Theoretical Investigation of a Free-Electron Maser Operating with a TEM Transmission Line

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***Abstract*—The possibility of using the transverse electric magnetic (TEM) transmission line in free-electron masers (FEM) is discussed. It is shown that at the centimeter and long-millimeter wavelengths such transmission lines allow one to combine the advantages of an open cavity and a waveguide-based resonator. A particular case of an FEM-based on the use of a shielded two-wire transmission line is investigated theoretically. A mathematical approach that allows one to calculate transmission-line parameters important to the FEM application is developed. It is based on the use of the integral equation technique and on a new representation of the Green function of the internal region of a circle, which was obtained in this paper. Numerical analysis of effective mode area, wave impedance, and attenuation constant was made for the odd TEM mode, which is excited in FEM operation. The FEM under research at Tel Aviv University was considered as an example. The frequency dependence of gain for an FEM operating in the linear regime was calculated. That the obtained gain value is much higher than the ohmic losses in the transmission line shows the possibility of using the TEM transmission line in this FEM.**

I. INTRODUCTION

FREE-ELECTRON lasers (FEL) and masers (FEM) [1]–[3] are powerful sources of electromagnetic radiation. Their principle of operation allows the construction of devices that can radiate an electromagnetic power in a wide spectrum from microwaves to the ultraviolet regime. Flexible tunability and high coherence of radiation make FEL's and FEM's attractive sources for scientific, medical, and future industrial applications.

As in conventional quantum lasers FEL's and FEM's use a resonator to generate electromagnetic radiation. The resonator is placed inside of a magnet stack, a so-called wiggler, that provides a strong periodic magnetic field. Fig. 1 shows schematically [4] two possible configurations: (a) an open Fabry-Perot cavity and (b) a waveguide resonator. Electrons passing through the wiggler oscillate and radiate an electromagnetic wave in a resonator mode that has a strong electric field component in the wiggling direction. The operating frequency is determined by the synchronism condition [5] and depends on the e-beam kinetic energy, period, and strength of the wiggler magnetic field and on the resonator parameters.

FEL's designed for the infrared, visible, and ultraviolet spectrum regions use the Fabry-Perot resonator operating normally in the fundamental free-space Gaussian mode. In this resonator configuration, the synchronism condition is always fulfilled, but the waist size of the Gaussian mode (for a fixed resonator Fresnel number) increases as the square root of the operating wavelength. At millimeter wavelengths and into far infrared regime, the waist size of the Gaussian mode becomes in many design cases larger than the desirable wiggler gap, and an open cavity is no longer suitable for use in an FEM.

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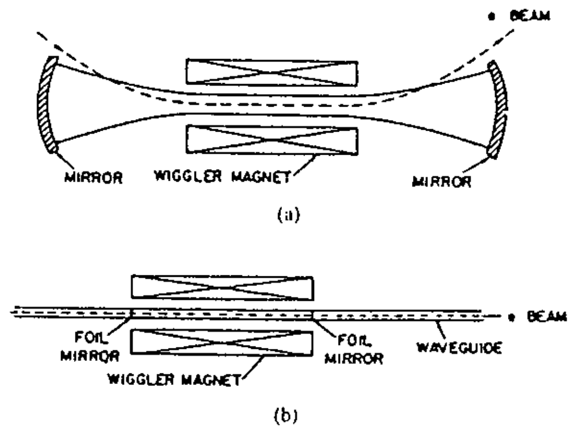

Fig. 1. Schematic drawing of the FEL resonator. (a) Open cavity configuration. (b) Waveguide cavity configuration.

This difficulty can be avoided by using a waveguide resonator. It allows one to reduce the transverse dimensions of an operating mode bellow the wiggler gap size, while attaining higher gain because of the improved filling factor. But in this case, another difficulty appears: The synchronism condition can be satisfied only for e-beam energies exceeding some value depending on waveguide sizes and transverse mode number. At millimeter waves and in the infrared regime, this problem does not occur as long as the wiggler gap is many wavelengths wide and the waveguide is overmoded. But at centimeter waves, the situation is more difficult because at such wavelengths the synchronism condition is satisfied (for conventional wigglers and moderate energy) only for waveguides having cross-sectional dimensions exceeding the wiggler gap.

