ELSEVIER

Nuclear Instruments and Methods in Physics Research A 393 (1997) 316–322

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Spontaneous emission in waveguide free-electron masers near waveguide cutoff

I.M. Yakover*, Y. Pinhasi, A. Gover

Department of Electrical Engineering and Physical Electronics, Faculty of Engineering, Tel-Aviv University, Ramat Aviv 69978, Israel

Abstract

In this work spontaneous emission is investigated in a waveguide free-electron maser, taking into account previously untreated interaction effects in the vicinity of the waveguide cutoff frequency.

Our study is based on the exact waveguide excitation equations, formulated in the frequency domain for a single electron moving in a planar magnetostatic wiggler. An analytical solution of the amplitude of the excited waveguide mode in the frequency domain was obtained using the Green function technique and allows us to calculate the spectral density of the radiated power and the time-dependent radiated field with good accuracy using a numerical inverse Fourier transform.

The obtained solution shows that for TE-modes the spectral density of the radiated energy tends to infinity at the cutoff frequency of a lossless waveguide. The character of this singularity is, however, such that the total radiated energy is finite. The radiated electromagnetic field in the time domain has the form of very long (of the order of tens of characteristic times on the scale of L_w/c, where L_w is the wiggler length and c is the speed of light) pulse, lagging behind the electron, at the carrier of cutoff frequency, in addition to two finite wave packets, corresponding to the two synchronism frequencies.

The results of a numerical calculation of the radiated energy spectral density and of the radiated electromagnetic field in the time domain are presented.

1. Introduction

Free-electron masers (FEM), operating at centimeter and millimeter waves, usually utilize waveguide resonators. The distinguishing feature of waveguides is that each of its eigenmodes has a cutoff frequency and, therefore, waveguide dispersion is nonlinear. It leads to the well-known fact that waveguide FEMs have in some range of e-beam energies two synchronism frequencies [1] (instead of one synchronism frequency for an open cavity configuration having linear dispersion).

In the vicinity of the waveguide cutoff frequency the group velocity of the electromagnetic wave, the wave number and the wave impedance of TM-modes tend to zero, and the phase velocity and the wave impedance of TE-modes tend to infinity. It should be expected that such waveguide properties should have a significant effect on the FEM radiation near the cutoff frequency. Analysis of super-radiant undulator radiation emission in a waveguide at beam line grazing conditions were published recently [2,3]. Nevertheless, to our best knowledge, these specific effects of FEM interaction near waveguide cutoff are not treated as yet.

In FEM oscillators, which are based on a waveguide resonator and intended to operate at the upper synchronism frequency, there may often be a situation in which the low synchronism frequency is close to the waveguide cutoff. In this case, parasitic FEL radiation near waveguide cutoff frequency may be excited and may interfere and draw away energy from the oscillation at the main frequency.

The purpose of this work is to study the electromagnetic radiation of a single electron or a short bunch of electrons moving in a planar magnetostatic wiggler taking into account the above-mentioned effects near the waveguide cutoff frequency. The emission from a finite bunch of electrons can be readily calculated by coherent or incoherent summation of the field or spectral energy density, respectively, that are calculated here, depending on the electron beam statistics.

* Corresponding author. Tel.: +972 3 6408246; fax: +972 3 6423508; e-mail: yakover@eng.tau.ac.il.

PII S0168-9002(97)00501-9

2. Waveguide excitation equations

Consider a regular metallic waveguide placed inside a planar wiggler, as shown in Fig. 1. Even though Fig. 1 depicts a rectangular waveguide, all the formulae presented in this section were obtained for a waveguide of an arbitrary cross section. It is assumed that the waveguide is matched at both ends, so that no reflected waves exist.

