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Numerical studies of resonators with on-axis holes in mirrors for FEL application

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An optical resonator with holes on-axis for applications in a FEL oscillator was recently proposed by Pantell et al. [Nucl. Instr. and Meth. A296 (1990) 638]. These authors presented an analytical approach based on approximating the field inside the resonator by a truncated expansion in Gauss–Laguerre modes. We perform further investigation of this configuration by using a Fox and Li type code which was recently applied to the analysis of a laser resonator with internal circular aperture. This numerical approach allows us to check the range of validity of Pantell's solutions, the parameters for which they constitute the lowest-loss mode and to map the transverse profile of the mode at different planes inside the resonator.

1. Introduction

A resonator with on-axis holes in the mirrors, is an effective method for eliminating space, weight and cost of bending magnets for directing the electron beam into the resonator of a FEL. According to the analysis of Pantell et al. [1], a mode can be built in a resonator which shows on one hand, a zero value for the field at the center of the holes and on the other hand a focusing property along the mid-part of the resonator, a desired property for enhanced electron beam overlap and correspondingly enhanced gain.

The use of holes in mirrors in order to couple out power from laser resonators was implemented a long time ago [3]. Although the configurations there were basically similar to those analysed here, the role and the requirements of the hole are basically different. In a hole coupled resonator, the hole serves usually as the output coupling port and its losses are therefore considered as useful, so that the design is aimed at optimized output coupling via the aperture. In the device discussed here, the hole serves as an insertion means, the radiation escaping through it being essentially useless. The output coupling is in this case implemented by means of an additional element in the cavity, and the design requirements on the aperture are minimal losses compatible with the minimal size of the hole, which is determined by the diameter of the electron beam.

In ref. [1] it was shown that by a suitable combination of low order Gauss–Laguerre modes, a field distribution on the resonator can be constructed with the requirement of cancellation of the field at the center of the mirror ("hole avoiding" effect). The same mode combination filled the volume at the center of the resonator as required for FEL gain optimization. This analytical approach, however, did not take into account diffraction effects of the holes and finite sizes of the mirrors. Furthermore, the existence of these distributions as lowest-loss eigenmode solutions of the entire cavity was not proved.

In this work we undertake a numerical analysis of the proposed configuration. We show that the hole-avoiding distributions are valid for a limited range of parameters. We check the mode shapes and losses for a range of values of both the mirror diameter and hole sizes. The shape of the mode is investigated also at planes inside the cavity. In the next section we briefly describe the numerical procedure and following we present some selected results and discussion on the role of the different resonator variables.

2. Computational procedure

Our investigation is based on a Fox and Li type code which was recently applied to the analysis of a laser resonator with an internal aperture [2]. The work was limited to cylindrically symmetrical patterns, since the lowest resonator mode we are interested in is expected to be such. The resonator consists of two finite mirrors, of diameter $2a_1$ and $2a_2$, and curvature radius equal to R_1 and R_2, respectively. The mirrors' separation along the resonator axis is L. Circular holes of radius h_1 and h_2 exist at the center of mirror 1 and mirror 2, respectively.

For the analysis, it is assumed that the mirror surface be perfectly reflecting, the dimensions of the resonator be larger compared to the wavelength λ of the light, and the resonator length L be much larger than the mirror's diameter. In our analysis we do not consider the edge effect of the hole. Therefore the resonator eigenmode is determined from the scalar formulation of Huygen's principles. The propagation of the field inside the resonator is governed by the general Fresnel–Kirchhoff integral and takes the following form in the case of cylindrically symmetrical field distribution:

$$U_n(r_n) = \int_0^{a_i} U_i(r_i)\rho_i(r_i)K_{in}(r_i,r_n)r_i\,dr_i, \tag{1}$$

with the kernel:

$$K_{in}(r_i,r_n) = j\frac{k}{L}J_0\left(\frac{k}{L}r_ir_n\right)\exp\left[-j\frac{k}{2L}(r_i^2+r_n^2)\right], \tag{2}$$

