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# Optimization of the electron-beam transport in the Israeli tandem FEL

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#### Abstract

Optimization of electron-beam transport in FEL devices is very important for efficient operation. A new simulation code for electron-beam transport in FELs was developed recently at Tel-Aviv University (TAU) in order to optimize the electron-beam transport. Equations of motion of an electron in the presence of magnetic and electric fields are solved numerically for various magnetic elements. The results of the simulation provided guidelines for setting appropriate currents on quadrupole magnets along the beam line. A comparison between calculated and measured beam diameters is provided. © 1998 Elsevier Science B.V. All rights reserved.

# 1. Introduction

Good electron beam transport through the resonator located within the wiggler is essential in order to enable energy exchange between beam electrons and the electromagnetic wave in the FEL resonator. Analysis of the electron-beam (e-beam) transport through the Tandem accelerator of the TAU FEL [1–3], using existing computer simulation codes such as "E-GUN" [4], "TRACE-3D" [5], "SPOT" [6] was not satisfactory since they do not include the wiggler fields. To solve this problem new programs, "QuadOpt" and "ELOP", were developed by the FEL-group at TAU.

Computer simulations of the electron transport in the FEL wiggler and through the quadrupoles were made using the "*ELOP*" program – especially developed for modeling e-beam transport through all electron-optics elements of the TAU FEL.

The electron beam line in the TAU Tandem FEL consists of 7 sections (see Fig. 1.). An injection section including a 50 keV e-gun, focusing and steering coils followed by a 1.0 – 3.0 MeV electrostatic accel*eration tube*. The next section includes a *diagnostic* screen (S1) and four quadrupoles (O1-O4) for e-beam focusing into the wiggler. The following section is a high-voltage terminal where a special waveguide resonator [7] is installed inside of a planar wiggler. Two diagnostic screens (S2, S3) are placed before and after the wiggler. Section 5 consists of four quadrupoles (Q5-Q8) which focus the beam into the electrostatic *decelerating* tube. The beam transport line is terminated by a *collector*, which will be used for e-beam energy recovery. The main parameters of tandem FEL are summarized in Table 1.

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Fig. 1. General view of the Tandem FEL at TAU.

Table 1		
TAU tandem	FEL	parameters

Accelerator	
Electron beam energy	1.4 MeV
Beam current	0.5 A
Wiggler	
Magnetostatic planar wiggler	
Magnetic induction	2 kG
Period length	4.44 cm
Number of periods	26

# 2. Electron transport through the wiggler

To calculate electron transport through the wiggler we used our "*ELOP*" program which solves the electron dynamics equations in the presence of constant, time-invariable magnetic and electric fields. This applies to coils, permanent magnets and magnet pairs, wigglers (which are made of permanent magnets), quadrupole magnets and other invariant configurations. The program calculates electron trajectories in a given set of electric and magnetic fields by use of Lorentz's force equation [8]:

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}z} = \frac{1}{\gamma} \bigg\{ -\frac{e}{mv_z} \bigg[ \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - \boldsymbol{v} \frac{\mathrm{d}\gamma}{\mathrm{d}z} \bigg] \bigg\}, \tag{1}$$
$$\frac{\mathrm{d}\gamma}{\mathrm{d}z} = -\frac{e}{mc^2 v_z} \boldsymbol{v} \boldsymbol{E},$$

where v is the electron velocity, e, m are the charge and the mass of the electron respectively, E, B are

Fig. 2. The Wiggler of TFEL.

matching magnets wiggler magnets

the time-invariant electric and magnetic fields,  $\gamma$  is the relativistic Lorenz factor.

The TAU FEL wiggler consists of 26 sets of magnets in a Hallbach configuration, where each magnet set is a period consisting of four permanent magnet pairs (see Fig. 2.). In addition to these magnets there are also "correcting" magnets near the entrance and exit of the wiggler and two "long" lateral focusing magnets one at each side of the wiggler which are required to eliminate angular deviation of the electron beam [3].

The magnetic field of a permanent rectangular magnet was calculated using a surface current model, in which each magnet is replaced by a rectangular loop of current [9].

longitudinal

magnets

#### 3. Electron transport through quadrupole lenses

Optimal transport of the electron beam through the wiggler requires special initial conditions at the wiggler entrance [10]. In order to determine the transport of e-beam through each quadrupole so as to provide these conditions we developed a computer procedure named "QuadOpt"; it determines the transport of the e-beam through quadrupoles, and enables determination of the values of the required quadrupoles current if a focused beam waist position and radius are given at the entrance and at the exit of the focusing system.

The "QuadOpt" program is based on the calculation of the transformation matrix of the electronoptical system, consisting of four quadrupoles separated by drift sections. The electron-beam current distribution is assumed to be Gaussian with an emittance  $\varepsilon = \pi r_{b0} \alpha_0$ ,  $r_{b0}$  and  $\alpha_0$  are the maximum beam radius and the beam divergence angle at the waist, respectively.

