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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

# Free electron maser oscillations near waveguide cutoff

L. Gilutin, A. Abramovich, M. Arbel, A. Eichenbaum, A. Gover, H. Kleinman, Y. Pinhasi, Y.M. Yakover*

*Department of Electrical Engineering and Physical Electronics, Faculty of Engineering, Tel-Aviv University, Ramat Aviv 69978, Israel*

**Abstract**

In waveguide-based FEMs there are two possible frequency radiation bands corresponding to the two intersections of the beam line with the waveguide dispersion curve. The low-frequency intersection point occurring near the waveguide cutoff frequency was studied by us experimentally. Special modifications of the existing TAU FEM facility were made in order to investigate FEM radiation near cutoff. Some rectangular waveguide components, used for the radiation coupling from the resonator to the detection system, were replaced by components based on a double-ridged waveguide having a lower cutoff frequency and a wider frequency band. FEM operation in the vicinity of the waveguide cutoff frequency was studied in free-running oscillator configuration using a unique experimental setup enabling time-dependent spectral measurements within a single radiation pulse. The observed FEM radiation was of good spectral purity without any significant mode competition. 

## 1. Introduction

In this paper we report the first experimental results of radiation near waveguide cutoff obtained with the TAU prebunched e-beam FEM. This FEM utilizes a straight section of rectangular waveguide WR-187 as a resonator, which together with a planar magnetostatic wiggler constitutes the FEM interaction region.

It is well known [1], that in waveguide-based FEMs, there are two possible frequency radiation bands, corresponding to the two intersections of the beam line with the waveguide dispersion curve. Until now our FEM was studied in the operation regime corresponding to the upper synchronism condition [2,3]. Our FEM parameters are such that the low synchronism frequency is close to the waveguide cutoff frequency. In particular, for a nominal voltage of 70 kV the low synchronism frequency *coincides* with the cutoff frequency (see Fig. 1). In this case, parasitic FEM radiation near the waveguide cutoff frequency may be excited; it may interfere with and provide radiation at the expense of oscillations at the higher band frequency.

In this work we present first results of FEM operation near the cutoff frequency carried out at lower (than nominal) beam voltage of 60.2 kV, for which the low band synchronism frequency is slightly (of the order of 6%) above the cutoff

* Corresponding author. Tel.: + 972 3 6408246; fax: + 972 3 6423508; e-mail: yakover@eng.tau.ac.il.

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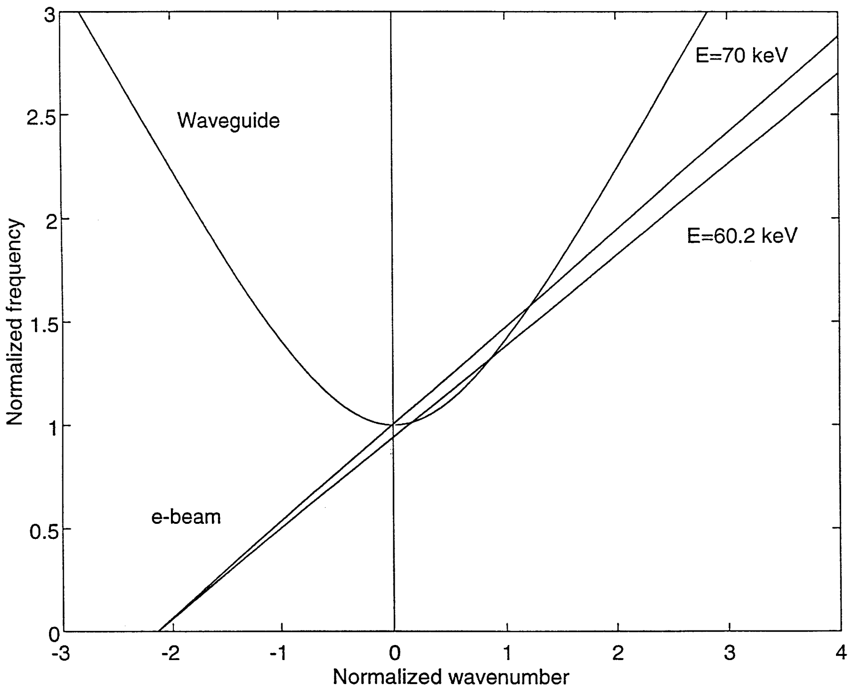