where $U_n(r_n)$, $U_i(r_i)$ are the field amplitude of the corresponding planes, $\rho_i(r_i)$ is the transmission function which multiplies the input field, J_0 is a first-kind zero-order Bessel function, k is the radiation wavenumber, and L is the propagation distance. This yields a single integral equation for the field amplitude having the general form of

$$\gamma U_1(r_1) = \int_{h_1}^{a_1} U_1(r)K(r,r_1)r\,dr, \tag{3}$$

where $U_1(r)$ is the eigenfunction and γ the eigenvalue. $K(r, r_1)$ is the kernel of the transformation of one round trip inside the resonator, implemented by successive applications of eq. (1). We focus our interest on the lowest mode only, namely, the one having the largest value of $|\gamma|$. This mode has the smallest losses, and the Fox and Li method will converge in general to its solution after a sufficient number of iterations. To confirm the validity of the analysis, we compared the results with earlier works on a similar system [3,4] and obtained very good agreement. In using the Fox and Li approach, we took advantage of an efficient quasi-fast Hankel transform (QFHT) algorithm for solving eq. (3) to speed up resonator calculations.

3. Results and discussion

In general, we found the shape of the transverse mode at the mirror plane to be very much dependent on the mirrors' and holes' sizes, cavity length and curvature of the mirrors. For a symmetrical resonator, the dependence can be condensed in three parameters: The cavity g factor and the Fresnel numbers N, and N_0 of the mirrors' and holes' sizes respectively [5]. In our calculations insofar, a cofocal configuration was assumed.

In fig. 1, a series of transverse mode patterns at the mirror's plane is shown, in which the hole size is varied, keeping the rest of the parameters fixed. For a small hole size ($N_0 = 0.001$), only a minor perturbation on the Gaussian is observed. As the hole size increased, a dip in the center of the field is created up to its total cancellation for $N_0 = 0.031$. Larger values of N_0 cause a renewed increase in the value of the field at the center, which can be explained as a Fresnel lensing effect caused by the diffraction of the field of one mirror into the plane of the other. For the examples displayed in fig. 1 the presence of two lobes at each side of the holes was unavoidable. The total losses as function of the hole size are presented in fig. 2. One observes that after a sharp rise in the loss, a range of parameters in the hole size is found for which the slope of the loss increase is moderate. In fig. 3 the size of the mirrors is varied keeping the rest of the parameters fixed. One observes here gradual decrease in the axis field for increased

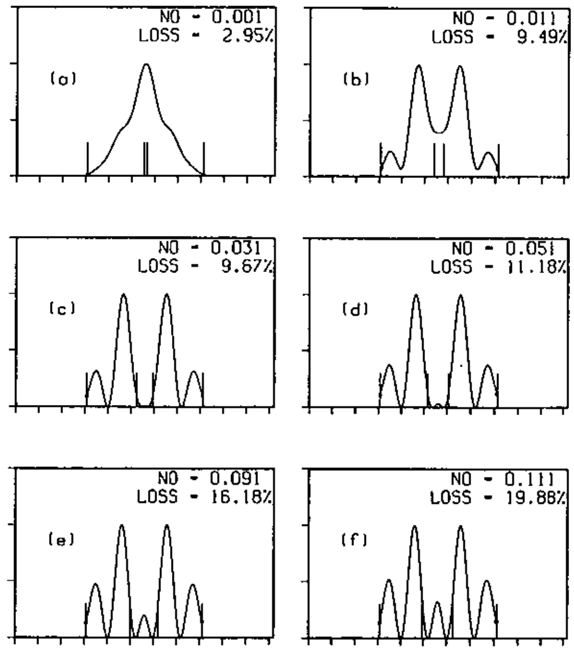

Fig. 1. Intensity profiles at the mirror's plane for different hole diameters and rest of resonator's parameters unchanged. N_0 is the Fresnel number of the hole, the Fresnel number of the mirrors is $N = 1.6$, and $g = 0$ (cofocal configuration).

