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Study of waveguide resonators for FEL operating at submillimeter wavelengths

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Abstract

Theoretical investigations of waveguides for FEL operating at submillimeter wavelengths were made. Several types of overmoded waveguides, i.e. rectangular, circular, parallel curved plates waveguide and metal–dielectric waveguide, were studied in order to achieve the best gain/loss ratio in FEL operation.

Gain calculations were made in the low gain regime. Waveguide ohmic losses were calculated taking into account the anomalous skin effect and the influence of surface roughness.

Numerical calculations made for the parameters of the Israeli tandem electrostatic FEL show that a parallel curved plates waveguide and a metal–dielectric waveguide provide good gain/loss ratios up to frequencies of the order of 1000 GHz (while conventional rectangular and circular waveguides do not).

1. Introduction

This paper deals with the theoretical studies of several waveguide types for FEL operating at millimeter and submillimeter wavelengths. Our investigations were based on the parameters of the Israeli tandem FEL experiment: e-beam energy $E_k = 1.5$–4.5 MeV, e-beam current $I < 2$ A, wiggler period $\lambda_w = 44.44$ mm, number of periods $N_w = 20$, wiggler magnetic field $B_w = 2$–3 kG. Such studies were previously made in Ref. [1] for the FOM-FEL operating at the frequency of 200 GHz and in Ref. [2] for the Israeli tandem FEL for the operation at a frequency of 100 GHz. The aim of this work is to extend the studies made in Ref. [2] in order to find an optimal waveguide for the FEL operation at submillimeter wavelengths.

It is well known that a high operating frequency can be reached by increasing the e-beam kinetic energy E_k, for fixed wiggler parameters. This leads, however, to a very rapid decrease of FEL gain (at relativistic energies FEL small-signal gain falls as E_k^{-5}). Therefore at submillimeter wavelengths it is difficult to provide gain values larger than round trip waveguide loss, which is a necessary condition for FEL oscillation build-up. Consequently our investigations were carried out in order to determine which waveguides type allows one to reach the best gain/loss ratio.

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2. General consideration

2.1. Small-signal gain

Gain calculations were made using the following well known single-mode gain–dispersion equation [3,4] for the FEL operating in the linear regime

$$G(s) \equiv \frac{(s - \mathrm{i}\theta)^2 + \theta_{\mathrm{pr}}^2}{s[(s - \mathrm{i}\theta)^2 + \theta_{\mathrm{pr}}^2] - \mathrm{i}Q}, \tag{1}$$

where

$$Q = \frac{e}{8mc} \frac{I_0}{\gamma^3 \gamma_z^2 \beta_z^3} a_w^2 L_w^3 \frac{(k_z + k_w)^2}{\omega_s} \times \frac{Z_{EH}}{A_{\mathrm{em}x}} [J_0(\rho) - J_1(\rho)]^2, \tag{2}$$

is the gain parameter,

$$\theta_{\mathrm{pr}}^2 = \tilde{r}^2 \frac{e I_0}{\gamma \gamma_z^2 m \varepsilon_0 V_z^3 \pi r_e^2} L_w^2, \tag{3}$$

is the reduced space-charge parameter, $\tilde{r}$ is the plasma frequency reduction factor, r_e is the e-beam radius, $L_w = N_w \lambda_w$ is the wiggler length,

$$\theta = \left(\frac{\omega_s}{V_z} - k_z - k_w\right) L_w, \tag{4}$$

is the detuning parameter, and $a_w = eB_w/mck_w$, $\gamma_z =$

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$\gamma/\sqrt{1+a_w^2/2}$, $\beta=\sqrt{1-1/\gamma^2}$, $\gamma=1+E/mc^2$, $\beta_z=\sqrt{1-1/\gamma_z^2}$, $V_z=c\beta_z$, $\rho=(\omega_s/8\beta k_w c)(a_w/\beta\gamma)^2[1-\frac{1}{2}(a_w/\beta\gamma)^2]^{-3/2}$. J_0, J_1 are Bessel functions, $k_w=2\pi/\lambda_w$, k_z, Z_{EH} and A_{emx} are the longitudinal wave number, wave impedance and effective mode area of the operating mode respectively. The effective mode area is defined by

$$A_{emx} = \frac{Z_{EH}\int[\boldsymbol{E}(x,y)\times\boldsymbol{H}^*(x,y)]_z\,dx\,dy}{|E_x(0,0)|^2}, \tag{5}$$

where the integration is carried out over the waveguide cross-section and $E_x(0,0)$ is calculated on the waveguide axis, x is the wiggling direction, y is the direction of wiggler's magnetic field, z is the waveguide axis.