Fig. 1. Normalized frequency $\bar{\omega} = \omega/\omega_c$ as a function of normalized wave number $\bar{k}$:$\bar{k} = k_{10}/(\omega_c/c)$ for the waveguide $TE_{10}$ mode, and $\bar{k} = (\omega/V_z - k_w)/(\omega_c/c)$ for the e-beam mode.

frequency, as shown in Fig. 1. Such operation makes it easier to perform the first diagnostics of the FEM radiation near the low synchronism point. In future experiments, the operating voltage will be increased so as to approach the waveguide cutoff frequency.

## 2. Theoretical consideration

The theoretical study starts by analyzing the FEM small-signal gain, using the well known single-mode gain dispersion equation for an FEL operating in the linear regime [4,5]:

$$G(s) \equiv \frac{(s - \mathrm{i}\theta)^2 + \theta_{\mathrm{pr}}^2}{s[(s - \mathrm{i}\theta)^2 + \theta_{\mathrm{pr}}^2] - \mathrm{i}Q}, \tag{1}$$

where

$$Q = \frac{e}{8mc} \frac{I_0}{\gamma^3 \gamma_z^2 \beta_z^3} a_{\mathrm{w}}^2 L_{\mathrm{w}}^3 \frac{(k_z + k_{\mathrm{w}})^2}{\omega_s} \frac{Z_s}{A_{emx}} [J_0(\rho) - J_1(\rho)]^2, \tag{2}$$

is the gain parameter,

$$\theta_{\mathrm{pr}}^2 = \tilde{r}^2 \frac{eI_0}{\gamma \gamma_z^2 \; m\varepsilon_0 \; V_z^3 \pi r_{\mathrm{e}}^2} L_{\mathrm{w}}^2, \tag{3}$$

is the reduced space-charge parameter, $\tilde{r}$ is the plasma frequency reduction factor, $r_{\mathrm{e}}$ is the e-beam radius, $L_{\mathrm{w}} = N_{\mathrm{w}} \lambda_{\mathrm{w}}$ is the wiggler length,

$$\theta = \left( \frac{\omega_s}{V_z} - k_z - k_{\mathrm{w}} \right) L_{\mathrm{w}}, \tag{4}$$

is the detuning parameter, and

$$a_{\mathrm{w}} = \frac{eB_{\mathrm{w}}}{mck_{\mathrm{w}}}, \quad \gamma_z = \frac{\gamma}{\sqrt{1 + a_{\mathrm{w}}^2/2}},$$

$$\beta = \sqrt{1 - 1/\gamma^2}, \quad \gamma = 1 + \frac{E}{mc^2}, \quad \beta_z = \sqrt{1 - 1/\gamma_z^2},$$

$$V_z = c\beta_z, \quad \rho = \frac{\omega_s}{8\beta k_{\mathrm{w}} c} \left( \frac{a_{\mathrm{w}}}{\beta\gamma} \right)^2 \left[ 1 - \frac{1}{2} \left( \frac{a_{\mathrm{w}}}{\beta\gamma} \right)^2 \right]^{-3/2},$$

$J_0, J_1$ are the zero and first-order Bessel functions, $k_w = 2\pi/\lambda_w$, $k_z$,$Z_s$ and $A_{emx}$ are the longitudinal wave number, wave impedance and effective mode area of the operating mode, respectively, $B_w$ and $\lambda_w$ are the magnetic field and the period of wiggler, $E$ is the e-beam energy, $c$ is the speed of light. In our case, the operating mode is $TE_{10}$ mode, for which $Z_s = \varsigma_0 k_0/k_z$, and $A_{em} = ab/2$, where $k_0$ and $\varsigma_0$ are the wave number and wave impedance in free space, $a$,$b$ are the waveguide dimensions.

