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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

ELSEVIER

Mode-locked super-radiant free-electron laser oscillator

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Abstract

Evolution of the time domain fields and the spectral power of super-radiant radiation in a free-electron laser oscillator (e-beam pulses shorter than a wavelength) are investigated. We consider a finite train of N short bunches of electrons propagating through the undulator. The coherence of the synchrotron radiation emitted from the bunched beam grows with the number N of the e-beam pulses entering the interaction region. When N grows to infinity, the radiation becomes perfectly coherent at all harmonic frequencies of the pulse injection (bunching) frequency.

When the super-radiant emission takes place inside a resonator, the coherence of the emitted radiation is enhanced. Under the condition of mode-locking, the fields add in phase and the spectral energy distribution becomes narrow. When the finesse F of the resonator is small $F < N$, the spectral width of the out-coupled radiation emitted from the resonator is limited by N, and in the opposite case $F > N$, it will be limited by F. If the number of pulses N grows to infinity, the out-coupled radiation reaches a steady state of perfect coherence with reduced harmonic contents (determined by the Finesse of the resonator). There is no threshold for emission of this kind of coherent radiation.

1. Introduction

Electrons passing through a magnetic undulator emit partially coherent radiation called undulator synchrotron radiation [1]. The radiation from different electrons, which enter the undulator at random, adds up incoherently, unless the electrons are inserted as a single short bunch (shorter than the oscillation period of the emitted radiation) [2–7] or enter as a periodic train of bunches at the frequency of the emitted radiation [8–11]. Only in these cases do the electrons radiate in phase with each other (super-radiant) and the radiation is coherent.

Super-radiant emission from a short pulsed electron beam has recently been observed experimentally in synchrotron radiation [12,13], in Cherenkov radiation [14] and in undulator radiation [15–17]. Most recently Asakawa et al. reported the emission of enhanced super-radiant undulator radiation from a train of short electron beam pulses within a mm-wave cavity [18].

The purpose of this article is to present a rigorous time and frequency domain analysis of super-radiant undulator radiation by a train of electron beam short pulses inside a waveguide resonator (see Fig. 1).

This scheme is of interest as a realizable source of a long or even of a continuous train of coherent radiation pulses. Contrary to conventional laser oscillators, this kind of super-radiant oscillator does not have an oscillation threshold and can produce coherent radiation at any power level.

2. Excitation of the electromagnetic field

Our analysis is based on modal expansion of the total electromagnetic field presented as a superposition of transverse eigen-functions of the uniform cavity in which the radiation propagates [20]. In the angular frequency domain ω, the field of each mode can be written as:

$$\tilde{E}_q(r,\ \omega) = \tilde{C}_q(z,\ \omega)\tilde{\mathscr{E}}_q(x,\ y)\ \mathrm{e}^{+\mathrm{j}k_{zq}(\omega)z},$$

$$\tilde{H}_q(r,\ \omega) = \tilde{C}_q(z,\ \omega)\tilde{\mathscr{H}}_q(x,\ y)\ \mathrm{e}^{+\mathrm{j}k_{zq}(\omega)z}. \qquad (1)$$

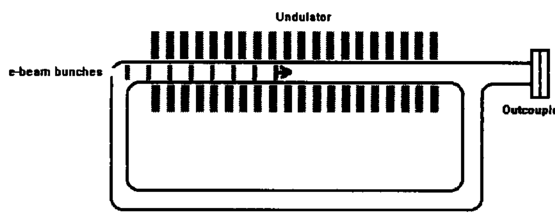

Fig. 1. Schematic illustration of a super-radiant free-electron laser oscillator.

