

Assignment Number 2

Deadline: beginning of class on April 2nd

1. Draw the $PPC(16)$ circuit using only $*$ -gates (that is, open the recursion).
2. Suggest three different encodings of $\Sigma = \{0, 1, 2\}$ using bits: binary encoding, unary, and one in which “2” has two encodings. For every encoding suggest a $*$ -gate and formulate the cost and delay of an adder based on a $CLA(n)$. Compute the cost and delay for $n = 8, 16, 32, 64, 128$, and compare the encodings.
3. Consider the following problem:

Input: A string $a_0, a_1, \dots, a_{n-1} \in \Sigma^n$ and a finite state machine $(S, \Sigma, \delta, s_0, F)$, where:

- S denotes the (finite) set of states;
- Σ denotes the input alphabet;
- a state-transition function $\delta : S \times \Sigma \rightarrow S$ (i.e., $\delta(s, a)$ is the new state reached from state $s \in S$ when given input $a \in \Sigma$);
- s_0 denotes the initial state; and
- $F \subseteq S$ denotes the set of final (accepting) states.

Output: The sequence of states s_0, s_1, \dots, s_n visited when a_0, \dots, a_{n-1} is read. Namely: $s_{i+1} = \delta(s_i, a_i)$.

Formulate this problem as a prefix computation problem. What is the alphabet that you use (how large is it)? What is the associative operator that you use? Analyze the cost and delay of a parallel prefix circuit that solves this problem.

4. Relate the carry look-ahead adder with the problem in Question 3. Can you explain the differences in the encoding, cost, and delay?
5. Consider the problem of “move to front” defined as follows:

Input: An alphabet Σ and symbols $z, x_0, x_1, \dots, x_n \in \Sigma$. The alphabet Σ contains a special symbol λ (which denotes “end of list”). The input satisfies:

- $x_n = \lambda$;
- the symbols x_0, x_1, \dots, x_{n-1} are distinct and do not equal λ .
- $z \in \Sigma - \lambda$.

Output: The symbols y_0, y_1, \dots, y_n defined by

$$y_0 = z$$

$$y_{i+1} = \begin{cases} x_i & \text{if } z \notin \{x_0, \dots, x_i\} \\ x_{i+1} & \text{otherwise} \end{cases}$$