Introduction to Digital Computers - Spring 1999

Assignment No. 5

Course homepage: http://www.eng.tau.ac.il/∼guy/Digital_Computers99/dc_home.html

Firm Deadline: June 2nd - before the beginning of the lecture.

Questions:

1. What does a two's complement adder output when an overflow occurs? Let:

   - $a[n-1:0], b[n-1:0] \in \{0,1\}^n$; and
   - $z = [a[n-1:0]] + [b[n-1:0]]$.

   Suppose that

   $$\langle c[n] \cdot s[n-1:0] \rangle = \langle a[n-1:0] \rangle + \langle b[n-1:0] \rangle.$$

   Prove that:

   (a)

   $$[s[n-1:0]] = z \pmod{2^n}$$

   (b) Suppose that $z \notin T_n$ and that $z > 0$. Write the smallest interval that z belongs to. Write a closed formula for $[s[n-1:0]]$. What is that interval that $s[n-1:0]]$ belongs to?

   (c) Suppose that $z \notin T_n$ and that $z < 0$. Write the smallest interval that z belongs to. Write a closed formula for $[s[n-1:0]]$. What is that interval that $s[n-1:0]]$ belongs to?

   (d) Write and execute a program that demonstrates your answers.

2. Unsigned subtraction. Let:

   - $a[n-1:0], b[n-1:0] \in \{0,1\}^n$; and
   - $z = \langle a[n-1:0] \rangle - \langle b[n-1:0] \rangle$

   Suppose that

   $$[c[n] \cdot s[n-1:0]] = [a[n-1:0]] - [b[n-1:0]].$$

   Prove that:

   (a)

   $$[s[n-1:0]] = z \pmod{2^n}$$

   (b) Suppose that $z \notin T_n$ and that $z > 0$. Write the smallest interval that z belongs to. What a closed formula for $[s[n-1:0]]$. What is that interval that $s[n-1:0]]$ belongs to?

   (c) Suppose that $z \notin T_n$ and that $z < 0$. Write the smallest interval that z belongs to. What a closed formula for $[s[n-1:0]]$? What is that interval that $s[n-1:0]]$ belongs to?

   (d) Write and execute a program that demonstrates your answers.

(e) Write a program in which the difference of two unsigned integer variables is stored in an unsigned integer variable. Try your examples from the previous item, and explain the results.

3. A two's complement adder can be used for unsigned addition simply because the sum string $s[n-1:0]$ output by a two's complement adder equals $\langle a[n-1:0]\rangle + \langle b[n-1:0]\rangle$. How should an overflow be detected during unsigned addition?

4. Let $\langle c[n] \cdot s[n-1:0]\rangle = \langle a[n-1:0]\rangle + \langle b[n-1:0]\rangle$. We know that an overflow occurs if and only if $c[n] \neq c[n-1]$. Prove that

$$c[n] \neq c[n-1] \text{ iff } (a[n-1] = b[n-1]) \text{ and } (a[n-1] \neq s[n-1])$$