This design difficulty prompted us to propose a shielded TEM wave transmission line as a basis for an FEM resonator, operating at centimeter and millimeter waves. Such a transmission line resonator combines the advantages of both an open cavity and a waveguide based resonator, i.e., the synchronism condition is satisfied for any e-beam energy because the TEM mode has no cutoff, and a shield limits

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the transverse dimension of the operating mode as desirable to obtain high filling factor and, consequently, higher gain. The gain is further increased because of higher magnetic field, which can be accepted by reducing the wiggler gap. Moreover, employment of the TEM transmission line as an RF cavity of the FEM enables tuning of the operating frequency of the device over a wide range (for example, by changing the acceleration voltage). We note that first results of an TEM-FEM experiment based on a similar transmission line were reported recently [6].

The shielded two-wire transmission line, which is well known and widely used in communication, is suitable for the proposed FEM application. This paper deals with the analysis of transmission line characteristics that are specific for the FEM application. On the basis of this analysis, the expected FEM parameters are estimated. The unique features of the proposed cavity enable designing compact and powerful FEM's tunable over wide frequency ranges at the cm and mm wavelengths.

II. FEM Consideration

Let us consider an FEM based on a shielded two-wire transmission line placed inside a gap of a planar wiggler, as shown in Fig. 2. The transmission line is formed of two identical wires of circular cross section, which are surrounded by a cylindrical metal shield. The axes of the wires are located in the diameter plane of the shield symmetrically with regard to its axis.

Let us assume that the e-beam axis coincides with the axis of the shield. The magnetic field of a wiggler forces the electrons to oscillate in the x-direction. In FEM operation, the odd TEM mode will be excited effectively. If the amplitude of electron oscillations is small, it can be supposed that the electric field of the operating mode does not change in the region occupied by electrons.

Taking into account all mentioned assumptions, the main FEM parameter, i.e., gain, can be calculated using the following well-known, single-mode gain-dispersion equation [7], [8] for the FEL operating in the linear regime[1]

$$G(s) \equiv \frac{(s - i\theta)^2 + \theta_{pr}^2}{s[(s - i\theta)^2 + \theta_{pr}^2] - iQ} \tag{1}$$

where

$$Q = I_0 \frac{e a_w^2 \varsigma_0 (k_0 + k_w)^2 L_w^3}{8mcA_{emx}\omega\gamma^3\gamma_z^2\beta_z^3}[J_0(\alpha) - J_1(\alpha)]^2 \tag{2}$$

is the gain parameter

$$\theta_{pr}^2 = \tilde{r}^2 \frac{cI_0}{\gamma_{z0}^2 \gamma m \varepsilon_0 V_{z0}^3 \pi r_e^2} L_w^2 \tag{3}$$

is the reduced space-charge parameter, $\tilde{r}$ is a plasma frequency reduction factor, and

$$\theta = \left(\frac{\omega}{V_{z0}} - k_0 - k_w\right) L_w \tag{4}$$

[1] Time dependence is presented as $\exp(i\omega t)$.

