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Section A

Transverse mode coupling and supermode establishment in a free-electron laser oscillator

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Abstract

A three-dimensional study of transverse mode evolution in a free-electron laser (FEL) oscillator is presented. The total electromagnetic field circulating in the resonator is represented as a superposition of transverse modes of the cavity. Coupled-mode theory is employed to derive a generalized 3-D steady-state oscillation criterion, from which the *oscillator supermode* can be found analytically in the linear gain approximation.

The oscillator supermode, which is the eigenmode solution of the oscillator at steady-state, keeps its transverse profile and polarization after each round-trip. Relations between the *oscillator supermode* and the *amplifier supermode* are discussed. It is shown that they are identical only when the feedback process is entirely non-dispersive and non-discriminating.

We employ a 3-D nonlinear simulation code based on the same transverse mode expansion to demonstrate the evolvement of transverse modes in the oscillator towards formation of a supermode in a simple example. The simulation shows that the steady-state result of the oscillation buildup simulation is identical to the supermode predicted by the analytical approach.

1. Introduction

Theoretical studies of nonlinear and saturation processes, taking place in the FEL oscillator have been carried out by the Nizhni-Novgorod research group [1–3], the university of California (Santa-Barbara) [4–5] and the University of Maryland (UMD) [6–9]. The works were carried out in the framework of a 1-D model, assuming a single transverse mode of electromagnetic radiation in the resonator.

In optical open resonators and overmoded waveguide cavities, a three-dimensional model of FEL interaction is required for adequate description of the oscillation built-up process. It was shown in Refs. [10,11], that there is a combination of transverse modes, which keeps such amplitude and phase relations, so that the field profile of the radiation field (except amplitude and phase) does not change along the interaction region ("an amplifier supermode"). In an oscillator configuration, the transverse dependence of the circulating radiation field is determined self-consistently by the amplification and feedback processes and evolves gradually into a steady-state distribution ("an oscillator supermode").

In this paper we present a coupled-mode analysis of radiation field excitation in FEL oscillators. The total field is represented in terms of transverse eigenmodes of the resonator in which the radiation propagates. The evolution of the radiation in the resonator into an oscillator supermode is studied in the linear and nonlinear regimes, employing analytical approach and a 3-D simulation code.

2. Analysis of a multi-transverse mode oscillator

In laser oscillators, usually many transverse modes can be excited simultaneously and may be coupled to each other. Consequently, one should employ a multi-mode analysis including feedback conditions in order to formulate the stability criterion for oscillations. Such a criterion is derived assuming that linear gain expressions can be still employed as the oscillator arrives to steady-state operation. This approach is similar to the one employed in general laser theory for estimating the threshold gain required for self-excitation and oscillation start-up, and for predicting the oscillation frequencies (*longitudinal modes*) in stable operation [12].

Assuming a uniform cross-section resonator (usually a waveguide), the total electromagnetic field at every plane z, can be expressed as a sum of a set of transverse (orthogonal) eigenfunctions $\mathcal{E}_q(x, y)$ with related amplitudes $C_q(z)$. At the entrance to the wiggler, the modes are assumed to have initial amplitudes $C_q(0)$ and the total field at $z = 0$ is given by:

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$$\tilde{E}(x, y, z=0) = \sum_q C_q(0)\mathscr{E}_q(x, y) . \quad (1)$$

Passing through the interaction region of the laser, the "slow varying" amplitude of each mode is $C_q(z)$, and the total electromagnetic field at the exit of the interaction region can be written as:

$$\tilde{E}(x, y, z=L_w) = \sum_q C_q(L_w)\mathscr{E}_q(x, y)\, e^{jk_{zq}L_w} , \quad (2)$$

where k_{zq} is the axial propagation constant of transverse mode q. Part of the field is coupled out through the resonator out-coupler and the remainder is reflected and fed back to the input of the FEL amplifier, as shown schematically in Fig. 1.

