COMPETITION IN CONGESTED SERVICE NETWORKS
WITH APPLICATION TO AIR TRAFFIC CONTROL PROVISION IN EUROPE

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ABSTRACT

A regulator controls a congestible network where service providers each control and set charges on part of the network. There are a limited number of network users, who also act strategically. We analyze how the network affects the competition, equilibrium charges and efficiency. Several regulatory strategies are explored including price-caps, mergers and facilitating the adoption of new technologies. The model is subsequently illustrated with a case study on air traffic control provision in Western Europe, in which it is shown that more drastic changes in the regulation are required in order to create a more cost efficient sector with increased capacity.

I. INTRODUCTION

Service providers face the continuous challenge of adapting capacities to the fluctuations in demand patterns of congestion sensitive customers, while maintaining a competitive advantage via appropriate pricing policies. As an example, air traffic demand has been estimated to increase continuously over the next two decades across the globe\(^1\), which requires sophisticated technologies to navigate aircraft accurately and safely through the skies otherwise the additional demand will not be served. An important aspect of air traffic control (ATC) draws from the network dimension whereby, in Europe, each section of the network is managed by a single service provider. However, flights may cross multiple sections of the airspace, hence need to be served by several providers seamlessly. In addition, given the multiple potential routes from origin to destination, it is clear that European service providers are also competing in certain markets. An important research question therefore draws from the implications of the tradeoff between the benefits of collaboration and the need to compete in serving the market.

\(^1\) [https://www.boeing.com/](https://www.boeing.com/) and [https://www.airbus.com/](https://www.airbus.com/)
The ATC market is one example in which service providers may compete over a geographically congested network. The service provider is both a monopolist which maximizes profits by setting capacities and charges for use of their own airspace and a competitor with their neighbors for transit traffic. The setting becomes more complicated because their customers, the network users, are non-atomistic in that each has market power by generating a non-negligible fraction of the total demand. The airlines maximize their profits by choosing the cheapest route to fly whilst internalizing self-imposed congestion costs. The extent to which airlines internalize congestion has been a subject of debate in both the theoretical (Brueckner, 2002; Brueckner and Van Dender, 2008) and empirical (Morrison and Winston, 2007; Rupp, 2009) literature. The major management issues caused by this complex market include high costs due to fragmentation of ATC service provision, slow technology adoption, lack of standardization in services and inefficient scale of operation (Baumgartner and Finger, 2014). Consequently, it is important to understand which regulatory policies, such as price-caps, technology R&D subsidies or horizontal integration, would promote efficiency.

Additional examples of competition across geographical service networks include rail, road and shipping services. In the rail sector, the infrastructure managers are the service providers and the railway companies serving passengers and freight are the non-atomistic customers. Similar management issues and regulatory policies may be considered, namely setting track access charges, harmonization of technical standards across the track network and efficient scale of operation. Consequently, the modeling approach developed in this paper could be applied to analyze the rail sector. Road network infrastructure is also divided into regional providers but the customers are independent and atomistic i.e. cars and trucks, choosing routes and frequencies assuming congestion levels as given. Efficiency issues in the road sector include high truck distance charges in Europe caused by the monopolistic power of each region (Mandell and Proost, 2016). In the US, regions use their monopoly power via state diesel taxes. Our model is relevant provided all road customers are affected by congestion in the same manner because they are atomistic. Similarly, in the shipping industry, the port managers are the service providers and the shipping companies serve the cargo market by choosing the frequency of port visits.

More generally, competition between service providers may be modelled using a directed flow network in which each arc represents service provided by a specific supplier and each customer demands path flows connecting specified origin-destination (OD) pair(s). In this research, we model such settings within a two-stage network congestion game. In a preliminary stage, the regulator sets the rules. In the first stage, each provider sets charges on the services
(arcs) they provide in order to maximize profits. Peak and off-peak pricing is also considered. In the second stage, each customer chooses the desired flows to minimize the sum of service charges and congestion-dependent operating costs, including the possibility of partial flows or not using any service when the associated costs are too high. The network structure matters because (1) decisions of one customer will impact the congestion levels of the other customers via network flows hence impact their choices and the size of the subsequent market; (2) the equilibrium price setting will depend on network effects related to whether routes demanded face horizontal competition and require one or multiple service providers.

