively long ($\bar{\theta}_{th} = 2.44$ and $R = 2.4$), but is quite warm at the third harmonic ($\bar{\theta}_{th} = 7.3$). Therefore, the third harmonic gain is further reduced relative to the fundamental harmonic gain. Our estimation for the gain at the fundamental is improved by using the FRED 2-D code to determine the guided mode size, which is then used to estimate an improved filling factor. In this case $FF_{og} = 0.38$, which provides much higher gain. The detuning curve resulting from the use of the improved filling factor is presented in fig. 2, together with the gain curve generated by FRED and the free space curve of fig. 1 (the fundamental) for comparison.

Table 1
The resulting FEL parameters for the three cases (MKS units)

Parameter	Case 1		Case 2		Case 3	
	1st har.	3rd har.	1st har.	3rd har.	1st har.	3rd har.
λ_w	0.114	0.114	0.083	0.83	0.064	0.064
N_w	237	237	325	325	422	422
$\bar{a}_w$	5.4	5.4	2.87	2.87	1.6	1.6
k_β	0.15	0.15	0.11	0.11	0.08	0.08
r_{b0}	3.4×10^{-4}	3.4×10^{-4}	4.0×10^{-4}	4.0×10^{-4}	4.6×10^{-4}	4.6×10^{-4}
γ_{0z}	364	364	658	658	1060	1060
λ_{2n+1}	4.3×10^{-7}	1.4×10^{-7}	9.6×10^{-8}	3.3×10^{-8}	2.85×10^{-8}	9.5×10^{-9}
ω_0 (fs)	1.4×10^{-3}	7.8×10^{-4}	6.4×10^{-4}	3.7×10^{-4}	3.5×10^{-4}	2.0×10^{-4}
ω_g (guided)	6.0×10^{-4}	–	4.6×10^{-4}	–	3.4×10^{-4}	–
FF_{fs}	0.11	0.27	0.43	0.7	0.776	0.96
FF_{og}	0.38	–	0.59	–	0.785	–
$\bar{\theta}_{th}(\Delta E/E)$	2.44	7.34	3.36	10.1	4.35	13.0
R	2.4	2.4	1.03	1.03	0.54	0.54
α_{2n+1}	0.48	1.45	0.45	1.34	0.36	1.08
$[JJ]^2_{2n+1}$	0.501	0.11	0.54	0.11	0.62	0.11
κ_{fs}	79.0	583.0	391.0	1912.0	744.0	2746.0
κ_{og}	278.0	–	536.0	–	753.0	–
θ_p	1.23	1.23	0.58	0.58	0.31	0.31
G_{max} (fs)	54.0	16	39.9	3.5	5.5	1.2
G_{max} (og)	1881	–	83.4	–	8.0	–
G_{max} (FRED)	682	–	72.2	–	7.9	–

We note at once in fig. 2 that the 1-D gain curve c resulting from use of the enhanced filling factor is nearly three times higher than that generated by the FRED code, curve b. We believe this discrepancy is due to the effects of diffraction. The interaction region is almost ten times longer than the input Rayleigh length for the guided mode solution realized with FRED, and the electron beam is small ($r_{b0} = 340$ μm) compared to the radius of the guided mode ($\omega_g = 600$ μm). Using as a model the work of Moore [17], we expect these scale differences to result in a guided mode gain length smaller than that of the one-dimensional theory using an enhanced filling factor. In Moore's notation, we have $\hat{a} \cong 1.0$ in this case, which is

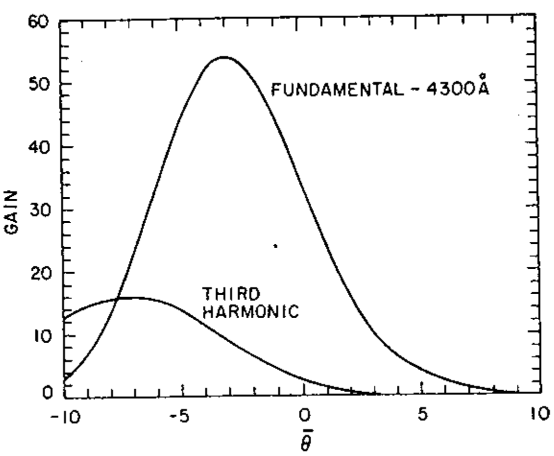

Fig. 1. Calculated single pass gain (P_{out}/P_{in}) as a function of resonance detuning parameter $\bar{\theta} = (\omega/v_{0z} - k_w - \omega/c)L$ for the fundamental (4300 Å) and third harmonic (1433 Å) in the Stanford Storage Ring FEL, obtained from solutions of the one-dimensional gain dispersion relation eq. (1) for free space input parameters of table 1, case 1.