2.2. Waveguide loss

Waveguide ohmic losses were calculated using perturbation theory having as a small parameter surface impedance of the waveguide walls $R_s=\sqrt{\pi f\mu/\sigma}$, where σ and μ are the conductivity and permittivity of wall, f is the frequency. At low frequencies the waveguide attenuation can be calculated taking the dc value of σ. At millimeter and submillimeter wavelengths this assumption is not valid. According to the results of Refs. [5,6], where highly precise measurements of attenuation of standard rectangular waveguides were carried out over a broad frequency range (25–200 GHz), such effects as anomalous skin effect, surface roughness and effects of waveguide machining quality should be taken into account. It was found [5,6] that due to the skin effect anomaly for copper at $f=35$ GHz the measured surface impedance is 1.13 times lrger than that calculated taking the dc value of σ. The investigations of surface roughness influence showed [5,6] that when the roughness r.m.s dimension is of the order of 1 μm the surface impedance is 1.3 times larger than that of a highly polished surface. Summarizing these results one arrives at an effective copper conductivity which is twice as small as its dc value, i.e. instead of $5.8\times10^7\ \Omega^{-1}\,\mathrm{m}^{-1}$ we have $2.9\times10^7\ \Omega^{-1}\,\mathrm{m}^{-1}$. The last value was used in this paper. We suppose that this value gives at submillimeter wavelengths the low limit of waveguide attenuation.

3. Rectangular waveguide

It is convenient for our consideration to classify eigenmodes of rectangular waveguide (see Fig. 1a) as LM-modes, having $H_x\equiv0$ (i.e. the magnetic field vector lies in the longitudinal cross-section of a waveguide), and as LE-modes, having $E_x\equiv0$. In a planar wiggler FEL, where the electrons are wiggling in the x-direction, only LM-modes are excited efficiently. The electric field of LM-modes has the form

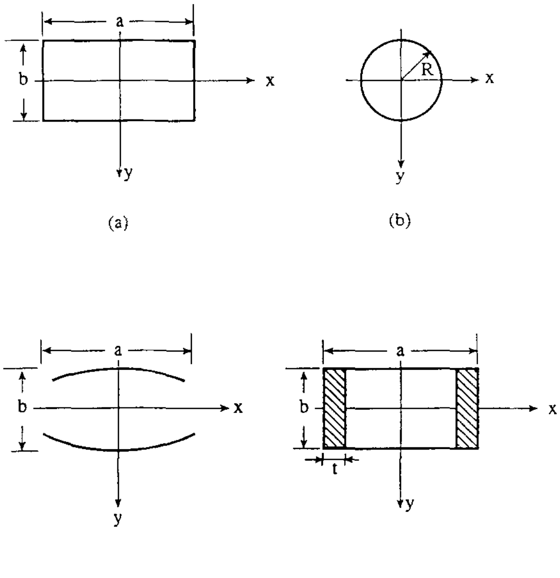

Fig. 1. Cross-sections of investigated waveguides.

$$E_{x,mn}=Z_{EH,mn}H_{y,mn}=U_0\cos(\alpha_{2m}x)\cos(\beta_{2n-1}y), \tag{6}$$

where U_0 is the amplitude of the electric field, $\alpha_m=m\pi/a$, $\beta_n=n\pi/b$, $Z_{EH,mn}=\zeta_0(k_0^2-\alpha_m^2)/k_0k_{z,mn}$ and $k_{z,mn}=\sqrt{k_0^2-\alpha_m^2-\beta_n^2}$ are the wave impedance and the longitudinal wavenumber of LM_{mn}-mode, $\zeta_0=120\,\pi\,\Omega$ and k_0 are the wave impedance and wavenumber in free space. Substituting Eq. (6) into Eq. (5) we obtain $A_{emx}=ab/\varepsilon_m\varepsilon_n$, $\varepsilon_m=2-\delta_{m0}$, δ_{m0} is Kronecker's delta.

