Computer Arithmetic - Spring 1998

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http://www.eng.tau.ac.il/~guy/Computer_Arithmetic/arith_home.html

Assignment Number 1

Deadline: March 19th, 16:00 (beginning of class)

1. Prove that the sum bit s_n depends on each of the input bits.

2. Prove a lower bound on the depth of a circuit that computes a value that depends on n inputs. Assume that the fan-in of each gate is at most d. Substitute the following values in the lower bound: $n = 64, 128$ and $d = 2, 3, 4$.

3. Consider the recurrence equation:

$$f(n) = \begin{cases} b & \text{if } n \leq 1 \\ a \cdot f(n/c) + bn & \text{if } n > 1 \end{cases}$$

   Prove that if $n = c^k$, for a positive integer k, then

$$f(n) = \begin{cases} O(n) & \text{if } a < c \\ O(n \log n) & \text{if } a = c \\ O(n^{\log_c a}) & \text{if } a > c \end{cases}$$

4. Consider the recurrence equation:

$$T(n) \leq \begin{cases} 0 & \text{if } n \leq 1 \\ \min_{n_1 \leq n/2}\{T(n - n_1) + n_1\} & \text{otherwise} \end{cases}$$

   Prove that $T(n)$ is minimized if $n_1 = n/2$ is chosen, and therefore, $T(n) = O(n \log n)$.

5. Prove that if $T(MUX_n) = \log n$, then $T(CSA_n) = O(\log^2 n)$.

6. Design an adder that is input two vectors $a, b \in \{0,1\}^n$ and computes s^0 and s^1 such that $< s^0 >= \langle a \rangle + \langle b \rangle$ and $< s^1 >= \langle a \rangle + \langle b \rangle + 1$. The delay of the adder should be $O(\log n)$ and the cost $O(n \log n)$.