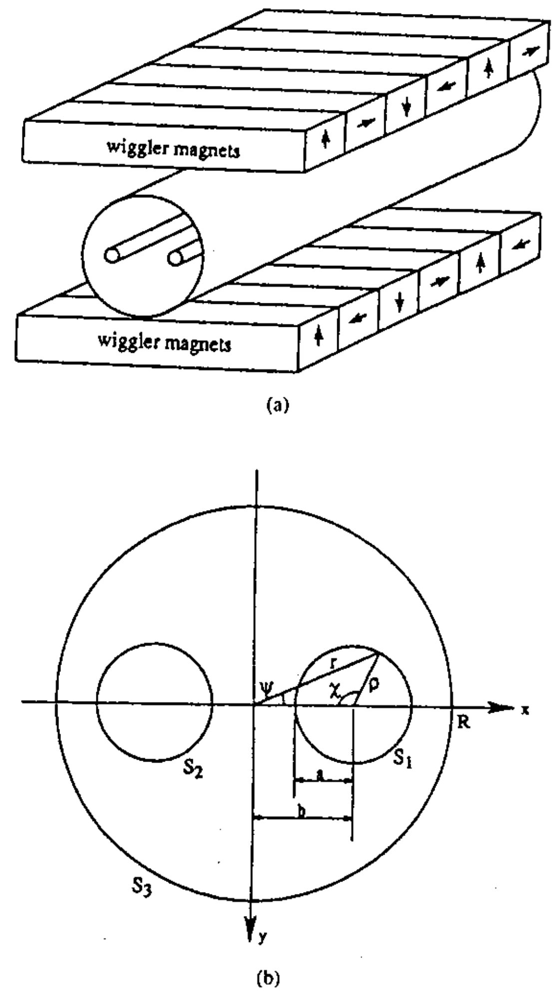

Fig. 2. Cross section of a shielded two-wire transmission line placed between the wiggler plates.

is the detuning parameter, $i = \sqrt{-1}$, r_e is the radius of e-beam, I_0 is the e-beam current, L_w is the wiggler length, ω is the angular frequency of the FEM radiation, $k_w = 2\pi/\lambda_w$, $k_0 = 2\pi/\lambda$, $\varsigma_0 = \sqrt{\mu_0/\varepsilon_0}$, μ_0, ε_0, c are, respectively, the wave number, the wave impedance, permeability, permittivity, speed of light in free space, e is the electron charge, m is the electron rest mass, J_0, J_1 are Bessel functions, and

$$a_w = \frac{eB_w}{k_w mc}, \quad \gamma_z = \frac{\gamma}{\sqrt{1 + a_w^2/2}}, \quad \beta = \sqrt{1 - 1/\gamma^2}$$

$$\beta_z = \sqrt{1 - 1/\gamma_z^2}, \quad \gamma = 1 + \frac{E_k}{mc^2}, \quad V_{z0} = c\beta_z$$

$$\alpha = \frac{\omega}{8\beta k_w c}\left(\frac{a_w}{\beta\gamma}\right)^2 \left[1 - \frac{1}{2}\left(\frac{a_w}{\beta\gamma}\right)^2\right]^{-3/2}.$$

B_w is the wiggler magnetic induction, E_k is the kinetic energy of electrons, and A_{emx} is the effective mode area of the operating mode. The effective mode area is determined by

$$A_{emx} = \frac{\int |E_x(x,y)|^2 \, dx \, dy}{|E_x(0,0)|^2} \tag{5}$$

where the integration is carried out over the transmission line cross section, and $E_x(0,0)$ is calculated on the transmission line axis where the e-beam passes.

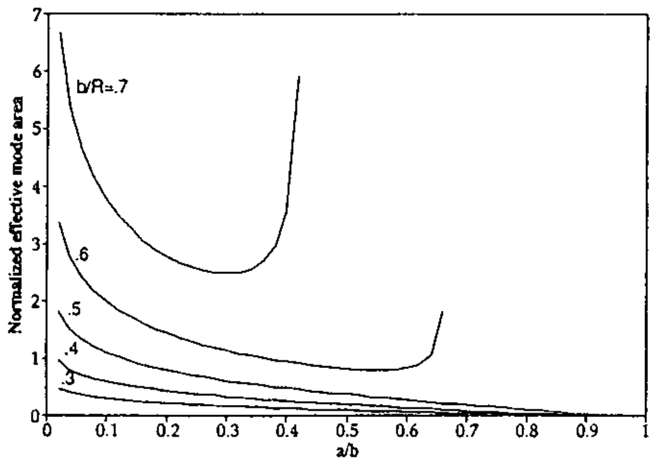

Fig. 3. Normalized effective mode area versus a/b.

