Strong stabilization of a wind turbine tower model in the plane of the turbine blades

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Abstract: We investigate the strong stabilization of a wind turbine tower model in the plane of the turbine blades, which comprises a nonuniform SCOLE system and a two-mass drive-train model (with gearbox). The control input is the torque created by the electrical generator. Using a strong stabilization theorem for a class of impedance passive linear systems with bounded control and observation operators, we show that the wind turbine tower model can be strongly stabilized. The control is by static output feedback from the angular velocities of the nacelle and the generator rotor.

Keywords: wind turbine tower, SCOLE model, two-mass drive-train model, impedance passive system, strong stabilization.

1. Introduction

Wind power has become an important source of clean energy, whose investment is expected to expand from 18 billion dollars worldwide in 2006 to 60 billion dollars in 2016, see Laks *et al.* [15] or [25]. Larger wind turbines are being employed further offshore because of restrictions on land use and to harvest more energy. The National Renewable Energy Laboratory offshore 5 MW baseline wind turbine model in Jonkman *et al.* [13] represents the current typical offshore turbines. Each blade of this model turbine weighs almost 18 tons and is 61.5 m long. Its nacelle (with the hub) weighs almost 300 tons. This turbine has a tower standing 87.6 m above the sea level with a base diameter of 6 m and with density decreasing with height.

From the above description, it is clear that offshore wind turbines with their towers are very flexible structures rather than rigid bodies. They are particularly susceptible to severe weather, turbulence, and other effects such as the wake impact of upwind machines, which can generate significant fluctuating loads and vibrations

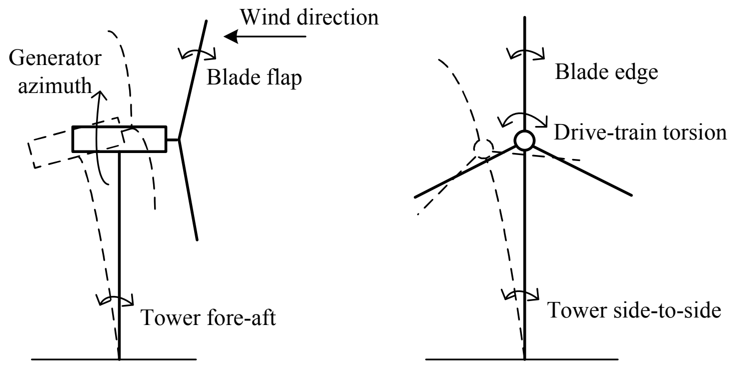

Figure 1: Schematic representation of a wind turbine tower. The left picture shows the vibrations in the plane of the turbine axis, while the right picture shows the vibrations in the plane of the turbine blades.

and thus cause fatigue. These fluctuating loads and vibrations will also reduce the life expectancy of other components such as the low speed shaft and the gearbox, and thus increase the maintenance cost of the turbine, which is a critical problem in the wind energy industry, see Anaya-Lara *et al.* [1], Heier [11] and [17].

There are plans to develop offshore wind turbines with maximal power up to 20 MW, which requires the rotor diameter to be over 252 m and this requires even higher towers to be built that will have to bear an even larger weight, see [21]. The structural solution is to make these towers more rigid and stiff to handle the increased weight and the increased fluctuating loads, which is very costly.

Thus, we see it as an important task to develop control techniques that may reduce the vibrations of the wind turbine tower and drive-train to increase their life expectancy, and to allow lighter, more flexible and hence cheaper towers to be built. Preferably this should be achieved with minimal additional investment into actuators: indeed, we propose to achieve the damping of the vibrations in the plane of the turbine blades (see the right side of Figure 1) by counteracting them via the generator torque, which in the case of a synchronous machine can be controlled via the q-axis current and hence can be adjusted very fast in the current control loop of the generator-side AC/DC converter. This control action needs to take place in the frequency band where the resonant frequencies of the tower are located, which is above 0.3 Hz, see [13, Table 9-1], while the main control action of maximum power point tracking (to optimize power capture) takes place at frequencies below 0.25 Hz, see [13, Figure 7-5 and Table 7-2], as dictated by the low-pass filter in Figure 2. Thus, the two control actions can be added without disturbing each other much, as shown in the proposed block diagram in Figure 2. The new control loop proposed in this paper is on the left edge in Figure 2, mainly the tower controller, while the remaining part is as in [13, Figure 7-5], see also [1, 15].