**Contributions**

The modeling approach is the first research known to the authors that considers general networks, demand for multiple OD pairs and oligopolistic markets in both stages of the game. It is also the first to model non-atomistic customers with market power in the second stage. Our network congestion model shows that service providers engage in competition selectively as a function of demand levels and network structure. Engaging in competition in some part of the network is worthwhile only when the demand level is sufficiently high, and this choice will exhibit interdependencies across different parts of the network. Equilibrium service charges are affected by the level of competition and congestion. Given this behavior, we analyze whether competition between service providers may lead to efficiency in the sense of minimizing total social costs and/or lower service charges for the benefit of the customers. Our setting enables an evaluation of multiple market design scenarios including the impact of deregulation, incentive based price-caps, different forms of co-operation between players and the introduction of new technology inducing capacity expansion, which in turn reduces costs for the players.

The results of the analysis suggest that in the presence of network effects, competition between service providers does not always reduce the service charge to the customers and may even increase total service charges along some routes, thus generating inefficiency from the point of view of overall social costs. Price-cap regulation discourages technology adoption, and generates inefficiency due to lower internalization of congestion costs and excessive congestion. Horizontal integration across service providers increases the internalization of congestion costs but also leads to higher charges with no incentives for technology adoption thus does not necessarily improve efficiency and is also not beneficial for the customers. Vertical integration, in the form of cooperation between some service providers and customers maintains the internalization of congestion cost levels and generates incentives for technology adoption, thus may improve efficiency. However, this is dependent on the regulator permitting the service
providers to charge for reduced congestion as a result of the adoption of new technologies. Understanding the likely behavior of service providers and their customers and the implication on overall social costs may help guide regulatory initiatives and highlight potential future institutional processes that may lead to improved market conditions.

**Related literature**

Since the pioneering work of Pigou (1920), there has been a substantial and well established literature analyzing the efficiency of congested service systems, including network congestion games. The standard approaches to analyze such settings include Wardrop equilibria (Wardrop 1952) and the potential game approach (Rosenthal 1973, Monderer and Shapley 1996), both of which consider atomistic and identical customers, each demanding an infinitesimal flow in the face of exogenous latency/congestion cost functions. A different approach assumes that competing customers are non-atomistic and demonstrate market power in that each customer controls a non-negligible fraction of the total flow (e.g., Brueckner 2002, Cominetti et al. 2009). The difference between the two approaches asymptotically vanishes as the number of non-atomistic customers increases (Haurie and Marcotte 1985).

Congestion and contracting in competitive service industries has also been addressed within the operations management literature (e.g., Cachon and Harker 2002, Netessine and Shumsky 2005, Allon and Federgruen 2007, Johari et al. 2010). Competition between service providers in the presence of congestion costs was analyzed in depth by Acemoglu and Ozdaglar (2007a, 2007b) and Perakis and Sun (2014). Acemoglu and Ozdaglar (2007a) consider a two-stage game in which profit-maximizing oligopolists compete by setting prices for travel on each of several alternative and parallel routes, all connecting the same OD pair, in the first stage. Atomistic and identical users choose one of these routes to minimize travel and congestion costs in the second stage. Acemoglu and Ozdaglar (2007b) extend this analysis to parallel-serial networks with a single OD pair, i.e. each parallel route may include several serial links. Instead of Bertrand competition, Perakis and Sun (2014) consider differentiated Cournot competition in the first stage and multimodal general Wardrop equilibrium (Dafermos 1982) in the second stage. Although not formulated using a network, the model of Perakis and Sun (2014) is analogous to a simple network having a single OD pair connected by parallel routes, as in Acemoglu and Ozdaglar (2007a). Consequently the network effects we identify are absent from these papers.

Our two-stage game of price competition between service providers in the presence of congestion is the first to consider general networks with oligopolistic markets in both stages of the game, i.e. allows for non-atomistic heterogeneous customers with market power in the
second stage who react to the first stage competitive pricing. Subgame perfect equilibria (Selten 1975) allow customers to consider self-imposed congestion across the various routes, potentially leading to interior point flows that do not occur with atomistic Wardrop equilibria. This is critical to the issue of existence of equilibria in the two-stage game when customers are heterogeneous, hence impacts the comparative conclusions we can draw from the analysis.

