factors needed to explain the dynamics or the transient behavior of the wavelength-modulated cw dye laser are contained in the theory and the observed effect can be understood on the basis of Eqs. (1)–(5).

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Equivalence of the coupled-mode and Floquet-Bloch formalisms in periodic optical waveguides*

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A comparison of two theories used to analyze distributed feedback lasers and periodic optical devices finds them, contrary to some claims, to be formally equivalent.

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The propagation and manipulation of optical modes in *periodic* optical waveguides has become a topic of considerable experimental and theoretical interest. Some of the more interesting phenomena which fall under this category are as follows: distributed feedback (DFB) lasers in dye media and in thin-film semiconductor configurations; optical Bragg filter; scattering of light from hypersound; Bragg amplifiers. These phenomena have been interpreted and explained in terms of a coupled-mode formalism[1–4] which is a direct descendant of a similar formalism introduced by Pierce[5,6] to describe traveling wave electron devices. Theoretical predictions of the theory have been borne out in the case of DFB lasers[7,8] and in a more detailed manner in the case of Bragg filters.[9]

A second point of view which can be used to describe periodic optical phenomena has been applied by Dabby, Kestenbaum, and Paek.[10] This method uses an expansion of the field in terms of Floquet-Bloch modes. A comparison of this method to the coupled-modes formalism showed that near the Bragg regime the two methods yield the same dispersion diagrams.[3,11]

A truncated form of Floquet-Bloch expansion has been applied to DFB lasers by Cordero.[12] In a series of recent papers[13–16] elaborating on this method Wang claims to avoid certain formal difficulties which he ascribes to the coupled-modes approach and in the process to gain some new insight as to how low-threshold DFB lasers ought to be fabricated.

It is the purpose of this paper to show that the truncated Floquet-Bloch approach is formally equivalent to that of the coupled-modes and that alledged differences are due merely to a misunderstanding in applying the boundary conditions.

Let us use as the focus of our remarks the structure shown in Fig. 1. It consists of a section of a periodic dielectric waveguide of length L and period a bounded on either side by semi-infinite uniform waveguides of the same material and height. This configuration can be considered as archetypal since by introducing gain into the periodic section we obtain the Bragg amplifier[17] or DFB laser and when the periodic section is passive (zero gain) it reduces to the case of the Bragg filter[9] and mirror.[3]

The form of the field amplitude solutions is indicated in the figures. In the coupled-mode approach one chooses to expand the field in the periodic waveguide in

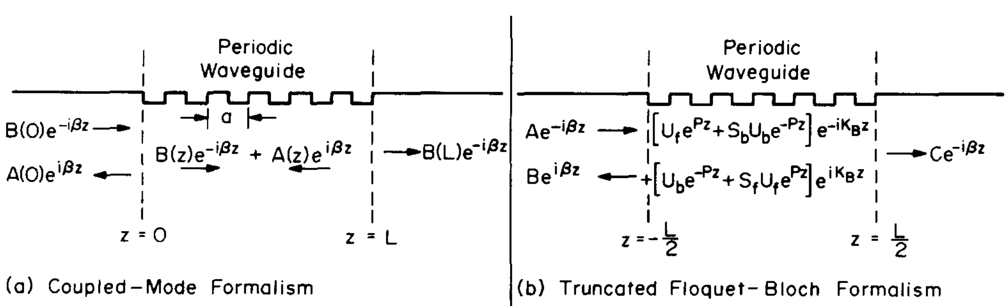

FIG. 1. The basic periodic waveguide model and the form of the assumed wave solutions in (a) the coupled mode formalism, (b) the truncated Floquet-Bloch formalism.

terms of the modes of the *unperturbed* (uniform) waveguide which constitute an orthonormal set.[18] When one substitutes the assumed solution into Maxwell's equations and limits the expansion to the two modes which are Bragg-coupled ($|\beta| \approx K_B$) the result is the coupled-mode equations[4]

$$\frac{dB(z)}{dz} = -i\kappa A \exp[i2(\beta - K_B + ig_0)z],$$
$$dA(z) = i\kappa B \exp[-i2(\beta - K_B + ig_0)z], \tag{1}$$

whose solutions are given in Ref. 2 for the case of a DFB laser and in Refs. 3 and 17 for the filter and amplifier configurations. Here β and g_0 are the propagation and gain constants, respectively, and $K_B = \pi/a$ is the Bragg wave number.

