Computer Arithmetic - Spring 1998

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Assignment Number 2

Deadline: beginning of class on April 2nd

1. Draw the $PPC(16)$ circuit using only $*$-gates (that is, open the recursion).

2. Suggest three different encodings of $\Sigma = \{0, 1, 2\}$ using bits: binary encoding, unary, and one in which "2" has two encodings. For every encoding suggest a $*$-gate and formulate the cost and delay of an adder based on a $CLA(n)$. Compute the cost and delay for $n = 8, 16, 32, 64, 128$, and compare the encodings.

3. Consider the following problem:

   Input: A string $a_0, a_1, \ldots, a_{n-1} \in \Sigma^n$ and a finite state machine $(S, \Sigma, \delta, s_0, F)$, where:

   - S denotes the (finite) set of states;
   - Σ denotes the input alphabet;
   - a state-transition function $\delta : S \times \Sigma \to S$ (i.e., $\delta(s, a)$ is the new state reached from state $s \in S$ when given input $a \in \Sigma$);
   - s_0 denotes the initial state; and
   - $F \subseteq S$ denotes the set of final (accepting) states.

   Output: The sequence of states $s_0, s_1, \ldots, s_n$ visited when $a_0, \ldots, a_{n-1}$ is read. Namely: $s_{i+1} = \delta(s_i, a_i)$.

   Formulate this problem as a prefix computation problem. What is the alphabet that you use (how large is it)? What is the associative operator that you use? Analyze the cost and delay of a parallel prefix circuit that solves this problem.

4. Relate the carry look-ahead adder with the problem in Question 3. Can you explain the differences in the encoding, cost, and delay?

5. Consider the problem of "move to front" defined as follows:

   Input: An alphabet Σ and symbols $z, x_0, x_1, \ldots, x_n \in \Sigma$. The alphabet Σ contains a special symbol λ (which denotes "end of list"). The input satisfies:

   - $x_n = \lambda$;
   - the symbols $x_0, x_1, \ldots, x_{n-1}$ are distinct and do not equal λ.
   - $z \in \Sigma - \lambda$.

   Output: The symbols $y_0, y_1, \ldots, y_n$ defined by

$$
\begin{aligned}
y_0 &= z \\
y_{i+1} &= \begin{cases} x_i & \text{if } z \notin \{x_0, \ldots, x_i\} \\ x_{i+1} & \text{otherwise} \end{cases}
\end{aligned}
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