In the general case, the reflection mirrors can produce inter-mode scattering and there may be cross-coupling between the reflected modes. After a round trip in the resonator the total field circulated back into the entrance of the interaction region is:

$$\tilde{E}(x, y, z=l_c) = \sum_q C_q(l_c)\mathscr{E}_q(x, y)\, e^{jk_{zq}l_c} . \quad (3)$$

l_c is the total round-trip length of the cavity, and $C_q(l_c)$ are the mode amplitudes after a round-trip in the resonator and are given by:

$$C_q(l_c) = \sum_{q'} \rho_{qq'}C_{q'}(L_w) , \quad (4)$$

where $\rho_{qq'}$ are complex reflection coefficients, expressing the intermode scattering of transverse mode q', to mode q, due to the resonator mirrors or any other passive elements in the entire feedback loop. The expression for the total circulated field is found from Eqs. (3) and (4):

$$\tilde{E}(x, y, z=l_c) = \sum_q \left[\sum_{q'} \rho_{qq'}C_{q'}(L_w)\right]\mathscr{E}_q(x, y)\, e^{jk_{zq}l_c} . \quad (5)$$

When the oscillator arrives to its steady-state regime of operation, the initial field at $z=0$ must be equal to the circulated field after a round-trip $z=l_c$, i.e.:

$$E(x, y, z=0) = E(x, y, z=l_c) . \quad (6)$$

By substituting in Eq. (6) the expressions (1) and (5) for the fields, and scalar multiplying both sides of the equation by the eigen-function $\mathscr{E}_q(x, y)$, one obtains the steady state oscillation condition:

$$C_q(0) = e^{jk_{zq}l_c}\sum_{q'} \rho_{qq'}C_{q'}(L_w) . \quad (7)$$

It was shown in the coupled-mode analysis of the FEL amplifier carried out in Refs. [10,11], that the amplitude of the transverse modes at the output of the FEL interaction region can be written in terms of the gain matrix $\mathbf{\Gamma}(L_w)$ of the FEL:

$$\boldsymbol{C}(L_w) = \mathbf{\Gamma}(L_w)\boldsymbol{C}(0) . \quad (8)$$

Substituting Eq. (8) in Eq. (7), we derive a set of equations for the amplitudes of the modes in steady-state operation:

$$C_q(0) = e^{jk_{zq}l_c}\sum_{q'} \rho_{qq'}\sum_{q''} \Gamma_{q'q''}(L_w)C_{q''}(0) . \quad (9)$$

The last set of equations (9) can be written in a compact matrix form:

$$[e^{j\mathbf{K}_z l_c}\boldsymbol{\rho}\mathbf{\Gamma}(L_w) - \mathbf{I}]\boldsymbol{C}(0) = 0 , \quad (10)$$

where the matrix $\mathbf{K}_z$ is a diagonal matrix with the wavenumbers k_{zq} on its diagonal. The condition for a nontrivial solution for $\boldsymbol{C}(0)$ is vanishing of the determinant:

$$|e^{j\mathbf{K}_z l_c}\boldsymbol{\rho}\mathbf{\Gamma}(L_w)\mathbf{I}| = 0 . \quad (11)$$

This is a generalized oscillation criterion for the case where a number of transverse modes are excited in the resonator. It is an extension to the criterion derived for single transverse mode laser oscillators [12] (where the gain, wavenumber and reflection coefficient are scalars), $\rho_q\Gamma_q(L_w)\, e^{jk_{zq}l_c} = 1$.