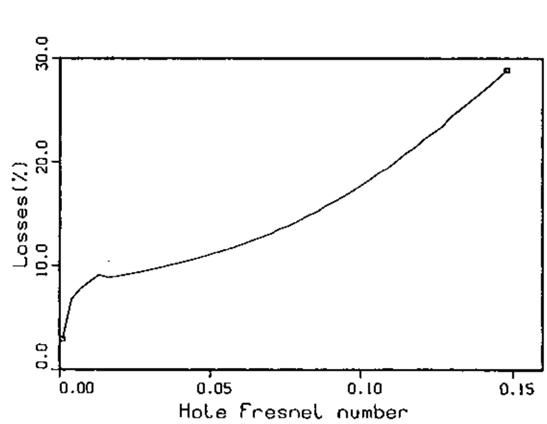

Fig. 2. Total losses as a function of the hole size for $N = 1.6$.

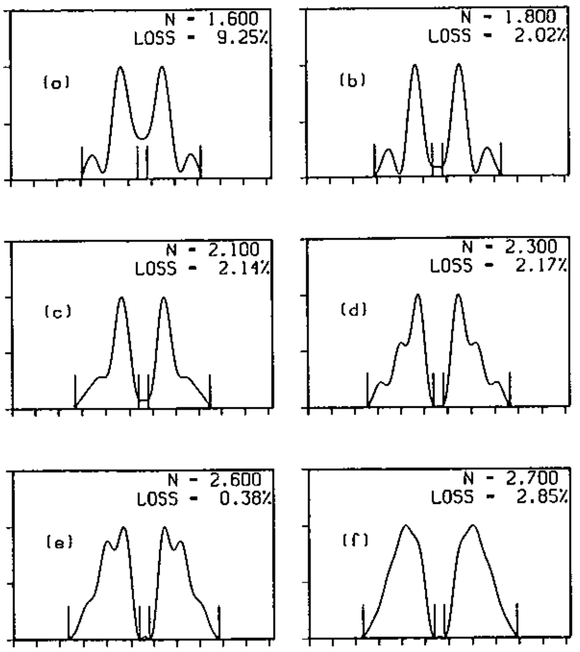

Fig. 3. Intensity profiles at the mirror's plane for different mirror's size and rest of resonator's parameters unchanged. N is the Fresnel number of the mirror, the Fresnel number of the holes is $N_0 = 0.031$, and $g = 0$ (cofocal configuration).

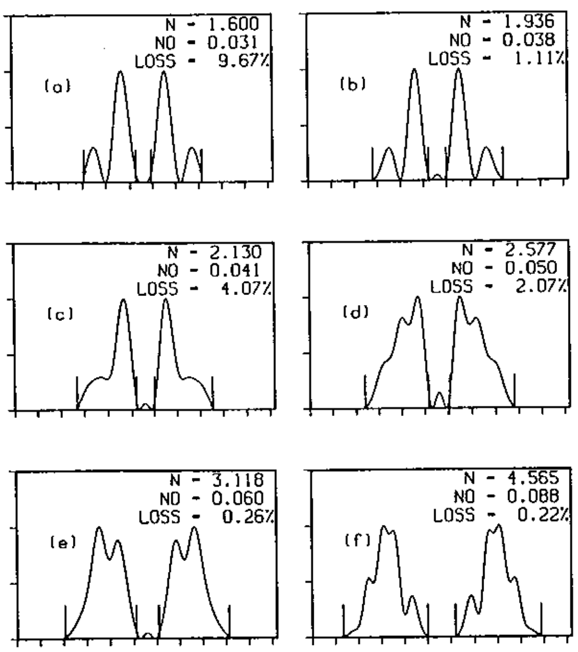

Fig. 4. Intensity profiles at the mirror's plane for different values of N and N_0, keeping the ratio N_0/N constant. These plots show the sensitivity of the profiles to radiation wavelength changes.