Using perturbation theory and field expression (6) we obtain the attenuation constant of LM_{mn}-mode

$$q=\frac{R_sk_0}{\zeta_0ak_{z,mn}}\left[\varepsilon_m+\frac{2a\beta_n^2}{b(k_0^2-\alpha_m^2)}\right]. \tag{7}$$

The calculated dependencies of the FEL gain on frequency are presented in Fig. 2 (solid curves). Calculations were made for $\mathrm{LM}_{01}(\mathrm{TE}_{01})$ mode and $a=b=10$ mm, $B_w=3$ kGs, $I=1$ A, $L_w=889$ mm and various values of e-beam energy E_k. The heavy solid curve represents the value $1/A$, where $A=\exp(-4qL_w)$ is the round trip power attenuation in a lossy waveguide. One may note that gain curves exhibit lower gain at higher operating frequencies where higher e-beam energy is needed. Consequently at frequencies larger than 400 GHz waveguide ohmic loss exceeds the FEL gain, making it imperative to increase in e-beam current or/and interaction length L_w in order to obtain net gain.

Note that the similar results we obtained for a circular waveguide having diameter of 10 mm and operating with the TM_{11} mode.

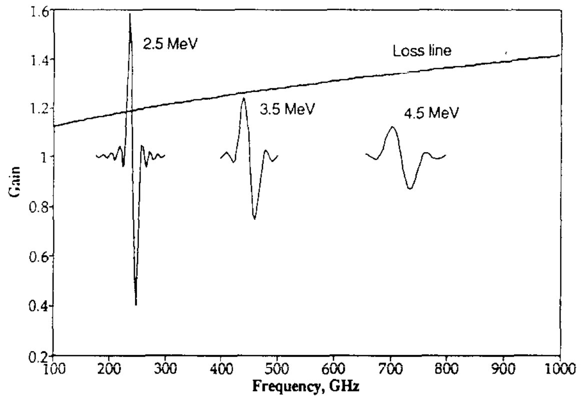

Fig. 2. Frequency dependences of gain (solid curves) and waveguide loss (heavy solid curve) calculated for a rectangular waveguide (LM_{01} mode).

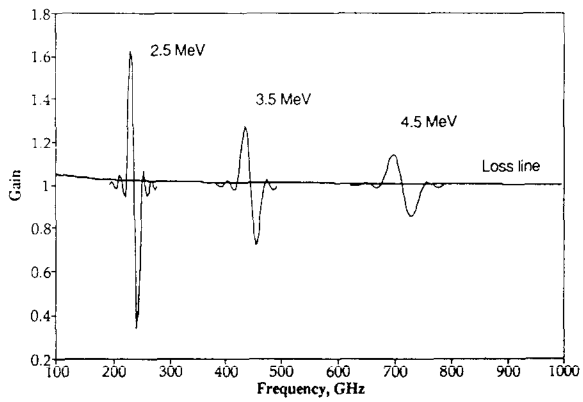

Fig. 3. Frequency dependences of gain (solid curves) and waveguide loss (heavy solid curve) calculated for a parallel curved plates waveguide (TE_{01} mode).

4. Parallel curved plates waveguide

The large ohmic losses (in comparison to gain) of rectangular and circular waveguides directed our investigations to unconventional waveguides. One of them is the parallel curved plates waveguide, shown in Fig. 1c. The experimental results of Ref. [2], where low attenuation of the order of 0.1 dB/m at $f \approx 100$ GHz was observed, allow us to assume that this waveguide can be suitable for FEL operation at submillimeter waves.

Analytical expression for eigenmodes were obtained in Ref. [7] in the form of Gaussian-Hermite functions. For the case of a large radius of curvature these expressions can be simplified, and the electric field profile of the TE_{01} mode has the form

$$E_x(x, y) = Z_{EH} H_y(x, y) = U_0 \cos\left(\frac{\pi y}{b}\right) e^{-x^2/w_0^2}, \tag{8}$$

where

$$w_0^2 = \frac{\sqrt{2Rb - b^2}}{\kappa_{01}},$$

$$\kappa_{01} = \frac{1}{b}\left[1 + \tan^{-1}\left(\frac{b}{\sqrt{2Rb - b^2}}\right)\right],$$

$$Z_{EH} = \frac{\zeta_0}{\sqrt{1 - (\kappa_{01} c/\omega)^2}}$$

is the wave impedance of the TE_{01} mode, and R is the curvature radius of the plates. Substituting the expression (8) into Eq. (5) we obtain $A_{emx} = \sqrt{\pi/8} w_0 b$.