3. The "supermodes" of the FEL oscillator

To derive the field profile of the steady-state eigenmodes of the oscillator (supermodes), we employ a linear transformation which transforms the coupled system of modes

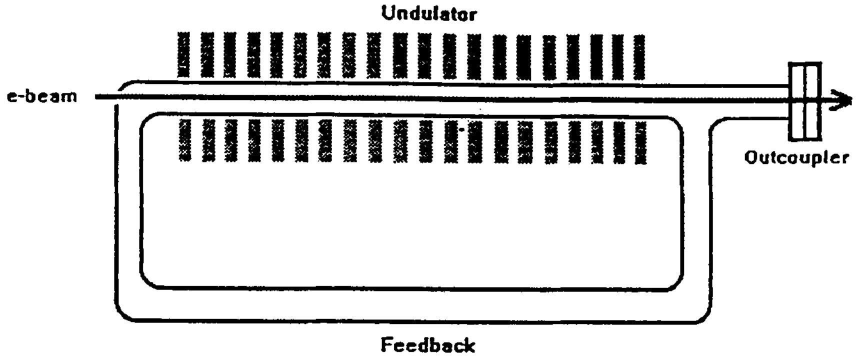

Fig. 1. Schematic illustration of a free-electron laser oscillator.

to an uncoupled one [13–14]. In every cross-section, each of the transverse (free-space or waveguide) modes can be written as a linear combination of a new set of uncoupled eigenmodes with amplitudes $\boldsymbol{U}(z)$. The relation between the two representations at $z = 0$ is given through the linear transformation:

$$\boldsymbol{C}(0) = \mathbf{T}\boldsymbol{U}(0)\,. \tag{12}$$

This transformation is used together with Eq. (10) to derive the steady-state condition for the supermodes:

$$\boldsymbol{U}(0) = \mathbf{T}^{-1}[\mathrm{e}^{\mathrm{j}\mathbf{K}_z l_c}\boldsymbol{\rho}\boldsymbol{\Gamma}(L_w)]\mathbf{T}\boldsymbol{U}(0)\,. \tag{13}$$

The above equation (13) is satisfied when the similarity transformation $\mathbf{T}^{-1}[\mathrm{e}^{\mathrm{j}\mathbf{K}_z l_c}\boldsymbol{\rho}\boldsymbol{\Gamma}(L_w)]\mathbf{T}$ produces a diagonal unit matrix. In that case the linear transformation $\mathbf{T}$ represents the superposition of cavity modes (of amplitudes $C_q(0)$) at $z = 0$ that keeps its transverse features every round-trip in the resonator.

Unlike the amplifier case, the supermodes of the oscillator do not, in general, keep their transverse profile unchanged along the resonator. Only when the feedback is non-dispersive, and all the modes are reflected indiscriminatively and without interscattering, the transformation matrix $\mathbf{T}$ is exactly the transformation required in order to derive the supermodes of the free-electron laser amplifier.

4. Transverse mode evolution in the FEL oscillator

In order to demonstrate the evolution of the electromagnetic radiation field in a multi-transverse mode free-electron laser oscillator into a supermode, we employ a three dimensional computer program simulating the FEL amplifier operation in the linear and non-linear regimes and an appropriate algorithm for feedback process thus simulating the oscillation built up process round-trip after round-trip until steady state is achieved. The FEL amplification code is based on a modal expansion of the total electromagnetic field in terms of the transverse waveguide modes as in Eq. (2). It solves self consistently a system of electron force equations and electromagnetic mode excitation equations [15].

We first show the calculation of the supermode in a specific example based on the analytical theory in the linear regime. The example presented here is of the electrostatic accelerator free-electron maser (FEM) now being developed in Israel [16,17]. Fig. 2 illustrates the small-signal gain curves of the fundamental TE_{01}, and the degenerate TE_{21} and TM_{21} modes, excited in the FEL amplifier operating in the linear regime. The results of single mode gain calculations (disabling coupling between the modes in the gain calculations) are given as dashed lines. Since the TE_{21} and TM_{21} modes have the same wavenumber, they operate in the same frequency range,