The extraction efficiency η_0 of the FEM can be estimated by the following formula [4]

$$\eta_0 = -\frac{\gamma}{\gamma-1}\gamma_z^2\beta_z^2\frac{V_{z0}}{\omega}\frac{\theta}{L_w}. \quad (6)$$

Using (6) one can estimate the extraction efficiency of the FEM and FEL directly from the bandwidth of the small-signal gain curve calculated in the linear regime. We should note that when the FEM is operating in the low-gain-Compton regime, the extraction efficiency giving in (6), can be found from the simplified expression $\eta_0 \cong 1/2N_w$, where N_w is the number of periods in the untapered wiggler [4]. Typically, the extraction efficiency is of the order of few percents if an untapered wiggler is used (see, for example, Table III). Shortening the wiggler, and thus improving efficiency, depends on the possibility of attaining gain values much larger than ohmic losses. Further efficiency enhancement can be achieved employing tapered wiggler [9] and depressed collector [10], [11] schemes (operation of FEL with total efficiency exceeding 30% was reported in [9]).

III. Transmission Line Analysis

As can be seen, in order to calculate the FEM gain, one needs to know the electric field distribution in the transmission line cross section. For the TEM mode, this problem reduces [12] to the solution of the two-dimensional Laplace equation

$$\Delta\varphi = 0 \quad (7)$$

which, for the case of the odd TEM mode, should be supplemented with the boundary conditions

$$\varphi|_{s_1} = -\varphi|_{s_2} = V, \quad \varphi|s_3 = 0. \quad (8)$$

These boundary conditions correspond to the oppositely charged (up to potentials $\pm V$) wires placed within a grounded shield.

This transmission line was analyzed in a number of works [13]–[15]. Standard transmission line parameters, such as wave impedance, capacitance per unit length, and attenuation constant, were computed in these papers. But the effective mode area for this transmission line was not calculated before.

We will solve (7) using the integral equation technique. Applying Green's theorem and taking into account boundary conditions (8), we get the following integral equation

$$\oint_{S_1} ds' K(\mathbf{r},\mathbf{r}')\sigma(\mathbf{r}') = V, \qquad \mathbf{r}\in S_1 \quad (9)$$

where

$$K(\mathbf{r},\mathbf{r}') = G(r,\psi;r',\psi') - G(r,\psi;r',\pi-\psi') \quad (10)$$

and

$$G(r,\psi;r,\psi') = \frac{1}{4\pi}\ln\left[\frac{R^2+r^2r'^2/R^2-2rr'\cos(\psi-\psi')}{r^2+r'^2-2rr'\cos(\psi-\psi')}\right] \quad (11)$$

is the Green function of the Dirichlet problem for the internal region of the circle [16] of radius R, σ is the surface charge density on the wire.

We solve the integral equation (9) using the Galerkin method expanding σ in a Fourier series

$$\sigma(\mathbf{r}) = \sum_\mu A_\mu\cos(\mu\chi), \qquad \mathbf{r}\in S_1 \quad (12)$$

where χ is the polar angle in the local polar coordinate system (ρ,χ), whose axis coincides with the axis of the wire [see Fig. 2(b)], A_μ are unknown coefficients, and the summation over μ is for $\mu = 0, 1, \cdots, M$.

As a result of this procedure, we get a system of linear algebraic equations (SLAE's)

$$\sum_{\mu'}^{M} A_{\mu'}T_{\mu,\mu'} = 2\pi\delta_{\mu,0} \quad (13)$$

having matrix elements

$$T_{\mu,\mu'} = a\int_0^{2\pi}d\chi\int_0^{2\pi}d\chi'\cos(\mu\chi)K(\mathbf{r},\mathbf{r}')\cos(\mu'\chi'), \qquad \mathbf{r},\mathbf{r}'\in S_1. \quad (14)$$

Here, $\delta_{\mu,0}$ is the Kronecker delta.

The main difficulty in calculating $T_{\mu,\mu'}$ is that the centers of the polar coordinate systems (ρ,χ) and (r,ψ) do not coincide. To avoid this difficulty, a new representation for the Green function (11) was found. The expression (11) can be rewritten as a power series, which for the case of $\rho \le b, \rho' \le b$ has the form

$$G(r,\psi;r',\psi') = \frac{1}{2\pi}\left\{-\ln\rho_> + \sum_{k=1}^{\infty}\left[\left(\frac{\rho_<}{\rho_>}\right)^k\frac{\cos k(\chi-\chi')}{k} - \left(\frac{rr'}{R^2}\right)^k\frac{\cos k(\psi-\psi')}{k}\right]\right\} \quad (15)$$

where

$$r = \sqrt{b^2+\rho^2-2b\rho\cos\psi}, \quad \rho_< = \min(\rho,\rho'), \quad \rho_> = \max(\rho,\rho').$$