A single electron moving in the planar wiggler can be represented by an electric current density

$$J(r,t) = -ev(z_i)\,\delta(x-x_i)\,\delta(y-y_i)\,\delta(z-z_i(t)), \tag{1}$$

where e is the charge of the electron,

$$v(z) = v_z(z)z_0 + \mathrm{Re}[v_\perp \exp(-\mathrm{j}k_\mathrm{w}z)],\quad v_\perp = -\mathrm{j}\frac{a_\mathrm{w}e}{\gamma}x_0 \tag{2}$$

is the instantaneous velocity of the electron, x_i, y_i, z_i are the coordinates of the electron position at a time t, x_0 and z_0 are the unit vectors along the x- and the z-directions, respectively, v_z is the axial velocity of the electron, $a_\mathrm{w} = eB_\mathrm{w}/(k_\mathrm{w}mc)$, $\gamma = 1 + E_\mathrm{k}/(mc^2)$, $k_\mathrm{w} = 2\pi/\lambda_\mathrm{w}$, B_w and λ_w are the magnetic field and the period of the wiggler, E_k is the kinetic energy of the electron, m is the electron rest mass and c is the speed of light in free space. We also assume that the electron oscillates in the x-direction, so that the transverse component $v_\perp$ of the electron velocity has only a x-component.

The electric field $\tilde{\mathscr{E}}(r,\omega)$ radiated by the electron can be found by solving the Helmholtz equation in the frequency domain:[1]

$$(\Delta + k_0^2)\tilde{\mathscr{E}}(r,\omega) = -\mathrm{j}k_0\varsigma_0\tilde{J}(r,\omega), \tag{3}$$

where k_0 and ς_0 are the wave number and the wave impedance of free space,

$$\tilde{J}(r,\omega) = -2e\frac{v}{v_z}\delta(x-x_i)\,\delta(y-y_i)\exp[\mathrm{j}\omega t(z)]u(\omega), \tag{4}$$

$$t(z) = \int_0^z \frac{\mathrm{d}z'}{v_z(z')}, \tag{5}$$

and $t = 0$ corresponds to the time moment when the electron enters the wiggler.

Solution of Eq. (3) was found by applying the Green function technique and it has the form

$$\tilde{\mathscr{E}}(r,\omega) = \mathrm{j}k_0\varsigma_0\int \ddot{G}(r,r';\omega)\cdot\tilde{J}(r',\omega)\,\mathrm{d}r', \tag{6}$$

[1] Values denoted by tilde are determined for $\omega > 0$ and are related (see Ref. [4]) to the Fourier transform by the following relationship: $\tilde{A}(r,\omega) = 2A(r,\omega)u(\omega)$, where $A(r,\omega)$ is Fourier transform of $A(r,t)$, $u(\omega)$ is the step function.

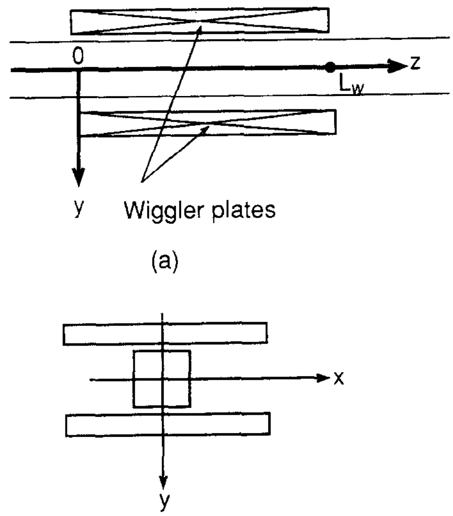


Fig. 1. Waveguide placed in a planar magnetostatic wiggler: (a) longitudinal cross-section, and (b) transverse cross-section.

where $\ddot{G}(r,r';\omega)$ is the solenoidal part of electric-type dyadic Green function of a waveguide. In further analysis we used the Green function representation, derived in Ref. [5] and having the form of a series in terms of waveguide eigenmodes

$$\ddot{G}(r,r';\omega) = \frac{\mathrm{j}}{2k_0\varsigma_0}\sum Z_q \begin{cases} E_q^+(r)\otimes E_q^-(r'), & z > z', \\ E_q^-(r)\otimes E_q^+(r'), & z < z', \end{cases} \tag{7}$$

where q is the eigenmode number, $Z_q = \varsigma_0 k_q/k_0$ and $Z_q = \varsigma_0 k_0/k_q$ is the wave impedance for TM- and TE-modes, respectively, $k_q = (1/c)\sqrt{\omega^2 - \omega_{q,c}^2}$ is the eigenmode wave number, $\omega_{q,c}$ is the cutoff frequency of the qth eigenmode, and E_q^+ and E_q^- is the electric field of qth eigenmode propagating in the positive and negative z-direction, respectively. These fields are normalized over the waveguide cross-section S as follows:

