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SIMULATION AND ANIMATION OF ELECTRON MOTION IN THE PONDEROMOTIVE POTENTIAL OF LASER BEATS

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The behavior of an electron in the beat wave field of two counter-propagating pulsed CO_2 laser beams, operating at different frequencies is usually studied by solving the equation of motion of the electron numerically. These solutions give the energy spectrum of the electrons at the end of the interaction. The present paper describes a computer program that animates the motion of the electron, giving a real-time picture of its motion during the entire interaction process. This program can be used to understand several trapping mechanisms of an electron moving in a ponderomotive field superimposed on a dc axial field. The first one is the case in which the electron is created inside the ponderomotive field. In this case the program can be used to show visually that such an electron can get trapped only if its energy is around the synchronization energy and its phase corresponds to the trap phase. In the second case the program can be used to understand the trapping mechanism for the case in which there is a temporal rise in the ponderomotive field. For the case in which an abrupt axial field jump is used to trap the electron in the ponderomotive wells, numerical simulations show periodicity in the relation between the trapping fraction and the energy of the electron. This periodicity and also the trapping mechanism can be clearly understood using the animated motion of the electron together with the varying ponderomotive potential. The program can also be used to understand the detrapping of trapped electrons in the case where there is a temporal fall of the field.

1. Introduction

The equation of motion of an electron in a combined axial and ponderomotive field does not have an analytical solution. This equation can only be solved numerically. When using such numerical solutions it is hard to display the whole picture of the fields combined with the position and energy of the electrons for complex dynamic mechanisms such as the ones described in refs. [1,2]. The need for a computer program that not only solves the equation of motion of an electron in a ponderomotive field, but animates its motion during the whole process rose when we could not explain an effect of periodic behavior of the electron trapping efficiency as a function of the injection energy. This effect appeared as a result of an ordinary numerical simulation made for the case in which trapping was achieved using an abrupt axial field jump [1,2]. The work began by developing a program that simulates the motion of an electron in a time independent ponderomotive field, then it was expanded to simulate cases in which the ponderomotive field varies with time and ended by developing a program that animates the motion of an electron for the complex case mentioned above

2. Theory

The governing equation of motion of an electron in a ponderomotive field superimposed on an axial electric field is the axial force equation [3,4]:

$$\frac{\mathrm{d}(\gamma m v)}{\mathrm{d}t} = -eE_{ax}(z) - eE_p(t) \times \cos\{(\omega_s - \omega_w)t - (k_s + k_w)z\}, \tag{1}$$

where E_{ax} is the externally applied axial field and E_p is the ponderomotive field, which for the laser beat electromagnetic wiggler is given by:

$$E_p = \frac{e\sqrt{\mu/\epsilon}}{\gamma_r mc^2}(\lambda_s + \lambda_w)|\hat{e}_s \cdot \hat{e}_w| \frac{\sqrt{P_s P_w}}{\pi^2 w_s w_w}. \tag{2}$$

$\hat{e}_s$, $\hat{e}_w$, P_s, P_w, w_s, w_w are the polarization unit vectors, powers and waists of the signal and wiggler waves respectively. The resonance electron velocity v_r is given by:

$$v_r = \frac{\omega_s - \omega_w}{k_s + k_w}, \tag{3}$$

and the resonance phase ψ_r is given by:

$$\psi_r = \sin^{-1}\left(\frac{E_{ax}}{E_p}\right). \tag{4}$$

Only electrons which are inserted into the ponderomotive field with an energy around γ_r and a phase around ψ_r are trapped inside the ponderomotive buckets. Electrons achieving the resonance energy γ_r within the interaction region do not get trapped since they follow open orbits in phase space. Instead of trapping, they experience the phase area displacement mechanism [3,4] and exchange energy in an opposite direction relative to