In the truncated Floquet-Bloch formalism of Cordero and Wang,[14–16] the solution is in the form of four waves (two fundamental components and two associated space harmonics) as shown in Fig. 1(b). Here the most relevant parameters are[15]

$$\begin{aligned}
&P = G + i\delta_{\text{eff}},\\
&G = \Gamma + iK,\\
&P^2 = (G + i\delta_{\text{eff}})^2 = (g_0 + i\delta)^2 + \kappa^2,\\
&\delta = K_B - \beta,\\
&S_f = se^{i\theta},\\
&S_b = se^{-i\theta},\\
&s = -i\kappa[G + g_0 + i(\delta + \delta_{\text{eff}})]^{-1}.
\end{aligned} \tag{2}$$

Since in both approaches one approximates a field expansion requiring an infinite number of modes by two modes and both solutions are made to satisfy Maxwell's equations, it would follow that both are equivalent. Indeed let us rewrite the coupled-mode solution of Fig. 1(a) as

$$E(z) = \{B(z)\exp[-i(\beta - K_B)z]\}\exp(-iK_Bz) + \{A(z) \times \exp[i(\beta - K_B)z]\}\exp(+iK_Bz). \tag{3}$$

It is clear that both forms are identical if we associate

$$B(z)\exp[-i(\beta - K_B)z] \rightarrow U_f e^{Pz} + S_b U_b e^{-Pz}, \tag{4(a)}$$

$$A(z)\exp[i(\beta - K_B)z] \rightarrow U_b e^{-Pz} + S_f U_f e^{Pz}. \tag{4(b)}$$

There still remains the important problem of the boundary conditions to be applied at the two extremities, $z = 0$ and $z = L$, of the periodic region. Kogelnik and Shank in their DFB laser analysis[2] use the condition $B(0) = A(L) = 0$. Stoll and Yariv[3] and Yariv and Yen,[17] in treating the filter and amplifier cases, respectively, have a finite input $B(0)$ and use the condition $A(L) = 0$. All of these conditions follow simply and straightforwardly from the form of the field solution in Fig. 1(a) when we require that the total field and its z derivative (the magnetic field) be continuous at both boundaries. Consider, for example, the boundary at $z = L$. The continuity conditions are as follows: [We have assumed that $|B'(z)| \ll \beta|B|$, which is true for the adiabatic case $\beta \gg \kappa$. This condition is amply fulfilled in all the experiments performed or discussed to date.]

$$\begin{aligned}
&B(L)e^{-i\beta L} + A(L)e^{i\beta L} = B(L)e^{-i\beta L},\\
&-i\beta B(L)e^{-i\beta L} + i\beta A(L)e^{i\beta L} = -i\beta B(i)e^{-i\beta L},
\end{aligned} \tag{5}$$

from which we obtain $A(L) = 0$. Wang finds this condition[15] "somewhat mysterious," yet using 4(b) and recalling that Wang's boundary is at $z = \frac{1}{2}L$ we find that the coupled-mode condition $A(L) = 0$ becomes $U_b/U_f = -S_f e^{PL}$. This is exactly the boundary condition used by Wang.[15]

As a further check we applied Wang's formalism to multiple boundary cases. The results agree with those previously derived by Chinn[19] who used the coupled-mode formalism.

We thus conclude that in spite of its considerably more complicated form and the large number of interrelated constants [see Eq. (2)] which it employs, the truncated Floquet-Bloch formalism is exactly equivalent to that of the coupled modes.

The question of assigning directions of power flow to the various wave components inside the perturbed region is not one that is relevant to any of the experiments performed to date in DFB lasers. The variables measured in the experiments are the input field $B(0)$, which is zero in a DFB laser, and the "output" fields $B(L)$ and $A(0)$. We find from the form of the fields in Fig. 1(a) and the boundary conditions discussed above that the field $B(z)\exp(-i\beta z)$ "matches" to the transmitted field at $z = L$ so that we choose to refer to it as the "forward" wave. In addition, as the periodic perturbation is decreased ($\kappa \rightarrow 0$) this wave goes continuously into the forward wave.