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$\tilde{\mathscr{E}}_q(x, y)$ and $\tilde{\mathscr{H}}_q(x, y)$ are complex vectors representing the transverse profile and polarization of the electric and magnetic fields of transverse eigenmode q, with wavenumber $k_{zq}(\omega)$. $\tilde{C}_q(z, \omega)$ is the Fourier transform of the mode's amplitude, which in the case of a propagating (non-evanescent) mode, satisfies the following excitation equation:

$$\frac{\mathrm{d}}{\mathrm{d}z}\tilde{C}_q(z, \omega) = -\frac{1}{4\mathscr{P}_q}\mathrm{e}^{-\mathrm{j}k_{zq}(\omega)z}\int\int\tilde{J}(r, \omega)\cdot\tilde{\mathscr{E}}_q^*(x, y)\,\mathrm{d}x\,\mathrm{d}y, \tag{2}$$

where $\mathscr{P}_q = \frac{1}{2}\Re\int\int[\tilde{\mathscr{E}}_{q\perp}(x, y)\times\tilde{\mathscr{H}}_{q\perp}^*(x, y)]\cdot\hat{z}\,\mathrm{d}x\,\mathrm{d}y$ is the normalization power of the qth propagating mode. For a radiation signal of finite energy, the spectral energy density carried by the excited propagating mode is given by:

$$\frac{\mathrm{d}\mathscr{W}_q(z)}{\mathrm{d}\omega} = |\tilde{C}_q(z, \omega)|^2\mathscr{P}_q. \tag{3}$$

3. Super-radiant emission from a train of e-beam bunches

Consider first the case of a train of electron bunches passing through a wiggler field, emitting super-radiant synchrotron radiation without a resonator. The electron beam is composed of N short bunches, entering the interaction region with time intervals T between each other. The duration of each bunch is small compared to the period of the emitted electromagnetic field. The current density is a product of the space-charge density and the velocity $\boldsymbol{v}$ field of the electrons:

$$J(r, t) = -\hat{q}\sum_{n=0}^{N-1}\boldsymbol{v}f(x, y)\delta[z - z_n(t)], \tag{4}$$

where $f(x, y)$ is the transverse space-charge distribution of the bunch of total charge $\hat{q}$. In the space-frequency domain the current density of the train of bunches is the Fourier transform of (4):

$$\tilde{J}(r, \omega) = \int_{-\infty}^{+\infty}J(r, t)\,\mathrm{e}^{+\mathrm{j}\omega t}\,\mathrm{d}t = -\hat{q}\sum_{n=0}^{N-1}\frac{\boldsymbol{v}}{v_z}f(x, y)\,\mathrm{e}^{+\mathrm{j}\omega t_n(z)}, \tag{5}$$

where

$$t_n(z) = nT + \int_0^z\frac{1}{v_z(z')}\mathrm{d}z' \tag{6}$$

is the time passed until the nth bunch, which entered the interaction region at time nT and moves at an instantaneous axial velocity $v_z(z)$, arrives at a point z. Substitution of the expression for the transverse current density $\tilde{J}(r, \omega)$ into Eq. (2), results in [21]:

$$\frac{\mathrm{d}}{\mathrm{d}z}\tilde{C}_q(z, \omega) = \frac{\hat{q}\zeta_q}{8\mathscr{P}_q}\sum_{n=0}^{N-1}\frac{1}{v_z}\mathrm{e}^{+\mathrm{j}[n\omega T+\int_0^z\theta_q(z')\,\mathrm{d}z']} \times\int\int f(x, y)\tilde{v}_\perp^{\mathrm{w}}\cdot\tilde{\mathscr{E}}_q^*(x, y)\,\mathrm{d}x\,\mathrm{d}y, \tag{7}$$

where we define:

$$\zeta_q \equiv \begin{cases} 1 & \text{for TE modes} \\ 1 - \dfrac{k_{\perp q}^2}{k_{zq}k_{\mathrm{w}}} & \text{for TM modes} \end{cases}$$

and $\theta_q(z) = \omega/v_z(z) - (k_{zq} + k_{\mathrm{w}})$ is the detuning at point z. $k_{\mathrm{w}} = 2\pi/\lambda_{\mathrm{w}}$, where λ_{w} is the wiggler period and $\tilde{v}_\perp^{\mathrm{w}}$ is the amplitude of the transverse wiggling velocity.

