Computer Arithmetic - Spring 1999
Assignment No. 1
Course homepage: http://www.eng.tau.ac.il/~guy/arith99/arith_home.html

Deadline: June 16th - before the beginning of the lecture.

Questions:

1. Prove Claim 8 in Lecture Notes #2 (FP division, part 2).
2. Draw a block diagram of a whole unsigned adder $CLA(n)$ based on a $PPC(n)$. Explain the functionality of each block. There is no need to deal with binary encodings.
3. The description of $CLA(n)$ given in class did not address the issue of how the alphabet $\{0, 1, 2\}$ is encoded. Three encodings are given in the Table below:

symbol	binary encoding	"hot one" encoding	"g & p" encoding
0	00	001	00
1	01	010	01
2	10	100	10 or 11

   (a) Design the blocks of each part of the $CLA(n)$ with respect to each encoding (gate level designs).
   (b) Which encoding do you think is best in terms of delay and in terms of cost?

4. Consider the following $2 \times k$ binary matrix

$$\begin{matrix} b_{k-1} & b_{k-2} & \cdots & b_0 \\ b_k & b_{k-1} & \cdots & b_1 \end{matrix}$$

   Assume that a bit in position (i, j) has a weight of $(-1)^i \cdot 2^j$, where the rows are indexed $[0 : 1]$ and the columns are indexed $[k - 1 : 0]$.

   Prove that the value represented by the matrix is in the range $\{-2^{k-1}, \ldots, +2^{k-1}\}$.

5. Prove that Booth$_1$ recoding replaces every block of consecutive ones with a block of $1, 0, \ldots, 0, (-1)$. Namely, if $x_n, \ldots, x_0 = Booth_1(b_{n-1}, \ldots, b_0)$, and $b_{i+1}b_i \ldots b_j b_{j-1} = 01 \cdots 10$, then $x_{i+1}x_i \ldots x_j x_{j-1} = 10 \cdots 0(-1)0$.
6. Consider a random binary string of n bits $b_{n-1} \cdots b_0$ (the bits are i.i.d. $\{0, 1\}$ random variables with $Pr(b_i = 0) = Pr(b_i = 1) = 1/2$). Let $x_n, \ldots, x_0 = Booth_1(b_{n-1}, \ldots, b_0)$. Let $w(b)$ denote the number of non-zero bits in b and $w(x)$ denote the number of non-zero bits in x. Prove or refute that

$$E[w(b)] > E[w(x)]$$