The focusing quadrupole system consists of four quadrupoles separated by equal distance d from each other. The quadrupole magnets focus the beam in one direction and defocus it in the perpendicular direction. Beam focusing in two directions is obtained by using two quads with opposite directed currents. When treating a quadrupole as an optical focusing element, its matrix is a function of the quadrupole current  $M_e(I)$  for the converging direction and  $M_d(I)$ for the diverging one. The full 4-quad x-direction transport matrix  $M_x$  and y-direction transport matrix  $M_y$  of the electron beam-line are given by

$$M_{x}(I_{1}, I_{2}, I_{3}, I_{4}) = D(d_{out}) \cdot M_{d}(I_{4}) \cdot D(d) \cdot M_{c}(I_{3})$$

$$\cdot D(d) \cdot M_{d}(I_{2}) \cdot D(d) \cdot M_{c}(I_{1}) \cdot D(d_{in}), \qquad (2)$$

$$M_{y}(I_{1},I_{2},I_{3},I_{4}) = D(d_{out}) \cdot M_{c}(I_{4}) \cdot D(d) \cdot M_{d}(I_{3})$$
$$\cdot D(d) \cdot M_{c}(I_{2}) \cdot D(d) \cdot M_{d}(I_{1}) \cdot D(d_{in}),$$

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where  $d_{in}$  is the input free-propagation length,  $d_{out}$  is the output free-propagation length, d freepropagation distance between quadrupoles. Using the "ABCD-law" for Gaussian beams, we may calculate the transformation of the Gaussian electron beam from the waist at the entrance position to the waist at the exit position

$$i\pi \frac{r_{b0out}^2}{\varepsilon} = \frac{A\left(i\pi \frac{r_{b0in}^2}{\varepsilon}\right) + B}{C\left(i\pi \frac{r_{b0in}^2}{\varepsilon}\right) + D},$$
(3)

where  $r_{b0in}$  and  $r_{b0out}$  are the waist parameters of the electron beam at the entrance position and at the exit position of the transport path. By operating with matrices  $M_x$  and  $M_y$  on the waist  $r_{b0in}$  and by comparing real and imaginary parts of Eq. (3), we get a set of four equations for the quadrupole currents  $I_{1,I_2,I_3,I_4}$ . Solving these equations we get the currents which enable to focus a Gaussian electron beam to the required size.

### 4. Numerical results

We used the "ELOP" program to calculate electron beam transport through the wiggler. Initially, we placed the correction magnets so as to obtain best electron transport through the wiggler (without betatron oscillations) for an electron that enters on the central axis (x = 0, y = 0) of the wiggler.

The simulation results show that the focusing point should be at distance  $\Delta_x$  and  $\Delta_y$  inside the wiggler and that the beam waist radii  $X_{b0}$  and  $Y_{b0}$  are different. The Gaussian beam parameters for two cases are summarized in Table 2 for two emittance parameters ( $\varepsilon = 3\pi \text{ mm mrad}$  and for  $\varepsilon = 10\pi \text{ mm mrad}$ ). The trajectory parameters thus

Table 2		
Electron beam parameters	and calculated required	quadrupole currents

Emittance (mm mrad)	Waist parameters (mm)			Quadrup	Quadrupole currents (A)			
	$X_{b0}$	$\varDelta_X$	$Y_{b0}$	$\Delta_Y$	$I_1$	$I_2$	$I_3$	$I_4$
3π	0.35	12	0.35	19	0.96	1.13	1.38	0.36
10π	0.70	11	0.68	23.3	1.23	1.16	1.35	0.63



Fig. 3. Electron beam trajectories in FEL through the quads and wiggler (ELOP simulation).



Fig. 4. Electron beam spot viewed on the screen S1.



Fig. 5. Electron beam spot viewed on the screen S2.

obtained are used in the next calculation as input for the "QuadOpt" program in order to find the required currents for quadrupoles Q1–Q4. The results thus obtained are given in Table 2.

Finally, we used the "*ELOP*" program to obtain the results of 9-electrons transported through the whole system of the first 4 quads (Q1–Q4), wiggler and the second set of quads (Q5–Q8). The currents for the quadrupoles Q5–Q8 were determined in a similar manner. In Fig. 3 we illustrate the transport through all the quads and through the wiggler.

In experimental work on the TAU Tandem FEL [11] we set the quadrupole currents  $I_1-I_8$  in accordance to the values obtained in the simulations (Table 2) and we recorded the electron beam cross-sections that we obtained on the screens S1 at z = -2405 mm and S2 at z = -880 mm. Fig. 4 shows the e-beam cross-section shape as recorded on screen S1; it is nearly round with a diameter of about 13 mm. Fig. 5 shows the e-beam cross-section and 8 mm in y-direction. These results are in fair agreement with simulation results given in Fig. 3.

# 5. Conclusions

This paper presents a method of e-beam transport optimization using a Gaussian beam model for the electron beam. This method provides detailed information on e-beam transport along the beam line and inside of the wiggler. The method can be applied to any linear time independent beam line elements and e-beam parameters by use of correct matrices for the beam-line elements.

A new electron transport simulation code, "*ELOP*", is presented, which enables simulation of electron transport through various electron-optic elements such as quadrupoles focusing lenses, steering magnets, magnetostatic wiggler, and other electro- and magnetostatic elements.

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