$$\int_S |E_{q,\perp}^\pm(r)|^2\,\mathrm{d}s = 1. \tag{8}$$

Substitution of Eq. (7) into Eq. (6) leads to the general expression of the electric fields radiated by the electron at the exit of the wiggler:

$$\begin{aligned}\tilde{\mathscr{E}}_q^+(x,y,L_\mathrm{w};\omega) &= U_q^+E_q^+(x,y,L_\mathrm{w}) \\ &= -\frac{1}{2}Z_qE_q^+(x,y,L_\mathrm{w})\int_S\int_0^{L_\mathrm{w}} E_q^-(r)\cdot\tilde{J}(r,\omega)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,\end{aligned} \tag{9}$$

and at the entrance of the wiggler,

$$\mathscr{E}_q^-(x, y, 0; \omega) = U_q^- \boldsymbol{E}_q^-(x, y, 0)$$

$$= -\frac{1}{2} Z_q \boldsymbol{E}_q^-(x, y, 0) \int_S \int_0^{L_w} \boldsymbol{E}_q^+(\boldsymbol{r}) \cdot \tilde{\boldsymbol{J}}(\boldsymbol{r}, \omega)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z, \tag{10}$$

where $U_q^{\pm}$ are the amplitudes of the excited eigenmodes, $L_w = N_w \lambda_w$ and N_w are the length and the number of wiggler periods.

Further considerations are concerned only with forward waves. Substituting expression (4) into Eq. (9), and assuming v_z = const. and that the amplitude of the electron wiggling is small, we obtain[2]

$$U_q^+ = \frac{\varsigma_0 a_w e^2 L_w}{2 v_z \gamma} E_{q,x}^-(\bar{x}, \bar{y}, 0)\, S_q(\omega) u(\omega), \tag{11}$$

$$S_q(\omega) = \frac{Z_q}{\varsigma_0} \left[\frac{\exp(\mathrm{j}\theta^+ L_w) - 1}{\theta^+ L_w} - \frac{\exp(\mathrm{j}\theta^- L_w) - 1}{\theta^- L_w} \right], \tag{12}$$

where $\bar{x}$, $\bar{y}$ are the average transverse coordinates of the electron, and $\theta^{\pm} = \omega/v_z \mp k_w - k_q$ is the detuning parameter.

3. Spectral density of energy

The expression representing spectral density of energy was found using Wiener–Khinchine theorem and expression (9); it has the form

$$\frac{\mathrm{d}W_q^+}{\mathrm{d}\omega} = \frac{1}{2\pi\, \mathrm{Re}(Z_q)} |U_q^+|^2. \tag{13}$$

Excitation equation (11) shows that the amplitude of the radiated eigenmode and, consequently, its spectral energy density reach their local maxima at the synchronism frequencies, for which $\theta^+ = 0$. Another interesting range of frequencies that was not treated previously is that in the vicinity of waveguide cutoff. Analysis of Eq. (13) based on the excitation equation (11) and the expression of Z_q leads to a conclusion that in the vicinity of waveguide cutoff $\mathrm{d}W_q^+/\mathrm{d}\omega$ vanishes as k_q for TM-modes; it tends to infinity as $1/k_q$ for the TE-modes. At first glance, this singular behavior of a single electron radiation does not make physical sense, but $1/k_q = c/\sqrt{\omega^2 - \omega_{q,c}^2}$ is an integrable function of ω, and therefore, the total energy radiated by the electron stays finite.

[2] Note that these expressions are similar to those presented in Ref. [2,3]. However, our derivation is different and is based on more general Green function technique.

Numerical calculations were made for the parameters of the prebunched e-beam FEM operating at the Tel Aviv University [6]. This FEM utilizes a wiggler with $\lambda_w = 44.4$ mm, $N_w = 17$, $B_w = 300$ Gs, and operates in the TE_{10} mode of a rectangular waveguide having cross section dimensions $a \times b = 47.55 \times 22.15\ \mathrm{mm}^2$. We made calculations of spectral energy density for three typical energies: $E_k = 40$ keV, for which the FEM has no synchronism frequency, $E_k = 58$ keV has two synchronism frequencies, and $E_k = 90$ keV has one synchronism frequency (see Fig. 2 where the dispersion curves of the waveguide TE_{10} mode and e-beam lines are presented).