Neglecting the interaction effect of the radiation on the electrons in the bunch, it is assumed that the electrons in the beam all move at a constant (averaged over wiggler period) axial velocity $v_z(z) = v_{z0}$ and keep their initial detuning parameter θ_q constant along the wiggler. Consequently, the solution of the excitation Eq. (7) at the exit of a wiggler of length L_{w} is found to be:

$$\tilde{C}_q(L_{\mathrm{w}}, \omega) = \mathscr{A}_q\,\mathrm{sinc}\left(\tfrac{1}{2}\theta_qL_{\mathrm{w}}\right)\frac{\sin\left(\frac{1}{2}N\omega T\right)}{\sin\left(\frac{1}{2}\omega T\right)} \times\mathrm{e}^{\mathrm{j}\frac{1}{2}[\theta_qL_{\mathrm{w}}+(N-1)\omega T]}, \tag{8}$$

where

$$\mathscr{A}_q = \frac{\hat{q}\zeta_q}{8\mathscr{P}_q}\frac{L_{\mathrm{w}}}{v_{z0}}\int\int f(x, y)\tilde{v}_\perp^{\mathrm{w}}\cdot\tilde{\mathscr{E}}_{q\perp}(x, y)\,\mathrm{d}x\,\mathrm{d}y$$

and $\mathrm{sinc}(x) \equiv (\sin(x))/x$. In the case of two well-separated solutions, where the respective frequency bandwidths of the emission are smaller than the spectral range between the resonance frequencies, it is sufficient to use a first order approximation $k_{zq}(\omega) \simeq k_{zq}(\omega_{\mathrm{s}}) + (1/v_{\mathrm{g}})(\omega - \omega_{\mathrm{s}})$ of the dispersion relation of the propagating mode q. The time domain picture of the field of the mode emitted by the e-beam bunches near the synchronism frequency ω_{s} is found to be [7]:

$$E_q(x, y, z = L_{\mathrm{w}}, t) = \Re\left\{\frac{\mathscr{A}_q}{\tau_{\mathrm{sp}}}\sum_{n=0}^{N-1}\mathrm{rect}\left(\frac{t - t_{\mathrm{d}} - nT}{\tau_{\mathrm{sp}}}\right) \times\tilde{\mathscr{E}}_q(x, y)\,\mathrm{e}^{-\mathrm{j}[\omega_{\mathrm{s}}t - k_{zq}(\omega_{\mathrm{s}})L_{\mathrm{w}}]}\right\}. \tag{9}$$

It consists of N rectangular pulses modulating a carrier at frequency ω_{s}. The temporal duration of each of the pulses is the slippage time $\tau_{\mathrm{sp}} = L_{\mathrm{w}}/v_{z0} - L_{\mathrm{w}}/v_{\mathrm{g}}$, and the rate of

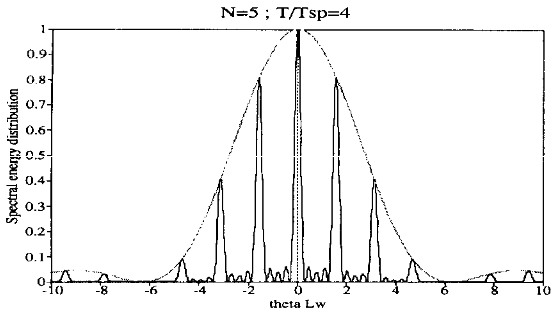

Fig. 2. The normalized spectral energy distribution of a super-radiant emission from $N = 5$ bunches. The duty-cycle of radiation pulses is $\tau_{sp}/T = 25\%$.

pulse appearance is given by the spacing T between the electron bunches. $t_d = \frac{1}{2}(L_w/v_{z0} + L_w/v_g)$ is the time delay of the center of the first pulse.

The spectral density of the radiation energy emitted by a train of N bunches is calculated from Eq. (8) according to Eq. (3):

$$\frac{\mathrm{d}\mathscr{W}_q(L_w)}{\mathrm{d}\omega} = |\mathscr{A}_q|^2 \mathscr{P}_q \operatorname{sinc}^2\left(\tfrac{1}{2}\theta_q L_w\right) \frac{\sin^2\left(\frac{1}{2}N\omega T\right)}{\sin^2\left(\frac{1}{2}\omega T\right)}. \tag{10}$$

It constitutes a comb of line-peaks separated by the free-spectral range $\Delta\nu_{\text{radiation}} = 1/T$ and weighted by a sinc^2 function with a main lobe width of approximately $\Delta\nu \simeq 1/\tau_{sp}$. The typical width of each line-peak is $\delta\nu_{\text{radiation}} = 1/(NT)$, becoming narrower as the number N of bunches increases. Fig. 2 shows a normalized line-shape of radiation energy emitted when $N = 5$ bunches pass through a wiggler. The duty-cycle of the emitted radiation in this example is chosen to be $\tau_{sp}/T = 25\%$.