The attenuation constant due to finite conductivity of the waveguide walls is given by

$$q_\sigma = \frac{R_s 2\pi^2}{\zeta_0 k_0 k_z b^3}, \tag{9}$$

and the attenuation constant caused by diffraction through the slot between plates is expressed by:

$$q_d = \frac{\kappa_{01}}{k_0 b \sqrt{\pi} \chi \exp(\chi^2)}, \quad \chi = a/\sqrt{2} w_0 . \tag{10}$$

Fig. 3 presents results of gain (solid curves) and waveguide loss (heavy solid curve) calculations made for $R = 16$ mm, $b = 10.7$ mm. The attenuation in this type of waveguide decreases with increasing frequency contrary to conventional waveguide (see Fig. 2). Therefore we obtain attenuation values smaller than FEL gain up to the frequencies of 1000 GHz.

5. Metal–dielectric waveguide

This waveguide (see Fig. 1d) was proposed in Ref. [8] as a promising transmission line for submillimeter wave applications. Its main property is that the operating mode is concentrated in the part of the waveguide cross-section which is free of dielectric, while the parasitic modes are concentrated inside the dielectric layers. This means that the operating mode excites very small surface currents on the waveguide walls and coupling between operating and parasitic modes is very weak. Therefore low attenuation and sustenance of the operating mode can be achieved in such a waveguide.

The electric field of LM_{m1} mode of metal-dielectric waveguide (MDW) is given by

$$E_x = U_0[k_z^2 + (\pi/b)^2] \frac{1}{N_m} \sqrt{2/b} \cos\left(\frac{\pi y}{b}\right) \times \begin{cases} \dfrac{\cos[\alpha_m^{(1)}(a/2 - x)]}{\bar{\varepsilon} \cos(\alpha_m^{(1)} t)}, & a/2 - t < |x| < a/2, \\[2ex] \dfrac{\cos(\alpha_m^{(2)} x)}{\cos[\alpha_m^{(2)}(a/2 - t)]}, & 0 < |x| < a/2 - t, \end{cases} \tag{11}$$

$\alpha_m^{(1)} = \sqrt{k_0^2\bar{\varepsilon} - (\pi/b)^2 - k_z^2}$, $\alpha_m^{(2)} = \sqrt{k_0^2 - (\pi/b)^2 - k_z^2}$, $Z_{EH} = \zeta_0[k_z^2 + (\pi/b)^2]/k_0k_z$ is the wave impedance of LM_{m1} mode, the longitudinal wavenumber k_z is the solution of the dispersion equation

$$\frac{1}{\bar{\varepsilon}}\alpha_m^{(1)}\mathrm{tg}(\alpha_m^{(1)}t) + \alpha_m^{(2)}\mathrm{tg}[\alpha_m^{(2)}(a/2-t)] = 0\,, \tag{12}$$

$\bar{\varepsilon} = \bar{\varepsilon}' + \mathrm{i}\bar{\varepsilon}''$ is the complex dielectric constant of side wall dielectric layers, t is the thickness of layers,

$$N_m = \frac{1}{\sqrt{2}}\left\{\frac{t}{\bar{\varepsilon}\cos^2(\alpha_m^{(1)}t)} + \frac{a/2-t}{\cos^2[\alpha_m^{(2)}(a/2-t)]} + \frac{\mathrm{tg}(\alpha_m^{(1)}t)}{\bar{\varepsilon}\alpha_m^{(1)}} + \frac{\mathrm{tg}[\alpha_m^{(2)}(a/2-t)]}{\alpha_m^{(2)}}\right\}^{1/2}, \tag{13}$$