In this paragraph we discuss the inherent limitations of our proposed controller. First of all, as already mentioned, we are only talking about vibrations in the plane of the turbine blades (see the right side of Figure 1), and this includes the vibrations

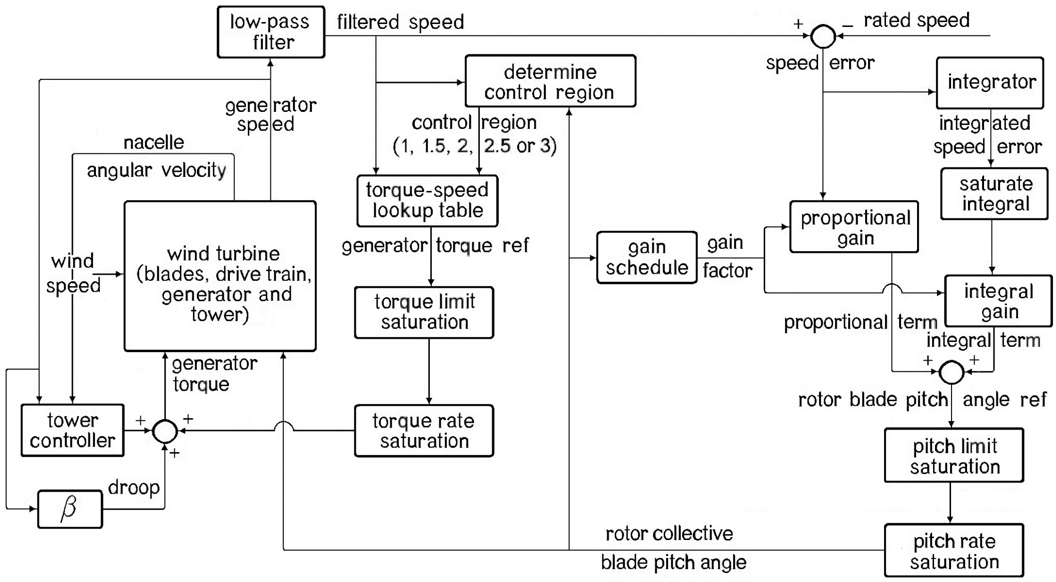

Figure 2: Block diagram of the baseline control system for a wind turbine, including the proposed tower vibration controller seen at the left edge.

in the low-speed shaft, that is part of our model. The vibrations in the plane of the turbine axis (see the left side of Figure 1) are easier to treat from the mathematical point of view, and this was done in our paper [27]. We anticipate that our controller would be used mainly in "region 3", that is, the range of strong wind speeds where the turbine is set to extract constant power (see [1], [13], [15], [22]) and the main task is to protect the turbine. In this region the low-pass filter from Figure 2 may be set to a lower corner frequency (such as 0.1 Hz), so that the main torque controller will not interfere much with our tower controller. We mention that, as always in practice, there is an upper limit to the range of frequencies that can be handled by the tower controller: this is dictated by the bandwidth of the current loop that controls the q-axis current of the generator, and this is of the order of 100 Hz. Our mathematical analysis does not take this limitation into account, but in practice we can only expect that the tower controller will suppress vibrations that correspond approximately to the first 20 eigenvalues of the tower system in the plane of the turbine blades. We think that vibrations at frequencies above 100 Hz are not significant for the wear of the tower and the drive train.

