Computer Arithmetic - Spring 1998

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Assignment Number 4

Deadline: beginning of class on May 21st

1. Consider the following $2 \times k$ binary matrix

$$\begin{array}{cccc} b_{k-1} & b_{k-2} & \cdots & b_0 \\ b_k & b_{k-1} & \cdots & b_1 \end{array}$$

   Assume that a bit in position (i,j) has a weight of $(-1)^i \cdot 2^j$, where the rows are indexed $[0:1]$ and the columns are indexed $[k-1:0]$.

   Prove that the value represented by the matrix is in the range $\{-2^{k-1}, \ldots, +2^{k-1}\}$.

2. Prove that Booth$_1$ recoding replaces every block of consecutive ones with a block of $1, 0, \ldots, 0, (-1)$. Namely, if $x_n, \ldots, x_0 = Booth_1(b_{n-1}, \ldots, b_0)$, and $b_{i+1} b_i \ldots b_j b_{j-1} = 01 \cdots 10$, then $x_{i+1} x_i \ldots x_j x_{j-1} = 10 \cdots 0(-1)0$.

3. Consider a random binary string of n bits $b_{n-1} \cdots b_0$ (the bits are i.i.d. $\{0,1\}$ random variables with $Pr(b_i = 0) = Pr(b_i = 1) = 1/2$). Let $x_n, \ldots, x_0 = Booth_1(b_{n-1}, \ldots, b_0)$. Let $w(b)$ denote the number of non-zero bits in b and $w(x)$ denote the number of non-zero bits in x. Prove or refute that

$$E[w(b)] > E[w(x)]$$

4. Let $a = a_{n-1} \cdots a_0$ and $b = b_{n-1} \cdots b_0$. Let $b'_{n'-1} \cdots b'_0 = Booth_2(b_n \cdots b_0)$, where $n' = \lceil \frac{n+1}{2} \rceil$. Define $\alpha''_j = \langle a \rangle \cdot b'_j + 3 \cdot 2^{n+1}$. Prove that

$$\langle a \rangle \cdot \langle b \rangle = 2^{n+1} + \sum_{j=0}^{n'-1} \alpha''_j \cdot 4^j \pmod{2^{2n}}$$

   Could we replace 2^{n+1} by 2^n, 2^{n-1}, or even 2^{n-2}? Why do we prefer 2^{n+1}?

5. Using the same notation of the previous question. Prove that $b'_{n'-1} \geq 0$.

6. Show how to Booth recode a number represented by a 2's complement binary string.

7. Show how to multiply $[a] \cdot [b]$ using Booth$_2$ recoding.