Figs. 3(a)–(c) show the value $|\varsigma_0 S_{10}(\omega)|^2/\mathrm{Re}(Z_{10})$ which is proportional to spectral energy density, calculated versus normalized frequency $\bar{\omega} = \omega/\omega_{10,c}$, where $\omega_{10,c}$ is the cutoff frequency of the TE_{10} mode. One may note that in the vicinity of the cutoff frequency $\mathrm{d}W_{10}^+/\mathrm{d}\omega$ tends to infinity. As for other frequencies, the spectral energy density reaches a significant values only near the synchronism frequencies (see Figs. 3(b) and (c)).

4. Radiated field in the time domain

In this section we present the results of simulations of the radiated electromagnetic field in the time domain. The time-dependent electric field at the exit of wiggler is given by the inverse Fourier transform

$$\mathscr{E}_q^+(x, y, L_w; t) = \mathrm{Re}\left[\frac{1}{2\pi} \int_0^{\infty} \tilde{\mathscr{E}}_q^+(x, y, L_w; \omega) \exp(-\mathrm{j}\omega t)\, \mathrm{d}\omega \right]. \tag{14}$$

Substituting expressions (9), (11) and (12) into Eq. (14) we obtain

$$\mathscr{E}_{q,x}^+(x, y, L_w; t) = E_{q,x}^+(x, y, 0) E_{q,x}^-(\bar{x}, \bar{y}, 0) \frac{\varsigma_0 a_w e^2 L_w}{2 v_z \gamma} \times \int_0^{\infty} S_q(\omega) \exp[-\mathrm{j}(\omega t - k_q L_w)]\, \mathrm{d}\omega, \tag{15}$$

where x and y are the coordinates of the observation point.

Figs. 4(a)–(c) display the results of the computation of the integral (15) showing the emitted field versus normalized time $\bar{t} = t/(L_w/c)$ for the same FEM parameters as in the previous section and for three values of e-beam energy: $E_k = 40$ keV (Fig. 4(a)), $E_k = 58$ keV (Fig. 4(b)), and $E_k = 90$ keV (Fig. 4(c)).

One can see that the radiation pulse starts at the time $\bar{t} = 1$, when a high-frequency radiation, having a group