Further transformations were made using the expressions following from the summation theorem of Bessel functions

$$\exp(iv\psi)r^v = b^v\Gamma(v+1)\sum_{k=0}^{v}\left(-\frac{\rho}{b}\right)^k\frac{\exp(-ik\chi)}{\Gamma(k+1)\Gamma(v-k+1)} \quad (16)$$

$$\frac{\exp(iv\psi)}{r^v} = \frac{1}{b^v}\sum_{k=0}^{\infty}\left(\frac{\rho}{b}\right)^k\frac{\Gamma(v+k)\exp(ik\chi)}{\Gamma(v)\Gamma(k+1)} \quad (17)$$

TABLE I
CONVERGENCE OF THE GALERKIN METHOD IN CALCULATING THE TRANSMISSION LINE PARAMETERS

	a/b								
	.2			.5			.8		
M	Z Ohm	q dB/m	$\bar{A}_{em}$	Z Ohm	q dB/m	$\bar{A}_{em}$	Z Ohm	q dB/m	$\bar{A}_{em}$
1	215.01	1.0964	.79634	105.06	1.1343	.38910	48.656	1.8502	.18021
2	215.01	1.0971	.79999	105.01	1.1450	.40305	48.468	1.9581	.21138
3	215.00	1.0975	.79886	104.62	1.1633	.37984	45.344	2.3259	.12885
4	215.00	1.0975	.79885	104.61	1.1634	.37860	45.311	2.3148	.12337
5			.79885	104.61	1.1634	.37828	45.235	2.3314	.11686
6						.37826	45.232	2.3310	.11587
7						.37825	45.230	2.3317	.11507
8						.37825	45.230	2.3317	.11489
9									.11478
10									.11475

where $\rho \leq b$ and $\Gamma(x)$ is the gamma function. As a result, the representation in (18), shown at the bottom of the page, of the kernel (10) of the integral equation (9) was achieved, allowing for the integration of (14) analytically. Solving the SLAE, one finds the surface charge density σ on a wire, allowing the calculation of transmission line parameters.

The effective mode area, given by (5), was found by calculating the electric field using the Green theorem and expressions (15)–(17). The result is expressed in terms of the SLAE solution in (19), shown at the bottom of the page, where

$$Z = \frac{\varsigma_0}{\pi a A_0} \tag{20}$$

is the wave impedance of the odd TEM mode.

The attenuation constant in the first order of perturbation theory having small parameters w_1 and w_2, is determined by formula [17]

$$q = \frac{\mathrm{Re}\,(w_1)\int_{S_3} ds_3|H_\tau|^2 + 2\,\mathrm{Re}(w_2)\int_{S_1} ds_1|H_\tau|^2}{2P_0} \tag{21}$$

where w_1 (w_2) is the surface impedance of the shield (wires), H_τ is the tangential component of the magnetic field found for $w_1 = w_2 = 0$, P_0 is the power through the cross section of the transmission line. Omitting the mathematical transformations,

$$\begin{aligned} K(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{2\pi}\Bigg[&\ln(2b/a) + \sum_{k=1}^{\infty}\frac{\cos k\chi \cos k\chi'}{k} - \sum_{k=1}^{\infty}\left(\frac{a}{2b}\right)^k \frac{\cos k\chi'}{k} \\ &- \sum_{k=1}^{\infty}\frac{a^k}{(2b)^k k\Gamma(k)}\sum_{m=0}^{\infty}\left(\frac{a}{2b}\right)^m \frac{\Gamma(k+m)}{\Gamma(m+1)}\cos(k\chi)\cos(m\chi') \\ &- \sum_{k=1,3,5\cdots}^{\infty}\left(\frac{b}{R}\right)^{2k}\frac{\Gamma^2(k+1)}{k}\sum_{m=0}^{k}\sum_{m'=0}^{k}\frac{(-a/b)^{m+m'}\cos(m\chi)\cos(m'\chi')}{\Gamma(m+1)\Gamma(m'+1)\Gamma(k-m+1)\Gamma(k-m'+1)}\Bigg] \\ &\boldsymbol{r},\boldsymbol{r}' \in S_1 \end{aligned} \tag{18}$$

$$A_{em} = \frac{4\varsigma_0 b}{Za\left[2A_0[(b/R)^2-1] - \frac{a}{b}A_1[1+(b/R)^2] - \sum_\mu A_\mu\left(\frac{a}{b}\right)^\mu\right]^2} \tag{19}$$