In [27] we have shown the generic strong stabilization of this wind turbine tower model Σ on its natural state space using electrical torque control and feedback from the angular velocities of the nacelle and the generator rotor. In this paper we remove the "generic" condition, see Theorem 4.2, that relies on a powerful strong stabilization theorem for a class of impedance passive systems due to Batty and Phong [2], see Theorem 3.4. We also rely on a powerful observability result of Guo

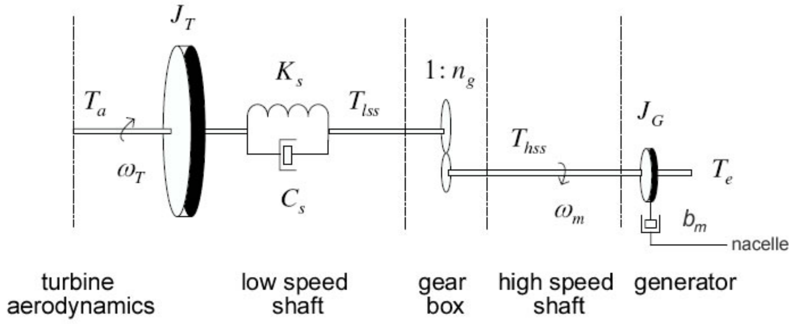

Figure 3: The two-mass drive-train model with gearbox. T_a is the active torque from the turbine and T_e is the electric torque of the generator, which acts between the rotor (connected to the high speed shaft) and the stator (connected to the nacelle).

and Ivanov [9]. Our main result is that the feedback

$$T_e = k\frac{1}{J} w_{xt}(l,t) + k\frac{1}{J_G}\left((\theta_m)_t(t) - w_{xt}(l,t)\right),$$

where $k > 0$, strongly stabilizes the wind turbine tower. For the meaning of the notation used above see Section 2.

In Section 2 we recall the modelling of this coupled system, based on earlier papers of ours. In Section 3 we recall some background on impedance passive systems, in particular the theorem of Batty and Phong mentioned earlier. In Section 4 we show the strong stabilization of the wind turbine tower model Σ by static output feedback as described above, using Batty and Phong's theorem.

2. Modelling the system

In Zhao and Weiss [26, 27], we have studied the well-posedness, controllability and strong stabilization of a wind turbine tower model. We decompose the system into one subsystem describing the vibrations in the plane of the turbine axis and another subsystem in the plane of the turbine blades, as shown in Figure 1. We have shown that the subsystem describing vibrations in the plane of the turbine axis can be strongly stabilized using either force control (with the velocity of the nacelle as measurement output), or torque control (with the angular velocity of the nacelle as measurement output). The force control can be obtained by modulating the turbine pitch angle, while the torque control can be obtained by an electrically driven mass located in the nacelle. We have modelled the wind turbine tower in the plane of the turbine axis as a nonuniform SCOLE system with either force control or torque control, as studied in Guo [8] and Guo and Ivanov [9]. In the plane of the turbine blades we were only able to prove generic strong stabilization, i.e., only for an open and dense set of parameters. In this paper, we remove the genericity condition, so that the controlled system will be strongly stable in the natural state space.

In [26, 27] we have modelled the wind turbine tower in the plane of the turbine blades as a non-uniform SCOLE system coupled with a two-mass drive-train model (see Figure 3), as studied in Hansen *et al.* [10], and in Wang and Weiss [22, 23], to take into account the effect of the vibration torque transferred from the turbine blades to the nacelle by the gearbox. This model Σ is described below:

$$\begin{cases} \rho(x)w_{tt}(x,t) + (EI(x)w_{xx}(x,t))_{xx} = 0, \quad (x,t) \in (0,l) \times [0,\infty), & (2.1) \\ w(0,t) = 0,\ w_x(0,t) = 0, & (2.2) \\ mw_{tt}(l,t) - (EIw_{xx})_x(l,t) = 0, & (2.3) \\ Jw_{xtt}(l,t) + EI(l)w_{xx}(l,t) = T_{lss}(t) + T_{hss}(t) - b_m(\theta_m)_t(t) - T_e(t), & (2.4) \\ (\theta_T)_{tt}(t) = \dfrac{1}{J_T}\big(T_a(t) - T_{lss}(t)\big) - w_{xtt}(l,t), & (2.5) \\ (\theta_m)_{tt}(t) = \dfrac{1}{J_G}\big(T_{hss}(t) - b_m(\theta_m)_t(t) - T_e(t)\big) + w_{xtt}(l,t), & (2.6) \\ \theta_k(t) = \theta_T(t) - \dfrac{\theta_m(t)}{n_g}, & (2.7) \\ T_{lss}(t) = K_s\theta_k(t) + C_s(\theta_k)_t(t) = n_g T_{hss}(t), & (2.8) \end{cases}$$

where the subscripts t and x denote derivatives with respect to the time t and the position x, respectively. The equations (2.1)–(2.4) are a non-uniform SCOLE model describing the movement of the tower and the nacelle, while the equations (2.5)–(2.8) are a two-mass drive-train model (based on [22]). They are coupled through the angular velocity of the nacelle and the total torque acting on the nacelle from the gearbox and from the electrical generator.

In the above equations w stands for the transverse displacement of the tower, l is the height of the tower, and EI and ρ are the flexural rigidity function and the mass density function. $m > 0$ and $J > 0$ are the mass and the moment of inertia of the nacelle. We assume that $\rho, EI \in C^4[0,l]$ are strictly positive. T_e is the electric torque control created by the electrical generator. θ_T and θ_m are the angles of the turbine rotor and generator rotor with respect to the nacelle. θ_k is the angular difference between the endpoints of the low-speed shaft. $J_T > 0$ is the rotational inertia of the turbine blades and other lows-speed components (for example the hub) while $J_G > 0$ is the rotational inertia of the generator rotor. $K_s > 0$ and $C_s \geq 0$ are the torsional stiffness and torsional damping coefficient of the low-speed shaft. T_a is the active torque from the turbine, which is a disturbance from our point of view. n_g is the gearbox ratio. b_m is the damping coefficient of the high speed shaft. Clearly $J > J_G$ (this fact will be used in the proof of the main result).

In this paper we can always assume $b_m > 0$, because even if the natural damping coefficient (expressing the viscous friction) b'_m is zero, we can ensure that $b_m > 0$ by adding droop control to the overall control system: choose $\beta > 0$ and set

$$T'_e = \beta\omega_m + T_e\,,$$

where $\omega_m = (\theta_m)_t$ and T'_e is the actual electric torque, see the left edge of Figure 2. The total torque acting on the nacelle due to electric torque and viscous friction is

$$T'_e + b'_m\omega_m = T_e + b_m\omega_m\,,$$

where $b_m = \beta + b'_m$, and this torque appears in (2.4) and (2.6). We mention that droop control is used in most electrical generators to help stabilize the system, see for instance Kundur [14], Zhong and Weiss [28].

It is of course debatable to which extent the above mathematical model is realistic. For example, it could be argued that a more accurate model would take into account the torque created by the weight of the tower and the moment of inertia of infinitesimal tower segments - see our discussion around formula (1.15) in [27]. Other, more complex beam models could be even more realistic, but more difficult to work with. Moreover, we have not taken into account the coupling between the vibrations of the blades and the vibrations of the tower. What we can say about this is that any model has its limitations but we hope that the feedback proposed in this paper would work also on the true system, because the true system is also impedance passive and our feedback is based on the idea of extracting energy out of the vibrations of an impedance passive system. In a practical implementation, our goal would be to introduce damping into the first few eigenvalues of the system (i.e., to move them to the left in the complex plane) and we believe that our model is accurate enough at relatively low frequencies to achieve this goal.