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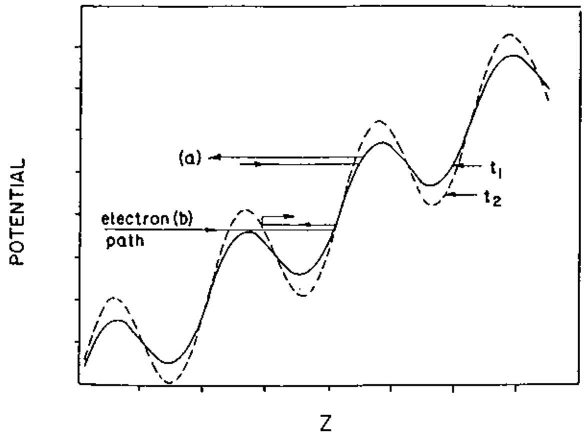

Fig. 1. Electron conservation of energy diagram.

the trapping process. Fig. 1 shows the conservation of energy diagram of an electron in the combined ponderomotive and decelerating dc fields. The diagram is shown as viewed in the rest frame of the ponderomotive potential. In this moving frame all the fields are static and the system is conservative. Electrons with a constant energy injected into a constant ponderomotive field are reflected and cannot get trapped in the ponderomotive buckets as shown in curve a. In order to trap the electrons inside the buckets either the amplitude of the laser fields has to rise gradually as shown in curve b or alternatively the energy of the electrons has to go down steeply during the approach of the electron. Such a change in the potential energy of the electron can only be attained by a force which looks time dependent in the moving frame of the beat wave rest frame. If the time variation of the field in this frame is fast relative to one period transit time, a nonconservative abrupt reduction of the electron energy takes place and consequently the electron is inserted into the trap. In fig. 2 we display the computer simulation results for

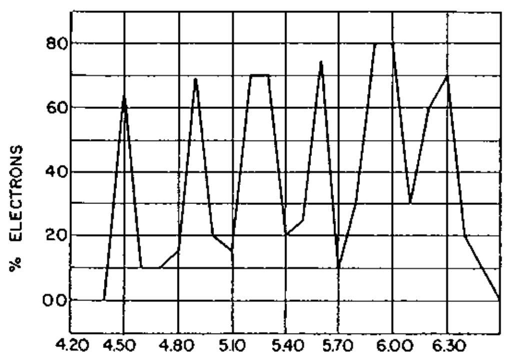

Fig. 2. Electron trapping efficiency against the relative electron energy using an abrupt axial field of 1000 V/m.

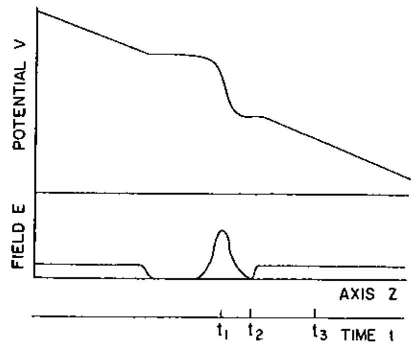

Fig. 3. Axial field and potential along the interaction length.

the trapping efficiency achieved by the spatial field variation shown in fig. 3 (which corresponds to a fast temporal field variation in the moving frame). The ponderomotive field was 230 V/m superimposed on a dc axial field of 65 V/m, with a maximum axial abrupt field of 1000 V/m, and the electrons were assumed to be injected without energy spread. In the following sections we will deal with the reason behind the periodic behavior of the efficiency variation as a function of electron energy.

3. Computer program

The computer program described in this paper runs on an IBM PC-AT personal computer and is written in Quick Basic version 4. The program solves the differential equation of motion of an electron (eq. (1)) and in parallel displays on the computer monitor screen its instantaneous position in the energy diagram in the frame of reference of the beat wave as shown in fig. 1. At every moment the program updates the ponderomotive field and the axial field, then it updates the position of the electron relative to these fields, giving a moving picture of the electron and the fields acting on it. The user of the program has the option to lower or raise the animation speed making it possible for him to view complex and fast processes in slow motion. Three different programs were written in order to simulate the following trapping mechanisms:

1) The first program animates the simple case in which both the axial and ponderomotive fields are time invariant. In this program the user has the option to change the relative initial energy of the electron making it possible for him to animate the simple trapping and PAD mechanisms.

2) In the second program the user has the option to choose a linear temporal rise or fall in the ponderomotive field in addition to the initial energy of the electron. This program helps the user to understand electron trapping mechanisms caused by a rising field as op-

posed to detrapping mechanisms caused by a falling field.

