Introduction to Digital Computers / Computer Structure - Spring 2003
Assignment No. 1

Firm Deadline: February 27th - before the beginning of the lecture

1. Answer one of the following:

   (a) Prove the **Absorption Law**:

   For every pair of elements a , $b \in B$,
   1. $a + a \cdot b = a$
   2. $a \cdot (a + b) = a$

   (b) Prove the **Associative Laws**:

   In Boolean algebra, each of the binary operations (+) and ($\cdot$) is associative. That is, for every a , b , $c \in B$,
   1. $a + (b + c) = (a + b) + c$
   2. $a \cdot (b \cdot c) = (a \cdot b) \cdot \mathrm{c}$

   (hint: prove that both sides in (1) equal $[(a + b) + c] \cdot [a + (b + c)]$.)

   (c) Prove the **DeMrorgan's Law**:

   For every pair of elements a , $b \in B$,
   1. $(a + b)' = a' \cdot b'$
   2. $(a \cdot b)' = a' + b'$

2. Consider the **Algebra of Sets** as defined in class. (i) How would you define the complement? (ii) Prove that the algebra of sets is a Boolean algebra.
   hints: (a) recall that $A = \{x : x \in A\}$, $A \cup B = \{x : x \in A \text{ or } x \in B\}$, ...
   (b) in order to prove distributive laws, show that $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ and vice versa.

3. Let f_d be the **dual** of a function $f(x_1, x_2, \ldots, x_n)$.

   (a) Show that $f_d = f'(x_1', x_2', \ldots, x_n')$

   (b) Find a three-variable function that is its own dual. Such a function is called *self-dual*.

   (c) Prove that for any function f and any two-valued variable A, which may or may not be a variable in f, the function $g = Af + A'f_d$ is a self-dual.

4. Consider a function $f : [0, 1] \to [0, 1]$. Suppose that $f(x)$ is monotone decreasing and that the derivative $f'(x)$ of $f(x)$ satisfies the following condition: $f'(x)$ is continuous, $f'(x)$ is strictly decreasing in the interval $[0, \alpha]$ and strictly increasing in the interval $(\alpha, 1]$. Moreover, $f'(\alpha) < -1$. Can you define threshold values $V_{low,out} < V_{low,in} < V_{high,in} < V_{high,out}$ so that C is an inverter according to Definition 3 (inverter in the bounded-noise model : page 9 on Lecture Notes)?