TABLE II
FEM PARAMETERS

Accelerator	
Electron beam energy	E_k = 70 keV
Beam current	I_0 = 1 A
Wiggler	
Magnetic induction	B_w = 300-500 Gs
Period	λ_w = 4.4 cm
Number of periods	N_w = 17

we present here only the resulting expression

$$q = \frac{1}{A_0\varsigma_0}\left[\frac{2\,\mathrm{Re}\,(w_1)a}{R}\sum_{k=1,3,5,\cdots}^{\infty}\Gamma^2(k+1)\left(\frac{b}{R}\right)^{2k}\right.$$
$$\cdot\sum_{\mu=0}^{k+1}\sum_{\mu'=0}^{k+1}A_\mu A_{\mu'}\left(\frac{a}{b}\right)^{\mu+\mu'}$$
$$\times\frac{(1+\delta_{\mu 0})(1+\delta_{\mu' 0})}{\Gamma(\mu+1)\Gamma(\mu'+1)\Gamma(k-\mu+1)\Gamma(k-\mu'+1)}$$
$$\left.+\,\mathrm{Re}\,(w_2)\sum_{\mu=0}^{\infty}(1+\delta_{\mu 0})A_\mu^2\right] \tag{22}$$

where

$\mathrm{Re}\,(w_1) = \sqrt{\omega\mu_0/2\sigma_1}$, $\mathrm{Re}\,(w_2) = \sqrt{\omega\mu_0/2\sigma_2}$, σ_1 (σ_2) is the conductivity of the shield (wires).

IV. Numerical Results

The computed results presented in Table I illustrate the stable and fast convergence of the developed method as the parameter M is increased [M is the number of cosines kept in series (12)]. Three transmission line parameters, the wave impedance Z, the attenuation constant q, and the normalized effective mode area $\overline{A}_{em} = A_{em}/\pi R^2$, calculated for $b/R = .5$, $R = 10$ mm, $f = 5$ GHz, are presented in this table. One can see that the convergence of Z is faster than of $\overline{A}_{em}$. The explanation for this is that the wave impedance Z and the attenuation constant q, unlike $\overline{A}_{em}$, are variational functionals [18], which are stationary at the exact solution of (9).

Studies of dependence of effective mode area on geometrical parameters of the transmission line are most important for FEM application. The dependence of $\overline{A}_{em}$ versus a/b, calculated for various b/R, is presented in Fig. 3. This figure shows that $\overline{A}_{em}$ decreases monotonically for $b/R \leq .5$ and has a minimum for $b/R > .5$. In the case of small a/b, the electric field concentrates strongly in the vicinity of the wires, while the electric field in the center of this transmission line is weak. So, $\overline{A}_{em}$ increases when $a/b \to 0$. In the limit $a/b \to (R-b)/b$, which can be reached only for $b/R > .5$, the electric field concentrates in the space between the wires and the shield, so $\overline{A}_{em}$ again increases.

As can be seen from (1), to achieve large gain, the effective mode area should be small. The curves of Fig. 3 show that values of $\overline{A}_{em}$ as small as desired can be reached by choosing $b/R \leq .5$ and by increasing a/b. It should be realized that the distance between the wire and the transmission line center must be larger than the sum of the e-beam radius and the amplitude of electron oscillations. This is required in order for the e-beam to pass through the transmission line without interception.