The ratio of the separation between peaks to the bandwidth of each radiation peak $\Delta\nu_{\text{radiation}}/\delta\nu_{\text{radiation}} = N$, measures the degree of radiation coherence. In the limit $N \to \infty$, the radiation becomes perfectly coherent at all harmonic frequencies of the pulse injection (bunching) rate. The temporal field then becomes a periodic signal with spectral power distribution:

$$\frac{\mathrm{d}P_q(L_w)}{\mathrm{d}\omega} = \left(\frac{2\pi}{T}\right)^2 |\mathscr{A}_q|^2 \mathscr{P}_q \operatorname{sinc}^2\left(\tfrac{1}{2}\theta_q L_w\right) \times \sum_{n=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi n}{T}\right). \tag{11}$$

This should be compared to the result obtained in Refs. [10,11], for the power emitted in a prebunched free-electron laser.

4. Super-radiant emission in a resonator

Now we consider the super-radiant emission of a finite train of e-beam bunches inside a waveguide resonator, the radiation is reflected by the end mirrors and is circulating in the cavity. In the frequency domain, the field of the transverse mode q inside the resonator is found from the summation of an infinite number of reflected waves:

$$\tilde{E}_q(x, y, z = L_w, \omega) = \tilde{C}_q(L_w, \omega)\tilde{\mathscr{E}}_q(x, y) \times \sum_{m=0}^{\infty} \rho^m \mathrm{e}^{\mathrm{j}mk_{zq}l_c}, \tag{12}$$

where ρ is the complex field combined reflectivity of all the mirror in the round-trip feedback loop and l_c is the round-trip length of the cavity. The summation (12) forms an infinite geometric progression, which can be written in the form:

$$\tilde{E}_q(x, y, z = L_w, \omega) = \frac{\mathscr{A}_q}{1 - \rho\, \mathrm{e}^{\mathrm{j}k_{zq}l_c}} \operatorname{sinc}\left(\tfrac{1}{2}\theta_q L_w\right) \times \frac{\sin\left(\frac{1}{2}N\omega T\right)}{\sin\left(\frac{1}{2}\omega T\right)} \mathrm{e}^{\mathrm{j}\frac{1}{2}[\theta_q L_w + (N-1)\omega T]} \tilde{\mathscr{E}}_q(x, y). \tag{13}$$

The spectral density of the out-coupled energy in a steady-state operation is found from Eq. (3):

$$\frac{\mathrm{d}\mathscr{W}_q^{\text{out}}(L_w)}{\mathrm{d}\omega} = |\mathscr{A}_q|^2 \mathscr{P}_q \frac{\mathscr{T}}{\left(1 - \sqrt{R}\right)^2 + 4\sqrt{R}\sin^2\left(\frac{1}{2}k_{zq}l_c\right)} \times \frac{\sin^2\left(\frac{1}{2}N\omega T\right)}{\sin^2\left(\frac{1}{2}\omega T\right)} \operatorname{sinc}^2\left(\tfrac{1}{2}\theta_q L_w\right), \tag{14}$$

where $R = |\rho|^2$ and $\mathscr{T}$ is the power transmission coefficient of the outcoupler. We notice that the spectral energy distribution of the transmitted wave is the line-shape of the radiation emitted by the N wiggling bunches (10), multiplied by a transfer function of the Fabry–Perot resonator. The maximum transmission of a Fabry–Perot resonator occurs when $k_{zq}l_c = 2m\pi$ (where m is an integer), which defines the resonant frequencies for the longitudinal modes of the resonator. The intermode frequency separation is $\Delta\nu_{\text{resonator}} = 1/t_r$, where $t_r = l_c/v_g$ is the round-trip time of the radiation. The width of the transmission peaks is given by $\delta\nu_{\text{resonator}} = \Delta\nu/F$ where $F = \pi\sqrt{R}/(1 - R)$ is the finesse of the resonator.

