Comment on "Compact Short-Wavelength Free-Electron Laser"

In a recent Letter [1] Chang and McDaniel offered a quantum-mechanical model in an attempt to explain the excessive Smith-Purcell radiation claimed to be measured in a recent experiment, which was reported later in [2]. This explanation suggests a quantum process of stimulated emission, which according to their model amplifies the spontaneous emission of bremsstrahlung radiation from electrons impacting on an optical grating. The purpose of this Comment is to express our opinion that this model and the conclusion about the feasibility of a compact x-ray laser based on this process are founded on erroneous assumptions.

The amplification is calculated in [1] on the basis of a Fermi "golden rule" computation. For an infinitely long interaction length this leads to an expression for the stimulated emission gain which is proportional to the "population inversion" factor:

$$G \propto f(\mathbf{k}_i) - f(\mathbf{k}_f) , \qquad (1)$$

where $f(\mathbf{k})$ is the momentum distribution function of the electron beam.

As is explained in detail in [3], the general free-electron-laser (FEL) gain expression reduces into (1) only under the condition that the homogeneous broadening of the emission line due to the finite interaction length is much narrower than the quantum recoil frequency shift and the inhomogeneous broadening linewidth [width of $f(k(\omega))$]. The homogeneous broadening linewidth is given by

$$(\Delta w)_{\mathrm{hom}}/w = 1/N_w , \qquad (2)$$

where N_w is the number of grating periods that the interacting electron passes through without losing coherence.

The recoil frequency shift is given by

$$(\delta w)_{\mathrm{rec}}/w = \hbar \omega^2/\gamma^3 v^3 m k_g , \qquad (3)$$

where v is the electron velocity, γ is the Lorentz factor, and $k_g = 2\pi/\lambda_g$ is the optical grating wave number.

For the parameters of the experiment reported in [2] $(\delta w)_{\mathrm{rec}}/w \cong 10^{-5}$ while for a reasonable estimate, $N_w \lesssim 10$, one obtains $(\Delta w)_{\mathrm{hom}}/w \cong 10^{-1}$. Consequently, $(\Delta w)_{\mathrm{hom}} \gg (\delta w)_{\mathrm{rec}}$. Therefore (1) is not the appropriate expression for calculating gain. The appropriate gain expression must be similar to the known homogeneous-broadening classical gain expression of the FEL, $d \sin[c^2(\omega - \omega_0)\tau]/d\omega$ [4]. It is not likely to result in any substantial gain at the low current intensity ($I < 5$ mA) used in the Smith-Purcell experiments [2].

Another questionable assertion in the Letter is based on an experimental finding [2] which is cited by the authors in order to support their model. According to that experiment, even electrons that pass far from the grating surface (1000 times the grating period) contribute to the measured radiation. This finding is highly debatable on its own, but even if we accept it, it is not clear at all how it corroborates the proposed theoretical model. Unless the beam was initially highly uncollimated (in the classical sense), electrons that pass more than a transverse quantum-wave-packet width $(\lambda_c L/2\pi)^{1/2} \simeq 20$ Å [3] away from the grating do not diffract off the grating material and consequently do not contribute at all to the striated wave function that is claimed to give a nonzero matrix element in the transition rate calculation in [1]. We also point out that the volume normalization of the electron wave function and particularly the extent of the wave function in the y dimension (perpendicular to the grating) in the matrix element calculation were chosen in the Letter based on vague and quite arbitrary considerations. These may have led to erroneous conclusions.

In conclusion, the problem of electrons diffracted by the crystalline material of an optical grating is an interesting new radiation process. The Letter does not provide a reliable analysis of this process. While the problem of the electrons' transport is indeed quantum mechanical in this effect (electron diffraction), the electron interaction with the radiation field is in the classical regime. Both the spontaneous and stimulated emission processes can be calculated with a proper classical or quantum-mechanical model which we intend to present in a future publication. Our preliminary calculations resulted in an estimate of spontaneous radiation emission parameters of 5×10^{-5} W/sr A or 2×10^{-5} photon/sr electron for a 100-keV electron-beam experiment under optimal conditions. This is 2 orders of magnitude less than the measured intensity reported in [3], suggesting that the spontaneous emission and certainly the stimulated emission [4] of the proposed new mechanism are not the dominant effects in the Smith-Purcell experiments. Finally, there is very little reason to believe that a substantial stimulated emission gain can be attained at low current intensities and that a compact x-ray laser can be constructed based on this mechanism.

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