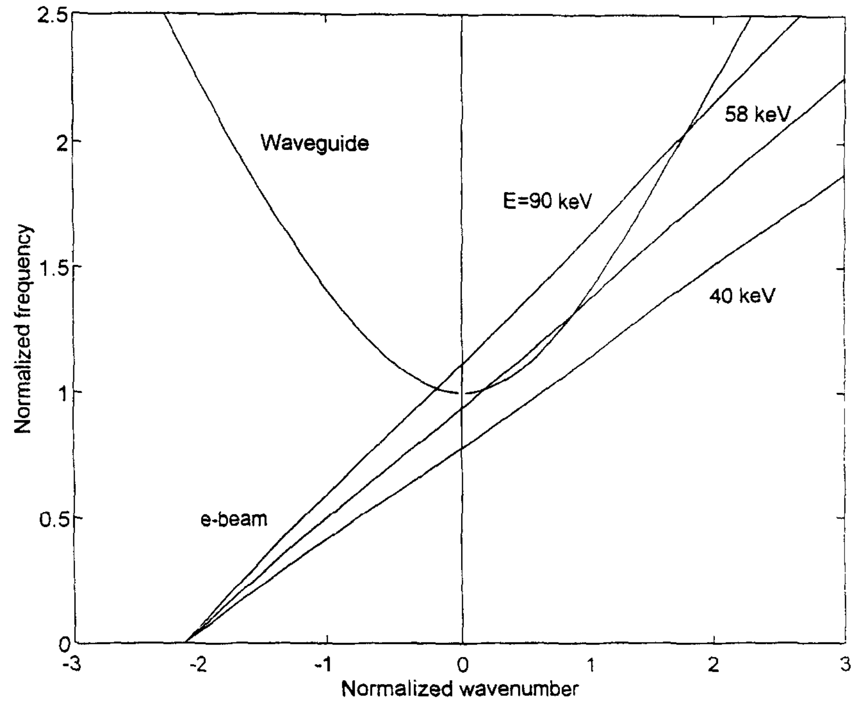


Fig. 2. Normalized frequency $\bar{\omega} = \omega/\omega_{10,c}$ as a function of normalized wave number $\bar{k}$: $\bar{k} = k_{10}/(\omega_{10,c}/c)$ for the waveguide