The effective mode area is found to be given by expression

$$A_{\mathrm{em}x} = bN_m^2\cos^2[\alpha_m^{(2)}(a/2-t)]/2\,. \tag{14}$$

The attenuation constant caused by the finite conductivity of the walls has the form where

$$q_\sigma = \frac{R_s}{2\zeta_0k_0k_z[k_z^2 + (\pi/b)^2]}(P_1 + P_2)\,, \tag{15}$$

where

$$P_1 = \frac{4\pi^2k_0^2}{b^3N_m^2}\left(\frac{[t + \sin(2\alpha_m^{(1)}t)/2\alpha_m^{(1)}]}{2\cos^2(\alpha_m^{(1)}t)} + \frac{\{a/2 - t + \sin[2\alpha_m^{(2)}(a/2-t)]/2\alpha_m^{(2)}\}}{2\cos^2[\alpha_m^{(2)}(a/2-t)]}\right), \tag{16}$$

$$P_2 = \frac{k_0^2[k_z^2 + (\pi/b)^2]}{N_m^2\cos^2(\alpha_m^{(1)}t)}\,. \tag{17}$$

The attenuation constant due to losses in side wall dielectric layers is

$$q_\varepsilon = \frac{\bar{\varepsilon}''/\bar{\varepsilon}'}{4k_zN_m^2}\left[\frac{k_0^2t}{\cos(\alpha_m^{(1)}t)} + (k_0^2\bar{\varepsilon}' - 2\alpha_m^{(1)2})\frac{\mathrm{tg}(\alpha_m^{(1)}t)}{\alpha_m^{(1)}\bar{\varepsilon}'}\right]. \tag{18}$$

Figs. 4 and 5 show results of numerical calculations made for MDW with $a = b = 10\,\mathrm{mm}$, $\bar{\varepsilon}' = 2$, $\bar{\varepsilon}''/\bar{\varepsilon}' = 0.001$. Calculations show that at wide frequency bands the operating mode of MDW has the following remarkable properties: its profile almost coincides with the profile of the LM_{11} mode of a conventional rectangular waveguide; the effective mode area depends weakly on frequency, and $A_{\mathrm{em}x}/ab \approx 0.2$ (see Fig. 4); and has very low attenuation (see heavy solid curves in Fig. 5). The gain curves are presented in Fig. 5 (solid curves). These results show that MDW can provide very good gain/loss ratios up to frequencies of the order of 1000 GHz.

It is necessary to note that rectangular waveguide with corrugated side walls has similar properties to metal–dielectric waveguides. Such waveguides have an advantage of charging effects due to stray electrons. On the other hand its fabrication involves finer technological processing, because the corrugation period must be much smaller than a wavelength; they may also be more susceptible to high RF field breakdown.

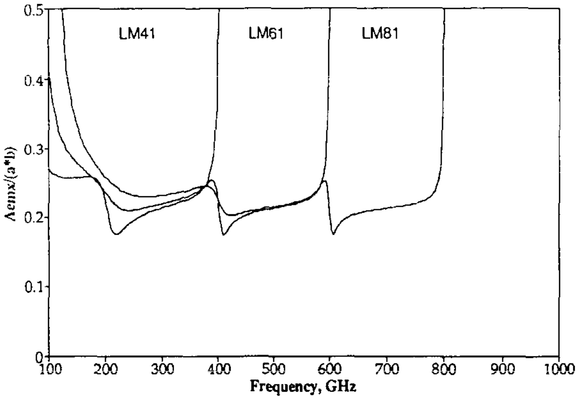

Fig. 4. Frequency dependencies of normalized effective mode area of the MDW eigenmodes.

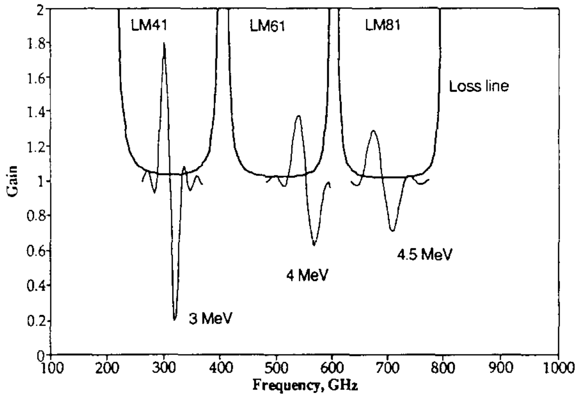

Fig. 5. Frequency dependencies of gain (solid curves) and waveguide loss (heavy solid curve) calculated for a metal–dielectric waveguide.

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