The paper is organized as follows: We develop the modeling approach in Section II and discuss several analytic results derived from the model in Section III. We present a case study of ATC in the European airspace in Section IV, including a literature search of the air traffic control provision literature. Section V presents the numerical results and Section VI draws conclusions and identifies potential future directions of research.

II. MODELING APPROACH

We consider a two-stage network congestion game in which the service providers set their charges in stage one and congestion sensitive customers choose their providers via path flows in stage two. The main focus of the model is to shed light on how first stage service providers choose to compete and their impact on the decisions of the second stage customers with respect to network paths, which ultimately impacts the preferred market design. We make use of two optimization models, one describing the simultaneous decisions of the first stage service providers and another defining those of the second stage customers, which represent a best response to each other and to the choices of the up-stream market. The set-up is summarized in Table 1. The customers (firms) want to satisfy demand for specific OD pairs. The OD pairs may consist of one to many arcs. Each arc is served by a single service provider. A set of arcs creating a path may therefore be served by a single service provider or require the collaboration of multiple providers. When one provider serves all arcs connecting an OD, we refer to this as captive demand. Hence, an arc may serve both captive demand and demand that is part of a path for which there are alternatives, which we call flexible demand.

### TABLE 1: Description of the model set-up

<table>
<thead>
<tr>
<th>Preliminary stage</th>
<th>Regulator chooses rules of the game for the service providers anticipating the outcome of stages 1 and 2; Regulator may allow horizontal or vertical agreements to be signed between service providers or between provider and customer respectively.</th>
</tr>
</thead>
<tbody>
<tr>
<td>First stage</td>
<td>Service providers set charges for each arc, anticipating the behavior of the customers and taking the behavior of other service providers as given.</td>
</tr>
<tr>
<td>Second stage</td>
<td>Customers choose the least cost path, taking charges over arcs as given as well as the volumes of network use created by all customers.</td>
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Prior to establishing an equilibrium outcome in the market, the regulator sets rules with respect to institutional form and price regulation in a preliminary stage. The regulator may also enforce legislation with respect to horizontal integration across service providers or vertical integration with customers. Finally, an a-priori agreement between service providers and customers with respect to technology adoption may be signed. Technology adoption is expected to reduce costs via capacity expansion. Decisions at this level are considered exogenous to the game defined hence create multiple model parameter scenarios to be assessed.

The network underlying the congestion game is composed of a set of origin, transit and destination nodes, and a set of arcs representing services offered. We use the following network definitions:

\[ P \] finite set of origin/destination nodes with indices \( o, d \)

\[ T \] finite set of transit nodes

\[ N \] set of all nodes, \( N = P \cup T \), with indices \( i, j \)

\( A_s \) set of arcs, \( A_s \subseteq N \times N \), owned by service provider \( s \)

\( A \) set of all arcs, \( A = \bigcup_s A_s \), with index \( a = (i, j) \)

\( \delta_a \) length of service over arc \( a \) (in units of distance or work)

\( W \) set of time windows with index \( w = 1 \) for peak and \( w = 2 \) for off-peak

for the service providers and customers we use the following definitions:

\( L \) finite set of customers with index \( l \)

\( S \) finite set of service providers with index \( s \)

\( s(a) \) service provider owning arc \( a \)

\( D_{lod} \) maximal demand of customer \( l \) for service from origin \( o \) to destination \( d \)

\( K_{law} \) maximal allowed flow for customer \( l \) over arc \( a \) during time window \( w \)

and we use the following definitions of costs and charges:

\( C^S_s \) service cost per length unit of arc controlled by service provider \( s \)

\( C^O_{la} \) operating cost per length unit for customer \( l \) over arc \( a \)

\( C^R_{law} \) loss per length unit for customer \( l \) over arc \( a \) during time window \( w \)

\( C^G_{ls} \) congestion cost per flow unit per length unit for customer \( l \) served by provider \( s \)

\( C^D_{od} \) outside option cost per flow unit of non-service from origin \( o \) to destination \( d \)

\( \tau_{sw} \) price-cap per length unit for service provider \( s \) during time window \( w \)

finally, we use the following sets of decision variables:

\( \tau_{sw} \) service provider \( s \)’s charge per length unit over arc \( a \) during time window \( w \)

\( f^l_{0daw} \) customer \( l \)’s flow via arc \( a \) within origin-destination \((o, d)\) during window \( w \)

\( f^R_{l0d} \) customer \( l \)’s non-flow from origin \( o \) to destination \( d \)

We model the first-stage service providers as profit maximizers choosing charges \( \tau_{sw} \). Each service provider \( s \) best responds to the choices of its competitors, taking as given the equilibrium
service flows $f_{lodaw}^*$ that will be chosen in the second stage of the game, thus leading to a sub-game perfect equilibrium. For service provider $s$, model (2.1) includes the profit function and a set of constraints in which the charges are price capped, to be included where relevant.

$$\begin{align*}
\text{Max}_{\tau_{saw}} & \sum_{a \in A_s} \sum_{w} (\tau_{saw} - C_{s(a)}) \theta_a \sum_{l} \sum_{od} f_{lodaw}^* \\
\text{s.t.} & \quad \tau_{saw} \leq \tau_{saw}^0 \; \forall a \in A_s, w \in W
\end{align*}$$

(2.1)

The customer cost function, equation (2.2), which is modelled in the second stage of the game with linear latency costs\(^2\) (Roughgarden and Tardos, 2002), is composed of several categories, all of which are impacted to some degree by the choices in the preliminary and first stages. For customer $l$, each length unit of flow over any arc $a$ generates an operating cost $C_{l(a)}^O$, a consumer surplus loss $C_{l(a)}^R$ during the less preferred off-peak time window, congestion costs $C_{l(a)}^G$ multiplied by the total arc flow including that of competing customers, and a service provider charge $\tau_{(a)aw}$. Hence the customer cost function is quadratic due to the congestion cost, which increases with total arc flow for each length unit a customer is served. Additionally, in order to account for elastic demand, there exists an outside option flow, $f_{lodaw}^T$, which represents the choice to reduce the service demanded below the maximal level $D_{lodaw}$, with cost $C_{lodaw}^T$ per flow unit. This opportunity cost will be preferred if the total costs of being served from the origin to the destination are too high.

$$\begin{align*}
\psi_l & \equiv \sum_{w} \sum_{a \in A} \left[ C_{l(a)}^O + C_{law}^R + C_{l(a)}^G (\sum_{i'=od} f_{i'odaw}) + \tau_{s(a)aw} \right] \theta_a \sum_{od} f_{lodaw} + \\
& \quad \sum_{od} C_{lodaw}^T f_{lodaw}^T
\end{align*}$$

(2.2)

We compare scenarios according to total social costs to be minimized in order to search for the most appropriate outcomes considering both sets of actors. The social cost function sums all customer costs, including service provider charges, minus service provider profits, so in total it equals the customer costs plus service provider costs.

Two alternative solutions are modelled for the second stage: either a system optimal outcome as described in equations (2.3) to (2.6), or a user equilibrium outcome in which only the objective function (2.3) is adapted as shown in (2.3') where each user minimizes its own costs only. In the system optimal approach, a central planner chooses the service paths and timing

\(^2\) Although congestion is generally highly non-linear when flows are close to the capacity, objective function (2.2) assumes that linear congestion costs are a reasonable approximation for equilibrium flows sufficiently far from the capacity levels of the arcs. We would also add that the reduced form cost function of a pure bottleneck congestion model is also linear in the ratio flow over capacity (Arnott et al.1993).
(peak or off-peak) for all customers simultaneously to minimize total customer costs, including operating and congestion costs, consumer surplus losses from being served in the off-peak, service provision charges and lost surplus for non-realized demand. The system optimal solution achieves efficiency in terms of minimizing total social costs when the service provider charge equals the service cost (per length unit).