TE_{10} mode, and $\bar{k} = (\omega/v_z - k_w)/(\omega_{10,c}/c)$ for the e-beam mode.

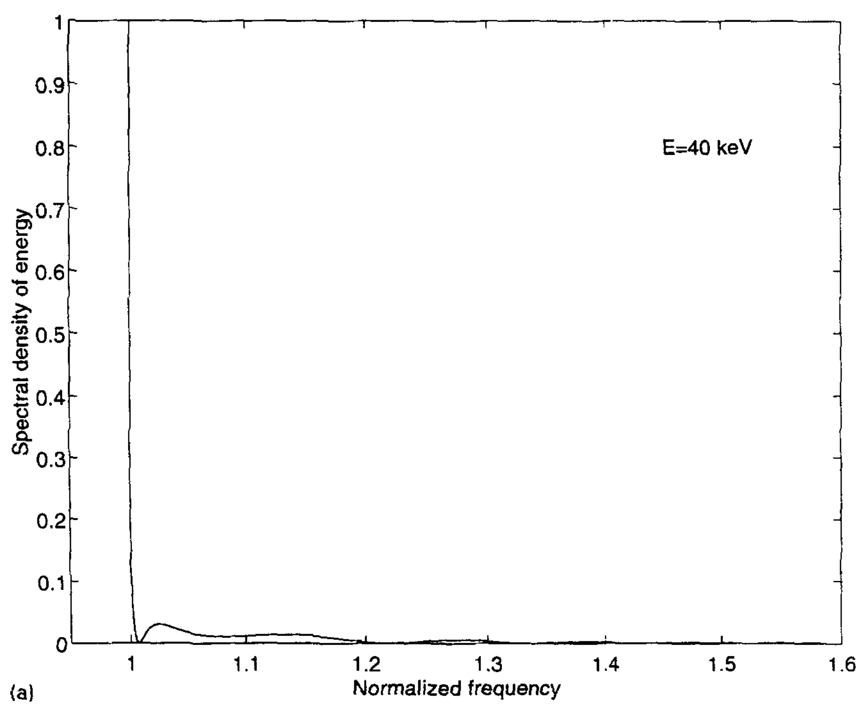
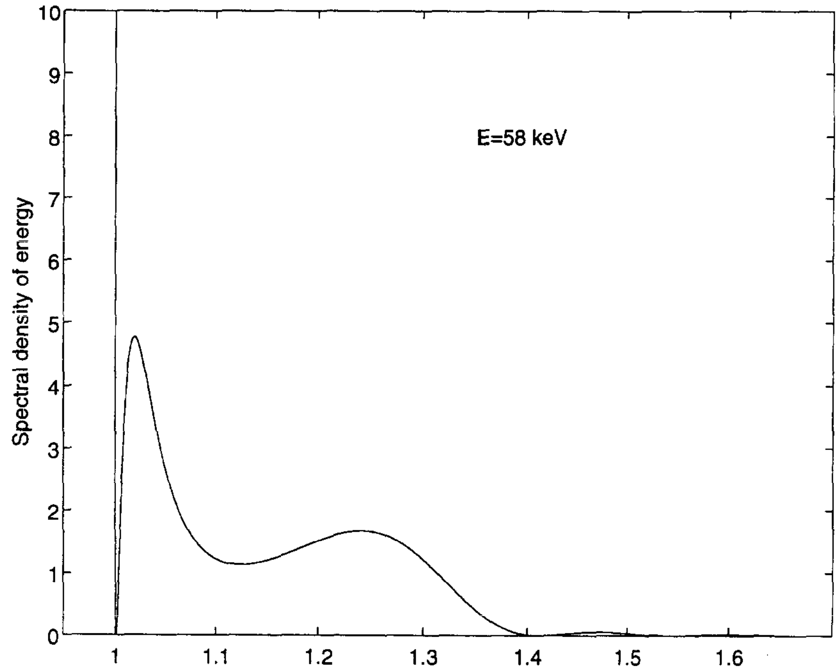
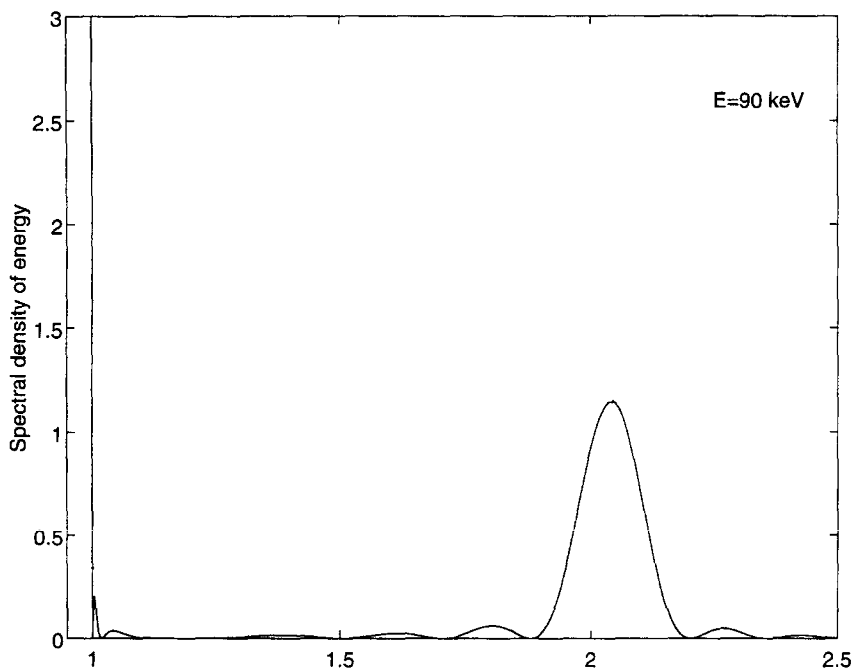