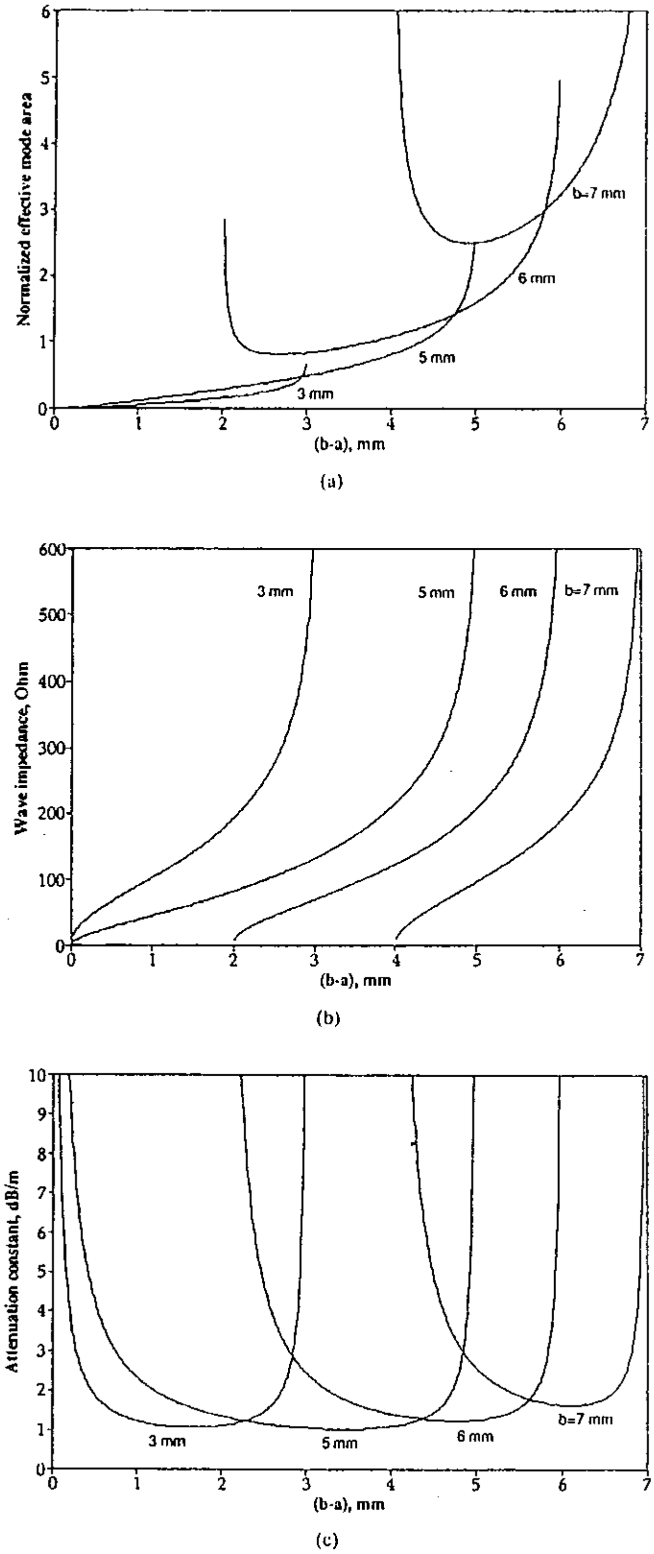

Fig. 4. Normalized effective mode area. (a) Wave impedance, (b) attenuation constant, and (c) versus the distance between wires.

TABLE III
COMPARISON BETWEEN THE ESTIMATED FEM PERFORMANCES WHEN EMPLOYING TWO-WIRE TRANSMISSION LINE OR RECTANGULAR WAVEGUIDE

		Two wire shielded transmission line	Rectangular waveguide
Waveguide characteristics	Dimensions	a=1 mm b=5 mm R=10 mm	22.15×47.55 mm^2
	Mode	TEM	TE_{01}
	Cut-off frequency	0	3.152 GHz
	A_{em}	300 mm^2	526.6 mm^2
FEM performances	FEM operating frequency (where maximum gain is obtained)	5.65 GHz	4.87 GHz
	Maximum gain	11.7	2.2
	Bandwidth (at 3 dB level)	≈300 MHz	≈300 MHz
	Extraction efficiency	≈3 %	≈3 %
	Saturation power	≈ 2 kW	≈ 2 kW