\[
\begin{align*}
\text{Min} & \quad f^T_l, f^T_T \sum_l \Psi_l \\
\text{s.t.} & \quad \sum_w \left[ \sum_{(o,i) \in A} f_{lod(o,i)w} - \sum_{(j,o) \in A} f_{lod(o,j)w} \right] + \sum_{(j,d) \in A} f_{lod(d,j)w} = D_{lod}, \quad \forall l \in L, \forall o, d \\
& \quad \sum_w \left[ \sum_{(j,d) \in A} f_{lod(d,j)w} - \sum_{(i,j) \in A} f_{lod(i,j)w} \right] + \sum_{(i,d) \in A} f_{lod(i,d)w} = D_{lod}, \quad \forall l \in L, \forall o, d \\
& \quad \sum_{(i,d) \in A} f_{lod(i,d)w} = 0, \quad \forall l \in L, w \in W, o, d, i \in N (i \neq o, d) \\
& \quad \sum_{od} f_{lod1} \leq K_{la1}, \quad \forall l \in L, \forall a \in A \\
& \quad f_{lodaw} \geq 0, f^T_{lod} \geq 0, \quad \forall l \in L, o, d \in N, a \in A, w \in W .
\end{align*}
\]

Constraint (2.4) sums the incoming less the outgoing flows to be equal to the (negative) demand at the (origin) destination and zero when using a transit point. The total flows are reduced by those that have been dropped via the outside option of not being served. Constraint (2.5) restricts the level of flows during the peak window on a per customer basis. This restriction may be removed if unnecessary, for example because congestion is not an issue. In other words, if all demand could be served in the peak window, constraint (2.5) would not be necessary. However, in many systems, agreements are drawn between firms and customers setting demand levels in the peak, for example through slot controls at an airport. Constraint (2.6) ensures non-negativity of the flows and non-flow.

The distinction between the user equilibrium and system optimum approaches may be intuitively understood as follows. In a user equilibrium, each customer chooses paths and time windows taking into account its own costs only and taking the flows of the other customers as given. Specifically, each customer \( l \) considers only its own congestion costs and ignores the external congestion costs imposed on the other customers. Hence the flows may be less balanced than those of the system optimal approach in which a system wide planner minimizes the sum of second stage costs including congestion. When congestion is sufficiently severe so that some demand opts out, low charges generate flows that are too high as compared to the system optimal approach, with the opposite effect with high charges. An efficient outcome that minimizes total social costs occurs in a user equilibrium only if the service provider charge is equal to the service cost plus external congestion costs (per length unit). Recall that service providers may have
market power depending on the network structure and customer demand. A monopolist service provider may set charges sufficiently above the service cost (per length unit) so that flows align to efficient levels from the point of view of minimizing total social costs of first and second stages. In contrast, depending on the level of competition generated by the network, competing service providers do not consider the entire system when choosing their charges. Consequently, their increasing reaction functions, which emerge from the differentiated Bertrand competition created by congestion, may lead their equilibrium charges to be higher than those set by a monopolist, which in turn would generate inefficiently low flows in some parts of the network. Our results in the following sections elaborate on the above.

Existence of equilibrium
The set of convex, quadratic customer cost objective functions with linear constraints ensures the existence of equilibrium in the second stage of the game, which is obtained by solving the Karush Kuhn Tucker conditions simultaneously (Kuhn 2014). Furthermore, since the first stage service provider charges affect only the right-hand side of these linear conditions, the second stage flows are piecewise linear functions of the first stage charges. Consequently, the first stage objectives are piecewise concave functions of the second stage equilibrium actions. For the first stage of the game, an equilibrium in mixed charging strategies always exists. Although an equilibrium in pure charging strategies may fail to exist, we show that it does exist in the simplified network analyzed in the next section. We also compute and analyze pure strategy equilibria for the more general network used in the air traffic control case study presented in Sections IV and V.

III. SERIAL-PARALLEL NETWORK ANALYSIS
After folding the second stage solution into the first stage of the game, we are able to draw several initial conclusions with respect to the sub-game perfect equilibrium outcomes given a simplified network under various regulation policies. We model three service providers and n customers on a serial-parallel network with multiple origins and destinations, as shown in Figure 1, with symmetric cost functions and a single time window.

