Introduction to Digital Computers - Fall 1997

Assignment No. 3

Course homepage: http://www.eng.tau.ac.il/~guy/Digital_Computers/dc_home.html

Deadline: March 29th

1. Define a ripple carry adder (RCA) of n bits. Prove its correctness. Write the (closed - not recurrence) equations for its delay and cost.

2. Consider a situation in which an n-bit adder of unsigned binary strings has only the outputs: sum $s \in \{0,1\}^n$, and carry-out bit $c_n \in \{0,1\}$.

   A two's complement adder requires the carry-bit c_{n-1} to generate the ovf signal.

   Show how an $(n+1)$-bit adder of unsigned binary strings can be used to construct an n-bit two's complement adder. Prove the correctness of your design.

3. Prove that the following operator $* : \Sigma \times \Sigma \to \Sigma$, where $\Sigma = \{0,1,2\}$, is associative.

$*$	2	1	0
2	2	2	2
1	2	1	0
0	0	0	0

   Why is the associativity of the operator $*$ required for the correctness of a parallel prefix circuit $PPC(n)$?

4. Suggest three different encodings of $\Sigma = \{0,1,2\}$ using bits. For every encoding suggest a $*$-gate and formulate the cost and delay of an adder based on a $CLA(n)$. Compute the cost and delay for $n = 8, 16, 32, 64, 128$, and compare the encodings.

5. Draw a parallel prefix circuit $PPC(16)$ using only $*$-gates.

6. Design a circuit in which every prefix $x_i * x_{i-1} * \cdots * x_0$ is computed using a separate circuit. Write the cost and delay equations of your design. Compare them with those of $PPC(n)$.