3. Feedback stabilization for a class of impedance passive linear systems

We shall need a strong stabilization result due to Batty and Phong [2]. Instead of just stating the result, we explain a bit the background and intuition behind it.

Passive systems are a class of dynamical systems that can dissipate energy but cannot produce energy. In particular, impedance passive systems exchange power with their environment at a rate that is the inner product between the instantaneous values of the input and the output signals. For the theory of impedance passive systems we refer to Staffans [19] and Staffans and Weiss [20]. For the closely related concept of port-Hamiltonian system see Lozano *et al.* [16] and in the linear infinite-dimensional context, we refer to the recent book Jacob and Zwart [12].

In this paper we consider only the very particular context of linear systems Σ described by the equations

$$\begin{cases} \dot{z}(t) &= Az(t) + Bu(t), \\ y(t) &= Cz(t) + Du(t), \end{cases} \tag{3.1}$$

with the input space U, state space X and output space U. Here A is the generator of a strongly continuous semigroup $\mathbb{T}$ on X, $B \in \mathcal{L}(U, X)$, $C \in \mathcal{L}(X, U)$ and $D \in \mathcal{L}(U)$. The functions $u \in L^2_{loc}([0, \infty); U)$, $z \in C([0, \infty), X)$ and $y \in C([0, \infty), U)$ are the input signal, state trajectory and output signal, respectively. The transfer function of Σ is $\mathbf{G}(s) = C(sI - A)^{-1}B + D$. The basic reference for such systems is Curtain and Zwart [7]. We define the operators Ψ by

$$(\Psi z_0)(t) = C\mathbb{T}_t z_0 \qquad \forall\, z_0 \in X. \tag{3.2}$$

It is clear that $\Psi \in \mathcal{L}\left(X, L^2_{loc}([0,\infty); U)\right)$.

Definition 3.1. The system Σ from (3.1) is *impedance passive* if it satisfies the following condition: if u, z and y are as in (3.1), then for any time $\tau \geq 0$, the following inequality holds:

$$\|z(\tau)\|^2 - \|z(0)\|^2 \leq 2 \int_0^\tau \mathrm{Re}\, \langle u(t), y(t) \rangle \mathrm{d}t. \tag{3.3}$$

We may regard (3.3) is an energy balance inequality: $E(t) = \frac{1}{2}\|z(t)\|^2$ is the energy stored in the system at the time t, and $\mathrm{Re}\,\langle u(t), y(t)\rangle$ is the incoming power of the system Σ at the time t.

The following proposition follows from a more general result in Staffans [18].

Proposition 3.2. *The system* Σ *from* (3.1) *is impedance passive if and only if the operator*

$$N = \begin{bmatrix} A & B \\ -C & -D \end{bmatrix}, \qquad \mathcal{D}(N) = \mathcal{D}(A) \times U,$$

is dissipative.

We mention that if the above operator N is dissipative then it is automatically m-dissipative. In a more general context (well-posed systems or system nodes) we would have to require m-dissipativity. If $D = 0$, then the condition that N is dissipative is equivalent to the facts that A is dissipative and $C = B^*$, as it is easy to verify (see also Curtain [4] for related operator inequalities).

Definition 3.3. The system Σ (or the semigroup $\mathbb{T}$) from (3.1) is called *strongly stable* if $\mathbb{T}_t z \to 0$ as $t \to \infty$, for all $z \in X$. The system Σ (or the semigroup $\mathbb{T}$) is called *exponentially stable* if its growth bound is negative.