Fig. 3. Spectral density of radiated energy (in arbitrary units) versus normalized frequency $\bar{\omega} = \omega/\omega_{10,c}$: (a) $E_k = 40$ keV, (b) $E_k = 58$ keV and (c) $E_k = 90$ keV.



(b)



(c)

Fig. 3. (*Continued*).

velocity close to the speed of light, arrives at the exit of the wiggler. Further pulse evolution depends strongly on the e-beam energy.

For energy values 40 keV, corresponding to the absence of a synchronism frequency (Fig. 4(a)), a precursor pulse is emitted in the time interval $1 < \bar{t} < \bar{t}_1$ (where

$\bar{t} = 1$ is the time moment when the fast photons propagating at speed c and generated at the wiggler entrance arrive to the wiggler exit and $\bar{t}_1 = c/v_z$ is the electron exit time from the wiggler). The modulated carrier pulse with frequency chirp, decreasing from very high frequencies down to the value corresponding to

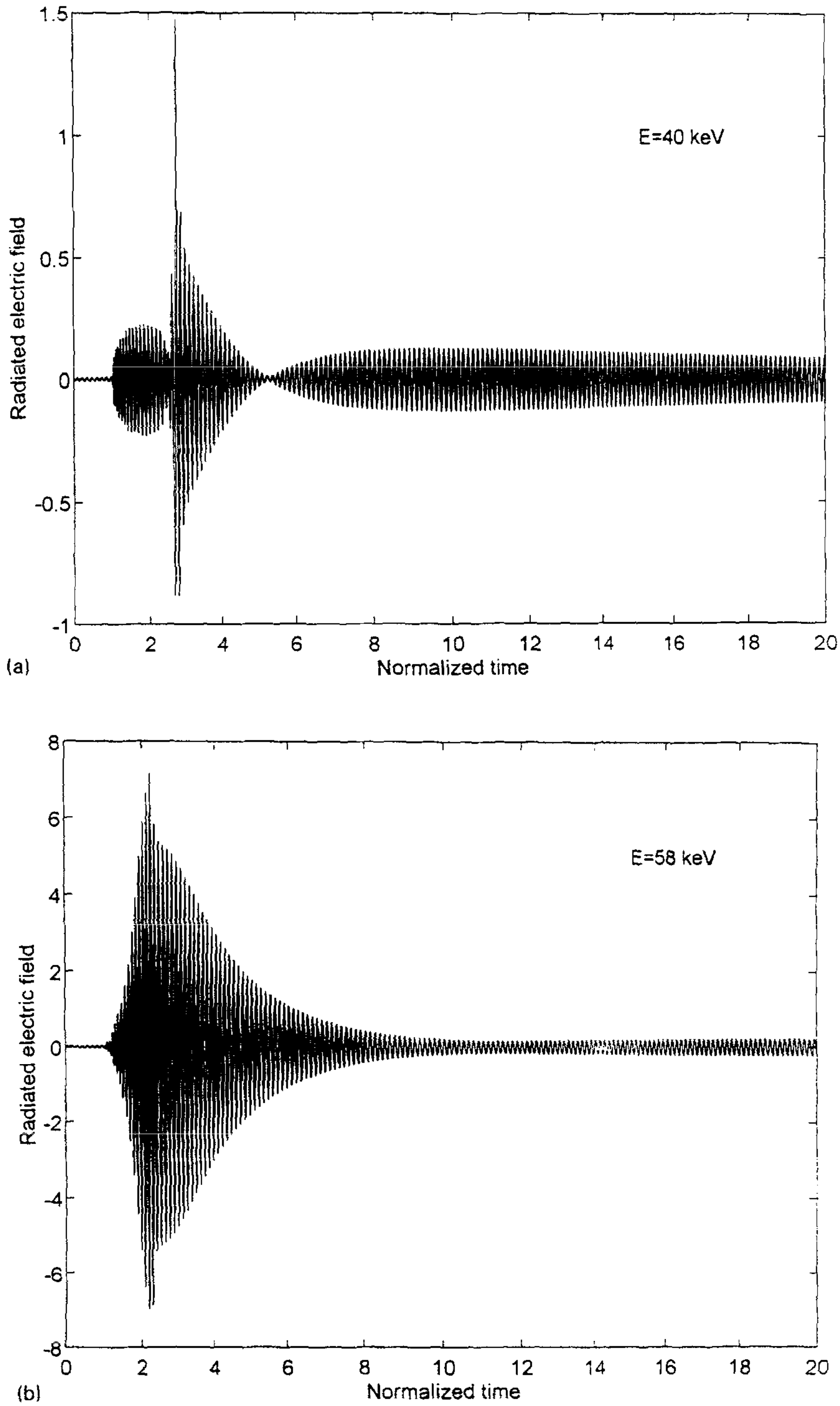


Fig. 4. Radiated electric field (in arbitrary units) at the exit of the wiggler versus normalized time $\bar{t} = t/(L_w/c)$: (a) $E_k = 40$ keV, (b) $E_k = 58$ keV and (c) $E_k = 90$ keV.