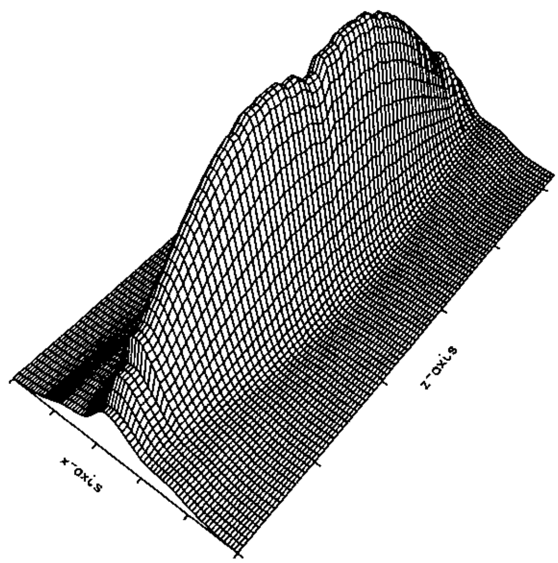

Fig. 5. Intensity distribution inside the resonator, for $N = 1.6$, $N_0 = 0.001$. The x-axis is along the diameter of the mirror.

mirror sizes. This effect can be explained by the need of the combination of several higher ideal-cavity modes in order to cancel the field at the center. These modes have wider transversal spread and require mirrors with larger diameter to be supported. The last profile ($N = 2.7$) has a shape of a kind similar to that presented in ref. [1].

A basic consideration in the implementation of the resonators of this type in a FEL system, is the sensitivity to wavelength variation, since tunability is usually required. In fig. 4 the wavelength change is represented in an increase in both N and N_0 while keeping their

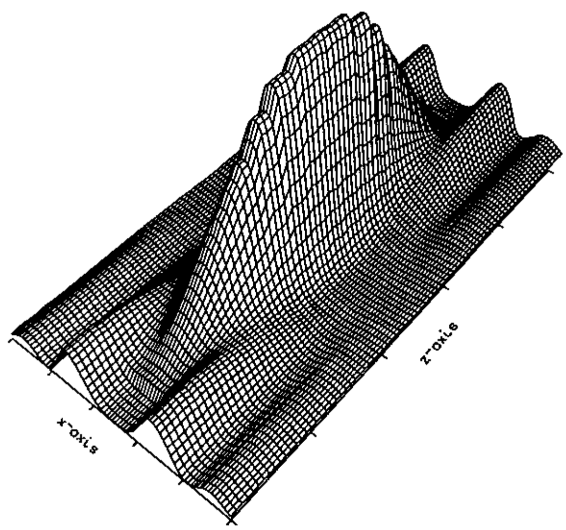

Fig. 6. Intensity distribution inside the resonator, for $N = 1.6$, $N_0 = 0.031$. The x-axis is along the diameter of the mirror.

ratio constant. In general a marked influence of the wavelength on shape and losses is noticed, however, a range can be found (cf. figs. 4a and 4b) in which the mode shape is preserved while keeping the losses low. Intensity distributions inside the cavity at different planes are shown in the 3D plots of figs. 5 and 6 which display two cases, one showing the hole-avoiding effect and the other not. In both cases a focusing effect is found towards the center of the cavity due mainly to the focusing action of the curved mirror. A striking effect is the fact that the focusing is larger for the case of a hole of larger size (fig. 6), a fact that can be explained again in terms of Fresnel lensing.

4. Conclusion

It is apparent that mirrors with holes on axis can serve as a potentially efficient electron beam insertion device. Our numerical simulation shows modes in which the power flux into the holes is practically avoided, and that do fill well most of the optical axis region inside the resonator. The existence of such a mode as the lowest-loss mode in the cavity depends, however, on specific relationships between the resonator's parameters, namely, the g factors and Fresnel numbers of mirrors and holes. We intend to continue the investigation and search systematically for criteria and optimal parameter design for best FEL performance.

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