Static output feedback stabilization of the system Σ from (3.1) means finding a feedback operator $K \in \mathcal{L}(U)$, such that the system with the input $u = Ky + v$, where v is the new input function, is stable in some sense. The state trajectories of the closed-loop system satisfy $\dot{z}(t) = A^K z(t) + B^K v(t)$, where $A^K = A + B(I - KD)^{-1}KC$ and $B^K = B(I - KD)^{-1}$. The operator A^K generates the semigroup $\mathbb{T}^K$. If K exists such that $\mathbb{T}^K$ is exponentially or strongly stable, then we call the original system *exponentially or strongly stabilizable* by static output feedback, respectively.

Many references have investigated impedance passive systems with the feedback operator $K = -kI$, where $k > 0$. This is often called collocated control. The idea behind this choice of K is that for $v = 0$, i.e., for $u = -ky$, the right-hand side of (3.3) is zero or negative and so the energy of the system is nonincreasing. Under certain conditions this feedback leads to exponential stability, see for instance Curtain and Weiss [5, 24] and the references therein.

Although exponential stabilization is the most desirable, often it is impossible to achieve. In this case, the best we can hope for is strong stabilization. Now we state a strong stabilization theorem for a class of impedance passive systems with collocated control. This result follows from Theorem 14 in [2] and it is a strengthening of the result in Benchimol [3]. A generalization will be given in Curtain and Weiss [6].

Theorem 3.4. *Suppose that the system* Σ *from* (3.1) *is such that* A *is m-dissipative with compact resolvents,* $C = B^*$ *and* $D = 0$.

Then for every $k > 0$, *if*

$$\{\, x \in X \mid \Psi x = 0 \text{ and } \|\mathbb{T}_t x\| = \|\mathbb{T}_t^* x\| = \|x\| \qquad \forall\, t > 0 \,\} = \{0\} \tag{3.4}$$

where Ψ *is the operator defined in* (3.2), *then the semigroup* $\mathbb{T}^k$ *generated by* $A^k = A - kBB^*$ *and its dual are strongly stable.*

Note that if (A, B^*) is approximately observable on $[0, \infty)$, i.e., the null-space of Ψ is only $\{0\}$, then (3.4) holds. Note also that the semigroup $\mathbb{T}^k$ corresponds to the feedback $u = -ky + v$, where v is the new input (typically zero).

4. Strong stabilization of the wind turbine tower model

In this section, we analyze the strong stabilization of the wind turbine tower model (in the plane of the turbine blades) Σ, described by (2.1)–(2.8), using Theorem 3.4.

First we formulate Σ as a state space system. The natural state of Σ, at the time t, is

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \\ z_5(t) \\ z_6(t) \\ z_7(t) \end{bmatrix} = \begin{bmatrix} w(\cdot, t) \\ w_t(\cdot, t) \\ w_t(l, t) \\ w_{xt}(l, t) \\ (\theta_T)_t(t) + w_{xt}(l, t) \\ (\theta_m)_t(t) - w_{xt}(l, t) \\ \theta_k(t) \end{bmatrix},$$

where z_1, z_2 are the transverse displacement and the transverse velocity of the tower, z_3, z_4 are the velocity and the angular velocity of the nacelle, z_5, z_6 are the angular velocities of the turbine rotor and generator rotor with respect to the earth and z_7 is the angular difference between the endpoints of the low-speed shaft.

The energy state space of Σ is

$$X = \mathcal{H}_l^2(0, l) \times L^2[0, l] \times \mathbb{C}^5. \tag{4.1}$$

Here

$$\mathcal{H}_l^2(0, l) = \{h \in \mathcal{H}^2(0, l) \mid h(0) = 0,\ h_x(0) = 0\},$$

where $\mathcal{H}^n$ ($n \in \mathbb{N}$) denote the usual Sobolev spaces. The natural norm on X (squared) is

$$\begin{aligned} \|z(t)\|^2 = & \int_0^l EI(x)|z_{1xx}(x, t)|^2 \mathrm{d}x + \int_0^l \rho(x)|z_2(x, t)|^2 \mathrm{d}x + m|z_3(t)|^2 + J|z_4(t)|^2 \\ & + J_T|z_5(t)|^2 + J_G|z_6(t)|^2 + K_s|z_7(t)|^2, \end{aligned}$$

which represents twice the physical energy of Σ.