Fig. 3 shows the line-shape of the spectral energy distribution of the out-coupled radiation when a super-radiant emission of a single bunch ($N = 1$) is circulating in the resonator. In that case the width of the line-peaks of the transmitted radiation is determined by the resonator quality. Improvement of the finesse of the resonator increases the spectral resolution between the peaks.

For the case of multi-bunching, line-peaks appear already in the spectrum of the radiation inside without feedback. The ratio between the separation of the line-peaks

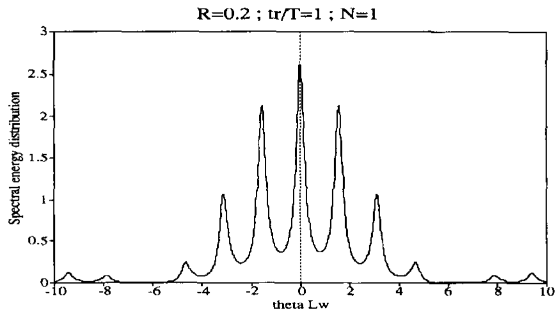

Fig. 3. The line-shape of the out-coupled radiation when synchrotron radiation from a single bunch is circulating in a resonator.

of the radiation emitted by the N wiggling bunches and the free-spectral range of the resonator $\Delta \nu_{radiation}/\Delta \nu_{resonator} = t_r/T$ determines whether and which of the peaks will be coupled out. When this ratio is equal to unity (i.e. the time T between the pulses is tuned to be equal to the round-trip time t_r of the radiation in the resonator), all of the radiation line-peaks are coupled out of the resonator. Under such a condition of "mode-locking" [19], the circulating waves add in phase resulting enhancement of the temporal coherence of the radiation.

Examination of the ratio between the line-widths of the radiation and those of the transmission curve of the resonator $\delta \nu_{radiation}/\delta \nu_{resonator} = (F/N)t_r/T$, shows that the bandwidth of the out-coupled radiation at each line becomes narrower as the finesse of the resonator increases. When the finesse of the resonator is small relative to the number of bunches $F < N$, the bandwidth of each line is limited by the number of bunches N. In the example demonstrated in Fig. 4a, the Finesse of the resonator was chosen to be $F = 1.8$, smaller than the number $N = 5$ of the e-beam pulses. The transmission characteristics of the resonator (which is illustrated in dashed line), is observed to have wide bandpass regions, corresponding to the poor finesse. The spectral width of the out-coupled radiation peaks (shown as a solid line) is narrower, determined by the number of bunches N. In the opposite case, shown in Fig. 4b, the finesse of the resonator $F = 14$ is higher than the number of pulses ($N = 5$) in the e-beam. The bandwidth of the spectral lines of the out-coupled radiation becomes narrow, and their energy peaks in the frequency domain are intensified, as the result of the enhancement of their temporal coherence.

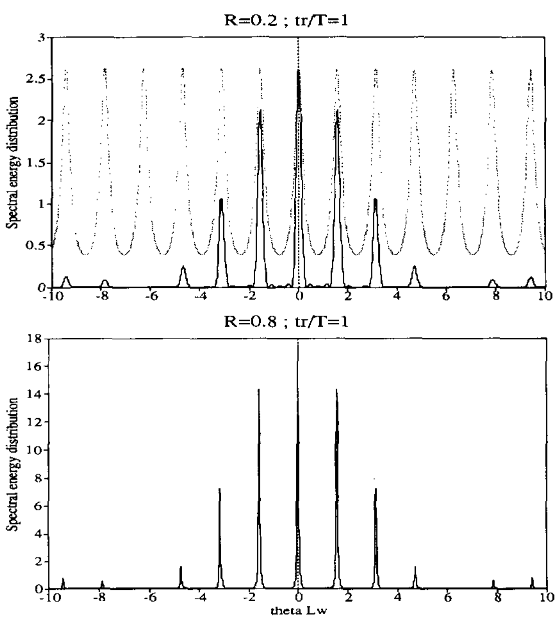

Fig. 4. The line-shape of the out-coupled radiation when a superradiant emission (of $N = 5$ bunches, duty-cycle $\tau_{sp}/T = 25\%$) is circulating in a resonator ($t_r = T$). (a) Finesse $F = 1.8$ smaller than the number of bunches $N = 5$. (b) Finesse $F = 14$ higher than the number of bunches $N = 5$.

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