By analyzing Eq. (2), we note that for lossless waveguides the wave impedance of TE modes, and consequently, the gain parameter $Q$ goes to infinity at the cutoff frequency. This fact indicates abnormal FEM behavior near waveguide cutoff. In order to work with finite gain values, we take into account the waveguide loss, using the following expression [6] for the complex propagation constant of the $TE_{10}$ mode:

$$k_z^2 = k_{10}^2 + \sqrt{\frac{2}{k_0 \varsigma_0 \sigma} \frac{\pi^2}{a^2 b}}\left(\frac{k_{10}^2}{\pi^2} + \frac{2b}{a} + 1\right)(1 + j),$$

$$k_{10} = \sqrt{k_0^2 - (\pi/a)^2}, \tag{5}$$

where $\sigma$ is the waveguide wall conductivity. Note that expression (5) does not lose its physical meaning near the cutoff frequency, while the widely used $k_z$ expression (see e.g. Ref. [7]) is not valid.

Fig. 2 shows the gain-frequency dependence calculated for the FEM parameters presented in the Table 1, and for a copper waveguide ($\sigma = 4 \times 10^7\,\Omega^{-1}\,m^{-1}$). One can see that the FEM gain curve has a region of abnormally high growth in the vicinity of the waveguide cutoff frequency in addition to two local gain maxima, corresponding to two different synchronism frequencies.

We also analyzed the spontaneous emission near the cutoff frequency, considering single electron radiation. This problem was solved in Ref. [8], and here we present only the main expressions, necessary for a better understanding. The spectral energy density, radiated by a single electron, has the form

$$\frac{dW_q^+}{d\omega} = \frac{1}{2\pi\ \mathrm{Re}(Z_q)}|U_q^+|^2 \tag{6}$$

where

$$U_q^+ = \frac{\varsigma_0 a_w e^2 L_w}{2 V_z \gamma} E_{q,x}^-(\bar{x},\bar{y},0) S_q(\omega) u(\omega) \tag{7}$$

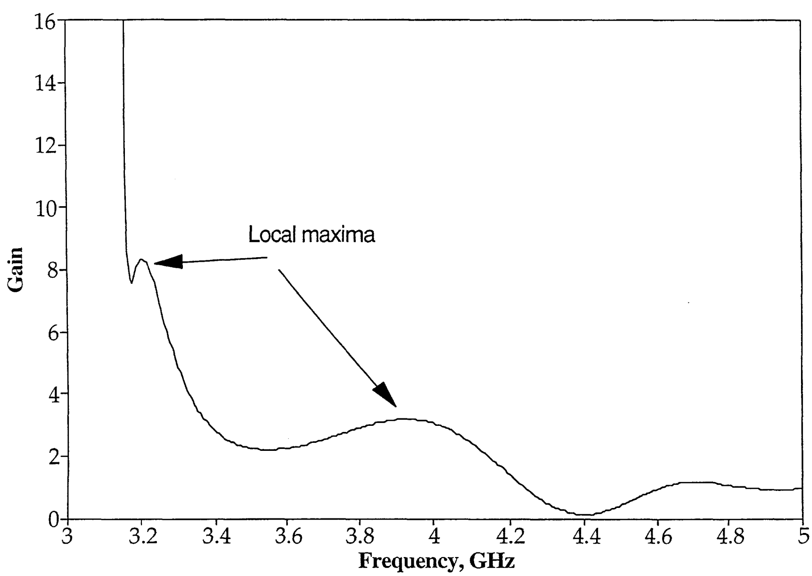


Fig. 2. Gain frequency response.

Table 1
Parameters of the FEM

| | |
|---|---|
| *Accelerator* | |
| Electron beam energy | $E_k = 60.2$ keV |
| Cathode e-beam current | $I_0 = 0.7$ A |
| *Wiggler* | |
| Magnetic induction | $B_w = 300$ Gs |
| Period length | $\lambda_w = 4.44$ cm |
| Number of periods | $N_w = 17$ |
| *Waveguide resonator* | |
| Rectangular waveguide | 2.215 cm × 4.755 cm |
| Mode | $TE_{01}$ |
| Interaction length | $L_w = 74.8$ cm |

is the electric field amplitude of the excited waveguide mode, propagating in the positive direction, $u(\omega)$ is the step function, and

$$S_q(\omega) = \frac{Z_q}{\varsigma_0}\left[\frac{\exp(j\theta^+ L_w) - 1}{\theta^+ L_w} - \frac{\exp(j\theta^- L_w) - 1}{\theta^- L_w}\right],$$

$$\theta^{\pm} = \omega/V_z \mp k_w - k_q. \quad (8)$$

Here $Z_q$ is the wave impedance of the operating mode, and $E^-_{q,x}$ is the transverse profile of its electric field.