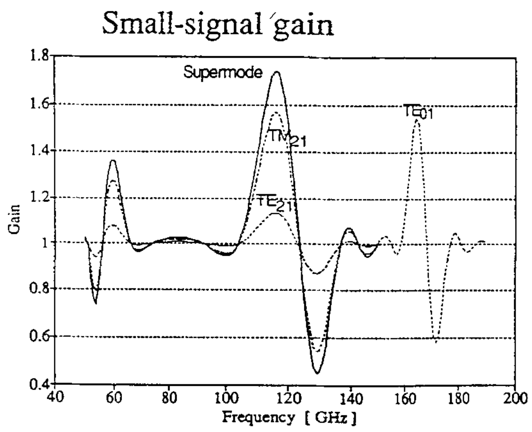

Fig. 2. Small-signal gain curves of the FEL amplifier.

and can strongly couple to each other. The gain curve of their resultant supermode is shown as a continuous line. This mode, found from couple-mode theory, was identified in this particular case as the linearly polarized LP_{21} mode of the rectangular waveguide [18], which is purely polarized in the wiggling dimension. Inspection of the gain curve reveals that the highest gain is obtained at a frequency $f = 116$ GHz, where the linearly polarized mode LP_{21} exhibits maximum gain.

We now report also a complete nonlinear numerical simulator of the process of radiation build-up in the FEL oscillator. Starting from a low level of initial power, the radiation obtained at the output of the FEL amplifier at each stage is fed back to its input, as described by Eq. (7), assuming that there is no cross-coupling between the modes due to the mirrors of the resonator. The phase shift for the degenerate TE_{21} and TM_{21} is assumed to be $2m\pi$ (the phase shift of the TE_{01} is determined by its wave number and the length of the feedback loop). Neglecting at this time multi-longitudinal mode competition, we assume operation at a single frequency corresponding to the maximum linear gain of the TE_{21} and TM_{21} modes and uniform power reflectivity of $\mathscr{R} = |\rho|^2 = 90\%$ for each of the transverse modes. Internal waveguide losses are neglected.

At the first round-trip, the fundamental TE_{01} mode and the degenerate TE_{21} and TM_{21} modes were assigned equal initial power and phase. The initial power was determined to be sufficiently small to avoid nonlinear effects on the first traversals. Graphs of the power carried by each of the individual modes relative to the total power circulating in the oscillator are shown in Fig. 3. The phase relation between the degenerate TE_{21} and TM_{21} modes is also drawn.

The evolution of the single-pass gain of the individual modes as a function of round-trip number is shown in Fig. 4. During several round trips, the radiation power is still small and the FEL is operating in the linear regime. The gain of the coupled TE_{21} and TM_{21} modes is self adjusted

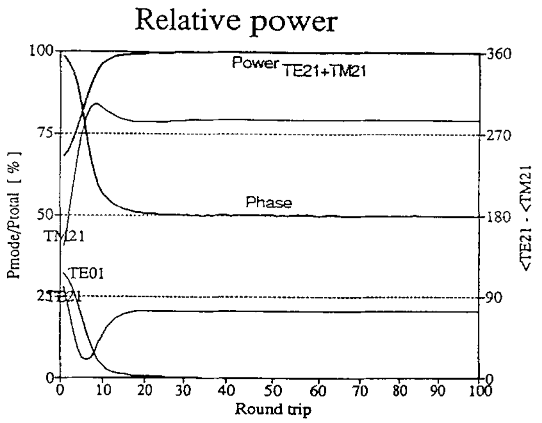

Fig. 3. Relative circulating power and phase evolution of transverse waveguide modes starting from equal power and same phase in an FEL oscillator.

until the power is shared in a combination that corresponds to the LP_{21} supermode. The non-synchronous fundamental TE_{01} mode does not contribute much to the interaction. As the circulated power grows, the oscillator enters the nonlinear regime, and the gain decreases. In this regime, the amplitude growth of the modes restrains until saturation is reached. Saturation is characterized by a constant FEL gain, equal to the transmission losses of the cavity (in the present simulation the gain $G = 1/\mathscr{R} = 1.1$). Observe that the phase difference changes until the TE_{21} and TM_{21} modes lock in anti-phase. This demonstrates the transverse mode evolution towards generation of the LP_{21} supermode, which is an anti-phase combination of the TE_{21} and TM_{21} modes. This supermode is the steady-state eigenmode of the FEL oscillator.