Some clarification is needed about Wang's claim that the DFB laser, *like conventional lasers*, requires reflections at boundaries in order to oscillate.[13,15] The geometry of Fig. 1(a) in which the periodic section possesses gain has been analyzed by Kogelnik and Shank using a coupled-mode approach.[2] Different aspects of this theory and its extension by Chinn[19] have been checked experimentally.[7,8,20] In this theory we interpret the form of Eq. (1) by saying that $B(z)$ is coupled to $A(z)$, and vice versa, by the periodic perturbation which is represented by κ. If one uses the Floquet-Bloch formalism one uses *ab initio* eigenfunctions of the periodic medium which by definition do not couple to each other.

A simple example of this statement can be found in the propagation of an electromagnetic wave through a paramagnetic (or ferromagnetic) medium. We are free to describe the propagation in terms of the two (counter) circularly polarized plane wave eigenmodes which do not "couple" to each other or in terms of the two linear polarized orthogonal plane waves which are "coupled" by the perturbation.

By "forcing" the assumed solution to satisfy Maxwell's equations and by insisting on the continuity of the transverse electric field and its derivative (i.e., the transverse magnetic field) at the boundaries, we get the same result from either point of view.[4]

So the problem of coupling and where it takes place is semantic. It depends on which set of (complete orthonormal) functions one chooses to expand the *total* field.

There still remains the important practical problem of how to fabricate DFB lasers so as to achieve the lowest possible threshold. We find by straightforward application of the coupled-mode formalism that the transition from a periodic waveguide to a uniform one is not essential to low-threshold operation in contrast to a finding by Wang.[15] The unpumped sections of a periodic waveguide extending beyond the gain region merely act as passive reflectors. In the limit $\kappa L \gg 1$ (L is the length of the unpumped sections) their reflectivity approaches unity[3] for frequencies ω such that $\Delta\beta(\omega) \leq \kappa$ (i.e., within the forbidden gap) and the laser threshold approaches zero. The presence of high losses in the unpumped region, which is the case in GaAs and dye lasers, tends to make the threshold similar to that which obtains in the geometry of Fig. 1(a) and which was treated by Kogelnik and Shank. This conclusion is supported by experimental data.[8]

In summary, the truncated Floquet-Bloch formalism is found to be *fully* equivalent to the conventional coupled-mode approach which has been used extensively in the analysis of Bragg optical devices (DFB lasers, periodic filters, and acoustic-optic interactions).

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Thermal blooming compensation using coherent optical adaptive techniques (COAT)

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Real-time compensation for thermally induced beam distortion has been demonstrated using a phase-adaptive aperture subdivided into 18 elements coupled with an 18-channel multidither servo system. The results were obtained using an argon ion laser and a nonlinear liquid cell placed in the near field of the transmitted beam. The system increased the peak beam irradiance by more than a factor of 4 at the highest power used and reduced the beamwidth by a factor of 5.4 from the uncorrected bloomed case. A comparison with predictions from existing theory is given.

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High-power optical beams propagating through the atmosphere can experience distortion from two sources: turbulence (a linear process) and thermal blooming (a nonlinear process). Real-time compensation for the linear propagation problem, turbulence-induced phase distortion, has already been demonstrated using a self-adaptive optical phased array.[1-3] This article describes the successful use of such a COAT[4] system to compensate for the nonlinear propagation problem of phase distortion induced in an absorbing liquid by a high-power laser beam.

The multidither-type COAT array used for these experiments is described elsewhere.[5] In such a system the radiation from each element in the array is tagged with a different low-index sinusoidal phase modulation ω_i (the dithers). Radiation from the array passes through the disturbed propagation path to a remote target. A sample of the interference pattern is reflected by a small glint on the target (i.e., an area of higher reflectivity unresolved by the array). This reflected signal contains *intensity*-modulated components at all possible sums and differences of the *phase* modulation frequencies ($n\omega_i \pm n\omega_j$). A bank of synchronous detectors fed by a simple intensity detector sorts out the signal components at the original frequencies and, after low-