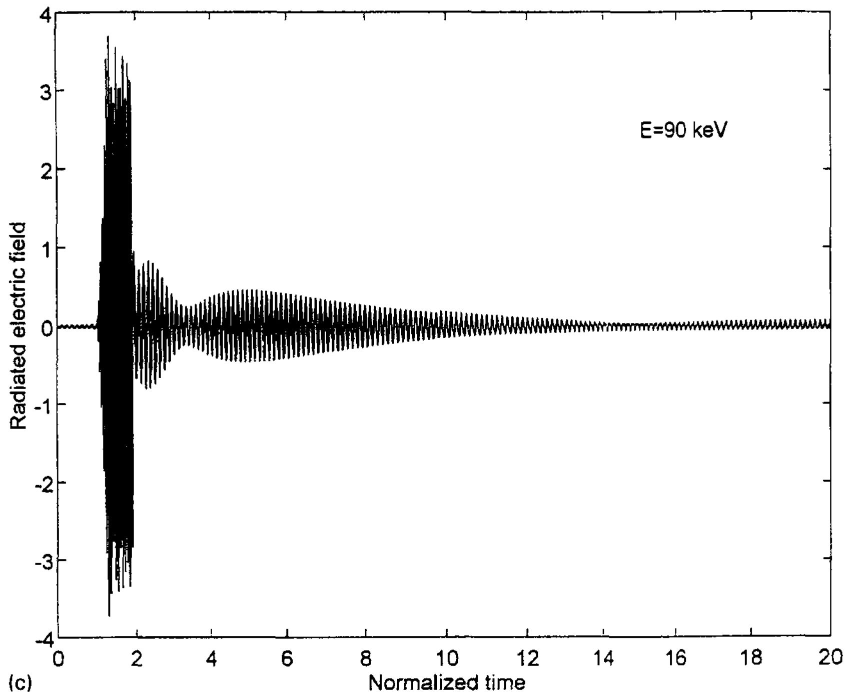


Fig. 4. (*Continued*).

minimum detuning parameter θ^+. At time $\bar{t}_1$, a high-intensity and short duration peak of the radiated field is observed, and at $\bar{t} > \bar{t}_1$ the radiated field has the form of a long decaying carrier pulse at the cutoff frequency. The reason for the long decay time is that near waveguide cutoff the group velocity is very small, so it takes a long time for the wave, radiated by the electron at the beginning of wiggler, to reach the wiggler end.

At an energy of $E_k = 58$ keV (two synchronism frequencies) in the time interval $1 < \bar{t} < \bar{t}_1$ the radiated field has the form of an increasing amplitude carrier pulse at the *higher* synchronism frequency (see Fig. 4(b)). When $\bar{t}_1 < \bar{t} < \bar{t}_2$ ($\bar{t}_2 = c/v_{10,1}$, $v_{10,1}$ is the group velocity of the TE_{10} mode corresponding to the lower synchronism frequency) the radiated pulse is at the *lower* synchronism frequency and its amplitude decreases. After $\bar{t} > \bar{t}_2$, one observes a long decaying carrier pulse of a relatively low amplitude at the cutoff frequency.

At an energy of $E_k = 90$ keV (producing only one syncronism frequency) the radiated field in the time interval $1 < \bar{t} < \bar{t}_1$ has the form of an almost rectangular carrier pulse at the synchronism frequency (see Fig. 4(c)). After $\bar{t} > \bar{t}_1$, the radiated carrier pulse is at the cutoff frequency, and its amplitude oscillates slowly with an increasing period and, as in the previous cases, decays to zero.

The reported simulations indeed indicate an enhanced excitation mechanism of FEM radiation near the cutoff frequency. Analyzing the presented results we conclude that because the spectral density of the radiated energy near waveguide cutoff tends to infinity, the radiated field in the time domain has the form of very long pulse, lagging behind the electron, at the carrier of cutoff frequency, in addition to finite wave packets corresponding to synchronism frequencies. Spontaneous emission near cutoff (where the group velocity of excited waveguide mode is close to zero) has a long slippage time and may affect the radiation buildup process in the FEM oscillator.

References

[1] B. Steinberg, A. Gover, S. Rushin, Phys. Rev. A 33 (1) (1986) 421.

[2] A. Doria, R. Bartolini, J. Feinstein, G.P. Gallerano, R.H. Pantell, IEEE J. Quantum Electron. QE-29 (1993) 1428.

[3] A. Gover, F.V. Hartemann, G.P. Le Sage, N.C. Luhmann, R.S. Zhang, C. Pellegrini, Phys. Rev. Lett. 72 (1994) 1192.

[4] Y. Pinhasi, Ph.D. Thesis, Tel Aviv, 1995.

[5] Kh. L. Garb, P. Sh. Fridberg, I.M. Yakover, Sov. Phys. Coll. 22 (3) (1982) 27.

[6] M. Cohen, A. Eichenbaum, M. Arbel, D. Ben-Haim, H. Kleinman, M. Draznin, A. Kugel, I.M. Yakover, A. Gover, Phys. Rev. Lett. 74 (19) (1995).