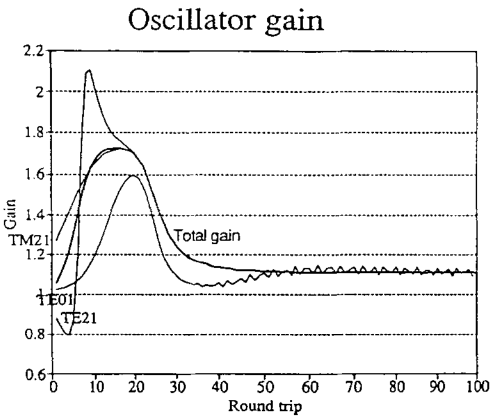

Fig. 4. Gain evolution of transverse waveguide modes starting from equal power and same phase in an FEL oscillator.

It is important to note that the process of the supermode build-up starts well before the onset of saturation. Contrary to the longitudinal mode competition process, which is an entirely nonlinear (saturation regime) effect [1–9], the transverse mode interaction process takes place also in the linear regime.

The simulation confirms numerically the prediction of the analytical model, that when the feedback is non-dispersive, the *oscillator supermode* at steady-state is identical to the *amplifier supermode*, and produces the same supermode solution (the LP_{21} mode) that was predicted with the analytical model.

References

[1] V.L. Bratman, N.S. Ginzburg and M.I. Petelin, Sov. Phys. JTEP 49 (1979) 469.

[2] V.L. Bratman, N.S. Ginzburg and M.I. Petelin, Opt. Commun. 30 (1979) 409.

[3] I. Kimmel and L. Elias, Phys. Rev. A 35 (1987) 3818.

[4] I. Kimmel and L. Elias, Phys. Rev. A 38 (1988) 2889.

[5] B. Levush and T.M. Antonsen, Nucl. Instr. and Meth. A 272 (1988) 375.

[6] B. Levush and T.M. Antonsen, Nucl. Instr. and Meth. A 285 (1988) 136.

[7] T.M. Antonsen and B. Levush, Phys. Fluids B 1 (1989) 1097.

[8] T.M. Antonsen and B. Levush, Phys. Rev. Lett. 62 (1989) 11488.

[9] B. Levush and T.M. Antonsen, IEEE Trans. Plasma Sci. 18 (1990) 260.

[10] Y. Pinhasi and A. Gover, Phys. Rev. E 51 (1994) 2472.

[11] Y. Pinhasi, A. Gover, R.W.B. Best and M.J. van der Wiel, Nucl. Instr.and Meth. A 318 (1992) 523.

[12] A. Yariv, Optical Electronics, 4th ed. (Holt, Rinehart & Winston, New York, 1991).

[13] W.H. Louisell, Coupled Mode and Parametric Electronics (Wiley, New York, 1960).

[14] J.H. Wilkinson, The Algebric Eigenvalue Problem (Clarendon, Oxford, 1965).

[15] Y. Pinhasi, M. Cohen and A. Gover, Int. J. Elect. 78 (1995) 581.

[16] M. Cohen, M. Draznin, A. Goldring, A. Gover, Y. Pinhasi, J. Wachtell, Y. Yakover, J. Sokolowski, B. Mandelnbaum, Y. Shiloh, G. Hazak, L.M. Levine and O. Shahal, Nucl. Instr. and Meth. A 341 (1994) ABS 57.

[17] A. Arensburg, M. Cohen, M. Draznin, A. Eichenbaum, A. Gover, Y. Pinhasi, V. Shterngartz, I.M. Yakover, J. Sokolovsky, B. Mandelnbaum, A. Rosenberg, J. Shiloh, G. Hazak, L.A. Levin, O. Shahal, M. Shapira, V. Granatstein, Nucl. Instr. and Meth. A 358 (1995) ABS 7.

[18] L. Lewin, Theory of Waveguides (Butterworths, London, 1975).