Introduction to Digital Computers - Fall 1997

Assignment No. 4

Course homepage: http://www.eng.tau.ac.il/~guy/Digital_Computers/dc_home.html

Deadline: April 5th

1. (15%) Consider a 2's complement adder. What do $[c_n, s]$ and $[s]$ equal when $ovf = 1$?

2. (15%) (sign-magnitude subtraction) Design a circuit with inputs $a, b \in \{0,1\}^n$ that outputs $s \in \{0,1\}^n$, $sign \in \{0,1\}$ such that

$$(-1)^{sign} \cdot \langle s \rangle = \langle a \rangle - \langle b \rangle$$

3. (50%) Consider the following two problems:
finding the computation path of a finite state machine

Input: A string $a_0, a_1, \ldots, a_{n-1} \in \Sigma^n$ and a finite state machine $(S, \Sigma, \delta, s_0, F)$, where:
- S denotes the (finite) set of states;
- Σ denotes the input alphabet;
- a state-transition function $\delta : S \times \Sigma \to S$ (i.e., $\delta(s,a)$ is the new state reached from state $s \in S$ when given input $a \in \Sigma$);
- s_0 denotes the initial state; and
- $F \subseteq S$ denotes the set of final (accepting) states.

Output: The sequence of states $s_0, s_1, \ldots, s_n$ visited when $a_0, \ldots, a_{n-1}$ is read. Namely: $s_{i+1} = \delta(s_i, a_i)$.

move to front

Input: An alphabet Σ and symbols $z, x_0, x_1, \ldots, x_n \in \Sigma$. The alphabet Σ contains a special symbol λ (which denotes "end of list"). The input satisfies:
- there exists an $i \in \{0, \ldots, n\}$ such that $x_i = x_{i+1} = \cdots = x_{n-1} = \lambda$ and the symbols $x_0, x_1, \ldots, x_{i-1}$ are distinct and do not equal λ.
- $z \in \Sigma - \lambda$.

Output: The symbols $y_0, y_1, \ldots, y_n$ are defined by

$$\begin{aligned} y_0 &= z \\ y_{i+1} &= \begin{cases} x_i & \text{if } z \notin \{x_0, \ldots, x_i\} \\ x_{i+1} & \text{otherwise} \end{cases} \end{aligned}$$

Formulate each problem as a prefix computation problem. What is the alphabet that you use (how large is it)? What is the associative operator that you use? Analyze the cost and delay of a parallel prefix circuit that solves each problem.

4. (20%) Relate the carry look-ahead adder with the first problem in Question 3. Can you explain the differences in the encoding, cost, and delay?