Geometrical parameters of the transmission line can be found from Fig. 4(a), where the dependence of $\overline{A}_{em}$ versus distance $(b-a)$ between the wire and the center of the shield, calculated for $R = 10$ mm and different b, are presented. Other parameters necessary for FEM resonator design are the wave impedance Z and the attenuation constant q [given in Fig. 4(b) and (c)]. The attenuation constant was calculated for a frequency of $f = 5$ GHz and $\sigma_1 = 10^7$ 1/Ohm · m, $\sigma_2 = 4 \cdot 10^7$ 1/Ohm · m. One can note that Z increases monotonically, while q has a minimum. It is important to note that q is close to its minimum over a rather large range of $(b-a)$.

Linear gain calculations were made for parameters of the FEM, which was recently operated at Tel Aviv University [19], [20] (see Table II). This set of parameters results in an amplitude of electron oscillations of the order of 3 mm. According to this, we have taken $b-a = 4$ mm and $b = 5$ mm. From the curves of Fig. 4, we obtain the following values for the transmission line parameters: $A_{em} \approx 300\,\text{nm}^2, Z \approx 200$ Ohm and $q \approx 1$ dB/m. The frequency dependence of the FEM gain expressed as a ratio is shown in Fig. 5. As we see at the operating frequency $f = 5.65$ GHz, the gain is much larger than ohmic losses of the transmission line, and successful FEM operation is possible.

To demonstrate gain enhancement when using a TEM transmission line, we also calculated the gain curve of the mode excited in an FEM when a 22.15 × 47.55 mm^2 rectangular waveguide is used. The effective mode area for this case, calculated from (5), is $A_{em} = 526.6$ mm^2. Here it is necessary to expand the gap of the wiggler resulting in reducing the magnetic induction to 300 Gs (instead of 500 Gs). Observe that the gain is much reduced because of the lower power filling factor and poor wiggler's strength. Table III compares estimated FEM performances when employing the two-wire shielded transmission line or rectangular waveguide.

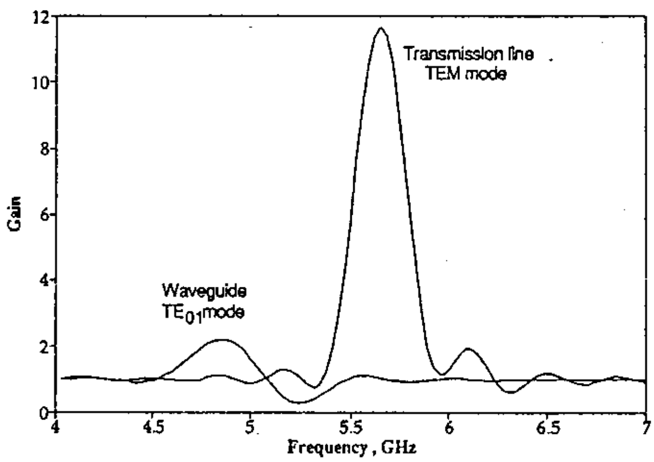

Fig. 5. FEM gain-frequency response.

V. CONCLUSION

We proposed and studied a free-electron maser operating with a shielded TEM transmission line. Such a transmission line at centimeter and millimeter wavelengths combines the advantages of an open cavity with those of a waveguide-based resonator, i.e., the lack of frequency cutoff in the transmission line enables FEM operation at any e-beam energy, and a shield allows reaching a strong electric field concentration in the region occupied by electrons (high filling factor). Continuous tuning of an FEM over a very wide frequency range is therefore possible.

A numerical analysis of the transmission line parameters was made on the basis of the integral equation technique; we obtained in this paper a new representation for the Green function of the internal region of a circle. We found that there is a wide range of geometrical parameters of the investigated transmission line in which the attenuation constant is close to its minimal value and the effective mode area is small, leading to high gain.

Linear gain calculations show that large gain values on the order of 10 can be achieved. These gain values are much above the ohmic losses in the transmission line. They allow for the design of a compact FEL oscillator with a short wiggler and, consequently, attain higher efficiency. The analyzed transmission line is, therefore, of great practical importance for high gain, continuous tuning FEM applications.

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