Analysis of Eqs. (6)–(8) shows that for lossless waveguides the spectral energy density tends to infinity as $1/k_z$ for the operating $TE_{10}$ mode, but $1/k_z = c/\sqrt{\omega^2 - \omega_c^2}$ ($\omega_c$–is the cutoff frequency) is an integrable function of $\omega$, and therefore the total energy radiated by the electron stays finite. Fig. 3 shows the frequency dependence of the spectral energy density calculated for our experiment parameters. One may note that the spectral density tends to infinity near the waveguide cutoff and reaches significant values only in the vicinity of synchronism frequencies; qualitatively this behavior is similar to gain frequency dependence.

## 3. The modified microwave system

The original microwave system design was intended for an FEM operating near the upper

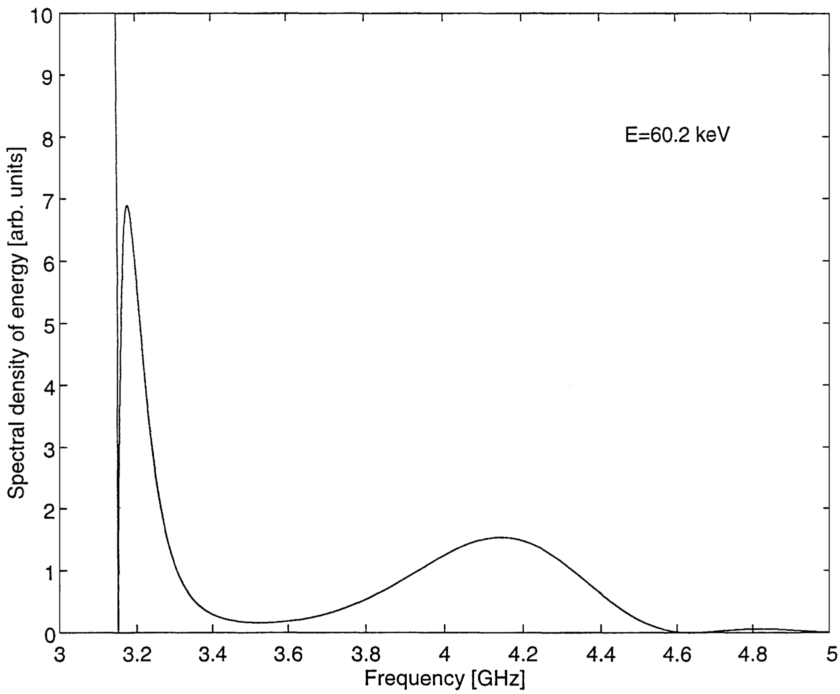


Fig. 3. Spectral density of radiated energy (in arbitrary units) versus frequency.

synchronism frequency, which is far from waveguide cutoff. It is based on standard rectangular waveguide WR-187 components, and it is, therefore, not suitable for FEM radiation study near the cutoff frequency. Therefore, modification of the FEM microwave system were made in order to investigate the FEM radiation near cutoff.

The straight waveguide section, which together with the wiggler constitute the FEM interaction region, and the waveguide bends, intended to separate the e-beam and the FEM radiation, were left without any changes (see Fig. 4). The rectangular waveguide components (such as power samplers, waveguide-coax adapters and matching loads), used for the radiation coupling from the resonator to the detection system, were replaced by double-ridged waveguide (WRD-250) components having a lower cutoff frequency and a wider frequency band. In order to obtain good matching, linear transitions between rectangular and double-ridged waveguide sections were employed. Also, additional matching sections in the form of dielectric pyramids were placed at the transitions in order to decrease reflections near the cutoff frequency.