3) The third program deals with the complex problem in which the axial field varies with time in the moving frame which corresponds to the spatial variation of potential shown in fig. 3 which is a model for the field variation in the experimental setup of refs. [1,2]:

$$E_{ax} = 500 \qquad 0 < t < t_1,$$
$$E_{ax} = 500/\cosh^2(5 \times 10^{-13}(t - t_1)) \qquad t_1 < t < t_2,$$
$$E_{ax} = 50 \qquad t_2 < t < t_3,$$

where E_{ax} is the axial field in V/m and t is the time in seconds.

In this program the user has control of the relative initial energy and phase of the electron. As will be shown in the next section the resultant trapping is a combination of a trapping mechanism and a subsequent detrapping mechanism.

4. Sample runs

In this section we will describe several runs that demonstrate the help of the animation program * in understanding the physics of complex processes. Each one of these runs simulates the actual parameters of the experiments made at our laboratory [2]. The constant parameters used in the simulation are given in table 1.

1) Run program FRM1 (animation speed = 50, initial energy = 0.5 eV). This exercise demonstrates trapping of an electron created inside the ponderomotive field. The electron moves with the ponderomotive wave while oscillating at the synchrotron frequency. Using any initial energy between 0 and 1.0 eV which is the trap depth in this case gives the same result, while using an initial energy higher than 1.0 eV demonstrates the PAD mechanism scenario in which the electron is reflected backwards by the ponderomotive potential. This last example represents a typical case in which trapping is impossible when an electron enters an existing time independent conservative field and ponderomotive potential.

2) Run program FRM2 (animation speed = 50, initial energy = 1.05 eV, ponderomotive field variation ratio with time = 0.05%/ns). This exercise demonstrates that when the ponderomotive field rises with time trapping is possible although the initial energy is higher than the trap depth of 1 eV. On the other hand using an initial energy of 0.95 eV and a ratio of −0.02%/ns causes a trapped electron to get eventually detrapped after several synchrotron cycles.

3) Run program FRM3 (animation speed = 50, initial energy = 3.6 eV, initial phase = 5 rad). This exercise shows a mechanism in which trapping is achieved by using a high axial field of 500 eV. In this case the electron remains trapped during the whole process in which the axial field varies as given in the previous section. On the other hand using the same initial energy with an initial phase of 6.1 causes trapping at the beginning (at time $0 < t < t_2$) followed by detrapping in the period when the axial field goes back to 50 V/m (at time $t_2 < t < t_3$). Using a combination of initial energy and phase as given in table 2 results in a periodic trapping fraction as a function of the initial energy of the electron as shown in the same table.

* Copies of the animation program are available upon request from the authors.

Table 1
Simulation parameters

Signal power	350 kW
Wiggler power	200 kW
Signal wavelength	9.2938 μm
Wiggler wavelength	10.591 μm
Laser beam waist	1.4 mm
Axial electric field	50 V/m
Resonance energy	1088 eV

Table 2
Initial conditions and trapping results from FRM3

Initial energy	$0 < t < t_2$		$0 < t < t_3$	
	Trapping initial phases	Trapping fraction at t_2	Trapping initial phases	Trapping fraction at t_3
3.6	0–0.1, 2.9–6.3	0.59	2.9–5.9	0.49
3.9	1.8–5.3	0.57	1.9–4.6	0.44
4.0	1.4–5.0	0.59	1.5–4.0	0.41
4.1	1.1–4.7	0.59	1.2–3.5	0.38
4.2	0.6–4.2	0.59	0.8–3.1	0.38
4.4	0–3.5, 5.6–6.3	0.70	0–2.3, 6.0–6.3	0.48
4.6	0–2.8, 3.4–6.3	0.94	0–1.6, 4.2–6.3	0.62

5. Conclusions

The examples described in this paper demonstrate that animation offers the researcher a new tool with additional dimensions for better understanding of the physics of many processes that are involved in the FEL problem. In particular we demonstrate its use for understanding the periodic behavior of trapping efficiency as a function of injection energy into the beat wave for the case of trapping by an abrupt axial field variation.

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