We use collocated sensors and actuators by choosing $C = B^*$ and $D = 0$, i.e., we define the output to be $y = B^* z$. Then the state space formulation of Σ from (2.1)–(2.8), with no disturbance (i.e., with $T_a = 0$) is

$$\begin{cases} \dot{z}(t) &= Az(t) + Bu(t), \\ y(t) &= B^* z(t), \end{cases} \tag{4.2}$$

where $u = T_e$ is the control input. The operators A and B are defined by

$$A\begin{bmatrix}\xi \\ \varsigma\end{bmatrix} = \begin{bmatrix} \xi_2 \\ -\rho^{-1}(x)\big(EI(x)\xi_{1xx}(x)\big)_{xx} \\ m^{-1}\big(EI\xi_{1xx}\big)_x(l) \\ -\frac{EI(l)}{J}\xi_{1xx}(l) + \frac{K_s(1+n_g)}{Jn_g}\varsigma_3 - \frac{b_m}{J}(\varsigma_2+\xi_4) + \frac{C_s(1+n_g)}{Jn_g}\kappa \\ -\frac{K_s}{J_T}\varsigma_3 - \frac{C_s}{J_T}\kappa \\ \frac{K_s}{J_G n_g}\varsigma_3 - \frac{b_m}{J_G}(\varsigma_2+\xi_4) + \frac{C_s}{J_G n_g}\kappa \\ \kappa \end{bmatrix}, \tag{4.3}$$

where $\kappa = \varsigma_1 - \frac{1}{n_g}\varsigma_2 - \frac{1+n_g}{n_g}\xi_4$,

$$\mathcal{D}(A) = \left\{ \begin{bmatrix}\xi \\ \varsigma\end{bmatrix} \in \left[\mathcal{H}^4(0,l) \cap \mathcal{H}^2_l(0,l)\right] \times \mathcal{H}^2_l(0,l) \times \mathbb{C}^5 \;\middle|\; \begin{array}{l} \xi_3 = \xi_2(l) \\ \xi_4 = \xi_{2x}(l) \end{array} \right\}.$$

$$B = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{J} & 0 & -\frac{1}{J_G} & 0 \end{bmatrix}^T.$$

It is clear that B is bounded, i.e., $B \in \mathcal{L}(\mathbb{C}, X)$.

We will need the following proposition from our paper [27]:

Proposition 4.1. *The generator A from* (4.3) *is m-dissipative with compact resolvents on the state space X.*

Our main result is the following:

Theorem 4.2. *The wind turbine tower model Σ from* (4.2) *is strongly stabilized on the state space X from* (4.1) *by the static output feedback $u = -ky + v$, where*

$$y(t) = B^* z(t) = -\frac{1}{J} w_{xt}(l,t) - \frac{1}{J_G}\left((\theta_m)_t(t) - w_{xt}(l,t)\right)$$

and v is the new input function. The feedback gain k may be any positive number.

Proof. We define the energy of the system Σ as $E(t) = \frac{1}{2}\|z(t)\|^2$. Then we have

$$\begin{aligned} \dot{E}(t) &= \frac{1}{2}\langle \dot{z}(t), z(t)\rangle + \frac{1}{2}\langle z(t), \dot{z}(t)\rangle \\ &= \operatorname{Re}\langle Az(t) + Bu(t), z(t)\rangle = \operatorname{Re}\langle Az(t), z(t)\rangle + \operatorname{Re}\langle u(t), y(t)\rangle. \end{aligned}$$

From (4.4) of our paper [27], we have $\mathrm{Re}\,\langle Az(t), z(t)\rangle = -b_m(\theta_m)_t^2(t) - C_s(\theta_k)_t^2(t)$, so that

$$\dot{E}(t) = \mathrm{Re}\,\langle u(t), y(t)\rangle - b_m(\theta_m)_t^2(t) - C_s(\theta_k)_t^2(t)\,.$$