The modified system was tested using a scalar network analyzer. We have measured the VSWR-frequency dependence for the straight waveguide section with and without linear transitions as mentioned above. The modified microwave system is matched well up to the cutoff frequency: at a frequency $f = 3.143\,\mathrm{GHz}$, which is slightly below cutoff ($f_c = 3.152\,\mathrm{GHz}$), the VSWR = 1.54, corresponding to a power transmission coefficient of $T = 95\%$; whereas the transmission of the original system drops drastically near the cutoff frequency: at $f = 3.15$, VSWR = 38.7, corresponding to a $T = 10\%$. The good matching levels which were achieved allows investigation of FEM operation near the waveguide cutoff both for oscillator and for amplifier configurations.

## 4. Experimental results

The experimental setup shown in Fig. 4 is similar to that used in Ref. [9] for mode competition studies in the high synchronism frequency band. The difference is only in the microwave system, in which the rectangular waveguide components (used for the FEM radiation outcoupling) were replaced by double-ridged waveguide components, as described above. The FEM radiation excited inside the resonator is outcoupled to a matched load through a diaphragm, having the form of rectangular opening placed between waveguide ridges perpendicularly to the electric field of the fundamental mode. A small part of the radiation is out-coupled from the resonator by a power sampler and is fed by

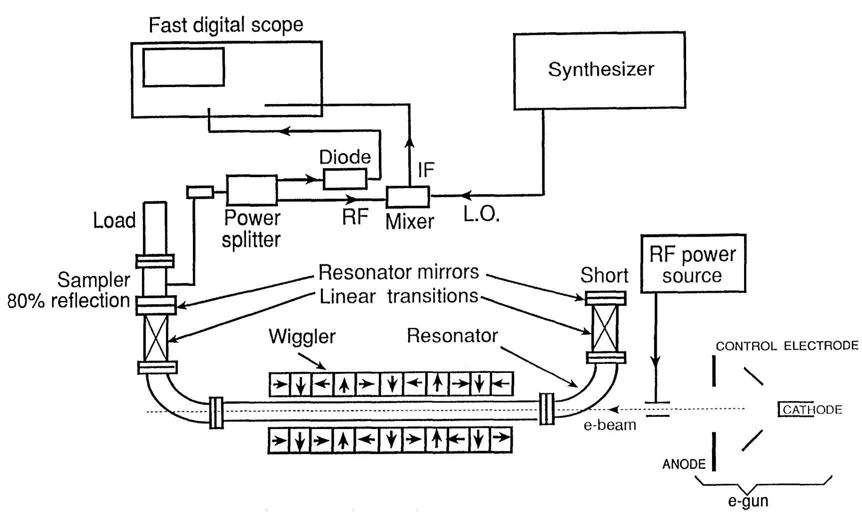


Fig. 4. Experimental setup.

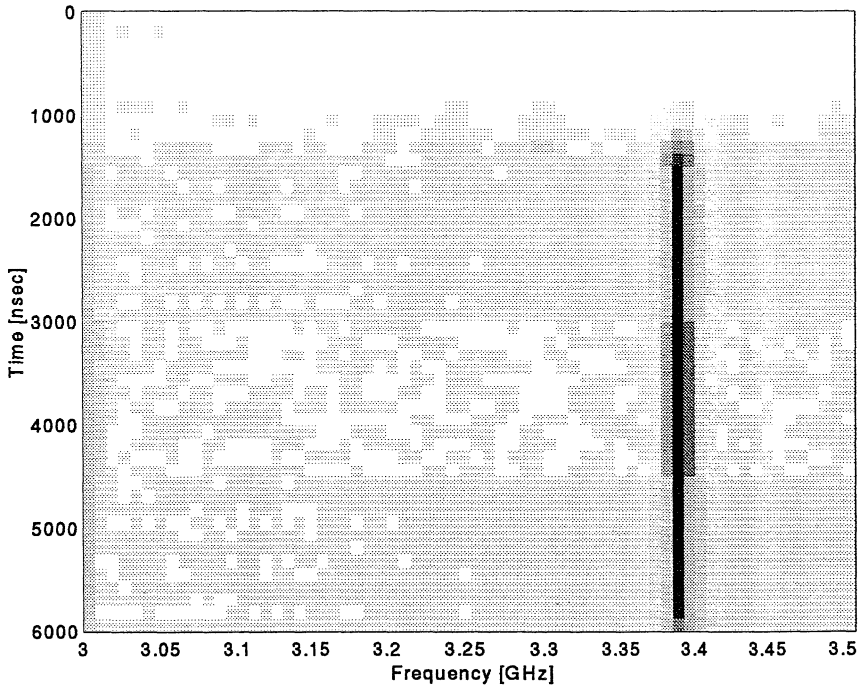


Fig. 5. Time evolution of the FEM radiation near waveguide cutoff frequency.

