Introduction to Digital Computers / Computer Structure - Spring 2003
Assignment No. 7

Firm Deadline: May 1st - before the beginning of the lecture

1. EE Students only:
   Answer question 41 in chapter 7 of the lecture notes.

2. Answer question 42 in chapter 7 of the lecture notes.

3. Prove the following statements for binary addition:

   (a) For every $n-1 \geq j \geq i \geq 0$: $\prod[j:i] = 1 \Leftrightarrow \langle A[j:i]\rangle + \langle B[j:i]\rangle = 2^{j+1} - 2^i$.

   (b) For every $n-1 \geq i \geq 0$: $\prod[i:-1] = 2 \Leftrightarrow \langle A[i:0]\rangle + \langle B[i:0]\rangle + C[0] \geq 2^{i+1}$.

4. We would like to design a combinational circuit that adds number represented in decimal representation.
   Input: $a[n-1:0], b[n-1:0] \in \{0,1,\ldots,9\}^n$
   Output: $s[n-1:0] \in \{0,1,\ldots,9\}^n$, $c[n] \in \{0,1,\ldots,9\}$
   Functionality:
   $$\sum_{i=0}^{n-1}(a[i]+b[i]) \cdot 10^i = c[n] \cdot 10^n + \sum_{i=0}^{n-1} s[i] \cdot 10^i.$$
   The individual decimal digits are input and output using the BCD (Binary Coded Decimal) format, where 4 bits are used to specify each digit (this is of-course wasteful, but very simple and useful).

   The object is to design a decimal adder. As usual we reduce the addition problem to the problem of calculating the carry bits.

   (a) Show that the depth of the reduction is constant and that its price is linear.

   (b) Define an appropriate associative operator $\odot$ that will enable you to utilize the design of PPC circuits seen in class in order to achieve a logarithmic delay and linear cost of your circuit. We suggest that you specify the $\odot$ operator using a few equations rather than a full table.

   (c) Draw a block diagram of your circuit.

   (d) Explain its correctness.

5. Let $\langle c[n] \cdot s[n-1:0]\rangle = \langle a[n-1:0]\rangle + \langle b[n-1:0]\rangle$. We know that an overflow occurs in two's complement addition if and only if $c[n] \neq c[n-1]$. Prove that
   $$c[n] \neq c[n-1] \text{ iff } (a[n-1] = b[n-1]) \text{ and } (a[n-1] \neq s[n-1]).$$