Thus, for every $\tau > 0$,

$$E(\tau) - E(0) = \int_0^\tau \mathrm{Re}\,\langle u(t), y(t)\rangle\,\mathrm{d}t - b_m\int_0^\tau (\theta_m)_t^2(t)\,\mathrm{d}t - C_s\int_0^\tau (\theta_k)_t^2(t)\,\mathrm{d}t\,. \quad (4.4)$$

Recall that we assume that $b_m > 0$ and $C_s \geq 0$. We have

$$y(t) = B^*z(t) = -\frac{1}{J}w_{xt}(l,t) - \frac{1}{J_G}\left((\theta_m)_t(t) - w_{xt}(l,t)\right). \quad (4.5)$$

Suppose $z_0 \in X$ is such that $\Psi z_0 = 0$ and $\|\mathbb{T}_t z_0\| = \|z_0\|$ for all $t > 0$. We consider the state trajectory of (4.2) corresponding to the initial state z_0 and $u = 0$. From $E(\tau) = E(0)$ and (4.4) we obtain that for every $\tau > 0$,

$$b_m\int_0^\tau (\theta_m)_t^2(t)\,\mathrm{d}t + C_s\int_0^\tau (\theta_k)_t^2(t)\,\mathrm{d}t = 0\,,$$

which implies that $(\theta_m)_t(t) = 0$ for all $t \geq 0$. Now from $\Psi z_0 = 0$ and (4.5), we have $w_{xt}(l,t) = 0$ for all $t \geq 0$. Here we have used that $J \neq J_G$ (because J is much larger than J_G). Then from (2.6) (with $T_e = 0$) we get that $T_{hss}(t) = 0$ for all $t \geq 0$. From (2.8) we get that $T_{lss}(t) = 0$ for all $t \geq 0$. From (2.5) (with $T_a = 0$) we get that $(\theta_T)_{tt}(t) = 0$ for all $t \geq 0$, so that $(\theta_T)_t$ is a constant. From (2.8) we have

$$K_s\theta_k(t) + C_s(\theta_k)_t(t) = 0\,. \quad (4.6)$$

But by differentiating (2.7) we have $(\theta_k)_t = (\theta_T)_t$ which, as we have seen, is a constant. The only solution of (4.6) for which $(\theta_k)_t$ is constant is $\theta_k = 0$. Hence, for all $t > 0$, we have $\theta_k(t) = 0$ and hence $(\theta_T)_t(t) = 0$.

Since the SCOLE model with output $w_{xt}(l,\cdot)$ is approximately observable (see Corollary 2.2 in Guo and Ivanov [9]), and since our system (2.1)-(2.4) is now a SCOLE model with output $w_{xt}(l,\cdot)$, we obtain that the tower states $z_1(t), z_2(t)$ and $z_3(t)$ are also zero for all $t \geq 0$. Thus, we have proved that for the combined system Σ we have $z(t) = 0$ for all $t > 0$. From Proposition 4.1 we know that A has compact resolvents on X. Clearly B is a bounded operator from $\mathbb{C}$ to X. Thus, the system Σ satisfies the conditions of Theorem 3.4. According to this theorem, Σ is strongly stabilized by the static output feedback $u = -ky + v$, for any $k > 0$. ■

5. Conclusions

We have obtained a strongly stabilizing static output feedback for the wind turbine tower model in the plane of the turbine blades, using electrical torque control and measurements of the angular velocities of the nacelle and the electrical generator. This feedback should be used in addition to the maximum power point tracking feedback, and it should reduce the vibrations of the tower. This research should help to use lighter, more flexible wind turbine towers, and extend the life expectancy of existing towers, by reducing the wear on the gearbox and the low-speed shaft.

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