a power splitter to a diode detector, and to a mixer. The mixer is heterodyned with a local oscillator signal. The IF output signal from the mixer is stored in a fast digital scope, having an analog bandwidth of 500 MHz and a sampling rate of 2 GSa/s. This allowed Fourier analysis on single radiation pulse, and determination of the time evolution of the spectrum of the radiated signal. The Fourier analysis was made on consecutive 128 ns time intervals (256 stored points), providing a frequency resolution of the order of 8 MHz.

The FEM radiation measurements were made in a free-running oscillator configuration for e-beam pulses of 5 μs duration. The local oscillator frequency was chosen to be 3 GHz; thus, the observed IF output signal corresponds to a FEM radiation spectral band from 3 GHz up to 3.5 GHz. This spectral range includes the waveguide cutoff and the lower-frequency band synchronism frequencies, leaving out the upper synchronism frequency band. The data recording starts 1 μs before the e-beam pulse is turned on, and continues for a total recording time of 6 μs.

Strong FEM radiation was observed at a frequency of 3.38 GHz; the estimated output power was of the order of 1 kW. Fig. 5 shows the time evolution of the radiation spectrum. One can see, that throughout the e-beam pulse, the FEM radiation has high spectral purity. The radiation frequency bandwidth was much smaller than the spacing between two neighboring resonator eigenfrequencies, which was of the order of 50 MHz. No significant mode competition was observed, while free-running oscillations, in the high synchronism frequency band, showed several resonator eigenfrequencies during oscillation buildup (see Ref. [9]. This fact leads us to the conclusion that at the observed eigenfrequency the FEM gain is much higher than at the neighboring eigenfrequencies. This differs from theoretical gain calculation results as shown in Fig. 2. It should also be pointed out that according to the gain curve (see Fig. 2), we expected the FEL radiation frequency to be close to the low-frequency band local maximum, while the observed radiation frequency is higher.

## 5. Conclusions

We presented first experimental and theoretical results of FEM radiation near the waveguide cutoff frequency, as obtained on the TAU prebunched e-beam FEM. High power FEM radiation near the waveguide cutoff frequency was observed. An unique experimental setup, enabling time-dependent spectral measurements on a single radiation pulse, was employed and allowed determination of observed radiation of high spectral purity without significant mode competition.

Comparison of the obtained experimental and theoretical results shows that a serious reexamination of the existing theoretical model is needed in order to explain the absence of mode competition near the cutoff, and of the fact, that the observed radiation frequency is higher than the calculated maximum gain frequency in the low-frequency band.

## References

[1] C. Brau, Free Electron Lasers, Academic Press, Boston, 1990.
[2] M. Cohen, et al., Phys. Rev. Lett. 74 (19) (1995) 3812.
[3] A. Abramovich et al., Nucl. Instr. and Meth. A 375 (1996) 164.
[4] Y. Pinhasi et al., Nucl. Instr. and Meth. A 318 (1992) 523.
[5] Y. Pinhasi, A. Gover, Phys. Rev. E 51 (1995) 2472.
[6] L. Lewin, Theory of Waveguides, Newnes-Butterworths, London, 1975.
[7] N. Marcuvitz (Ed.), Waveguide Handbook, Dover, New York, 1965.
[8] I.M. Yakover, Y. Pinhasi, A. Gover, Nucl. Instr. and Meth. A 393 (1997) 316.
[9] A. Abramovich et al., these Proceedings (19th Free Electron Laser Conf., Beijing, China, 1997) Nucl. Instr. and Meth. A 407 (1998) 87.