![Figure 1: Simplified network](image-url)
This simplified network allows the analysis of network effects of two types: (a) heterogeneous customers offering different origins and destinations, and (b) collaborating service providers with asymmetric market power: C has a monopolistic advantage in that all customers starting at origin 0 must use its services but may choose whether to use the services of A or B. Such asymmetric market power naturally arises in decentralized geographical service networks, for example in the air traffic control case study considered in Sections IV and V. Network effects are absent from the existing analysis in the literature without a network (Brueckner, 2002; Perakis and Sun, 2014), which is equivalent to assuming a single OD pair connected only by multiple parallel links. Our analysis using the simplified network emphasizes the insights emerging in the presence of network effects. Even for this network, the equilibrium charge setting will already be complex, as it will depend on the two types of network effects, namely asymmetric service collaboration and heterogeneous customers. Our conclusions in this section continue to hold whenever these effects are present, as demonstrated by the more involved air traffic control case study. The results are presented in the theorems below, summarized in Table 2.

**TABLE 2: Summary of results based on the simplified network**

| Asymmetric service collaboration: homogeneous customers from origin 0 to destination 4 using either the service collaboration of C and A or of C and B |
|---|---|
| Theorem I | Unregulated competition under asymmetric service collaboration |
| Corollary I | Price-cap regulation leads to inefficiency |
| Theorem II | Horizontal integration of collaborating providers (C,A) competing with provider B |
| Corollary II | Horizontal integration may coordinate charges and improve efficiency |

| Heterogeneous customers: flexible demand from origin 1 to destination 4, and captive demand from 1 to 2 and from 1 to 3 |
|---|---|
| Theorem III | Unregulated competition with heterogeneous customers |
| Theorem IV | Horizontal integration of parallel providers A and B |
| Corollary III | Merger cost savings do not lead to lower charges |
| Theorem V | Technology adoption by provider A and customer 1 |

Asymmetric service collaboration: homogeneous customers from origin 0 to destination 4 using either the service collaboration of C and A or of C and B

We start by showing that competition may lead to higher prices as compared to horizontal integration (A+C) due to the network effects arising from asymmetric service collaboration (C+A or C+B). As a result, asymmetric competition flows are less efficient from the point of view of minimizing total social costs. Consider the case where the (maximal) demand is equal to \(D\) per customer, for services from origin 0 to destination 4 using either the collaboration of service provider C and A or of C and B (later we consider the case where there is captive demand
on some of the arcs). The results depend on the outside option cost, $C^T$, being sufficiently high that all demand will choose to be served when the service provider charges are sufficiently low (specific conditions are stated in the Appendix). Using the piecewise concavity of the first stage objective as a function of the second stage equilibrium actions, we find the pure strategy solution outcome by analyzing the first order conditions (presented in the Appendix).

**Theorem 1: Unregulated competition under asymmetric service collaboration**

**Case 1: low demand**

$$D < \frac{54c^T - 3c^O - 3c^S}{288(n+1)c^G}$$ \hspace{1cm} (3.1')

$$\tau^\text{comp}_A = \tau^\text{comp}_B = C^S + \frac{n+1}{2n} C^G (2nD)$$ \hspace{1cm} (3.1.1.1)

$$\tau^\text{comp}_C = C^S + (C^T - 3C^O - 3C^S) - 4(n + 1)C^G D$$ \hspace{1cm} (3.1.1.2)

$$f^\text{comp}_{l,A} = f^\text{eff}_{l,A} = \frac{1}{2} D + \frac{\tau^B - \tau^A}{2(n+1)c^G}, \forall l$$ \hspace{1cm} (3.1.1.3)

$$f^\text{comp}_{l,B} = f^\text{eff}_{l,B} = D - f^\text{comp}_{l,A}, \forall l$$ \hspace{1cm} (3.1.1.4)

$$f^\text{comp}_{l,C} = f^\text{comp}_{l,A} + f^\text{comp}_{l,B} = f^\text{eff}_{l,A} + f^\text{eff}_{l,B} = D, \forall l.$$ \hspace{1cm} (3.1.1.5)

**Case 2: excessive demand**

$$D \geq \frac{54c^T - 3c^O - 3c^S}{288(n+1)c^G}$$ \hspace{1cm} (3.1)

$$\tau^\text{comp}_A = \tau^\text{comp}_B = C^S + \frac{36}{288}(C^T - 3C^O - 3C^S)$$ \hspace{1cm} (3.1.2.1)

$$\tau^\text{comp}_C = C^S + \frac{108}{288}(C^T - 3C^O - 3C^S)$$ \hspace{1cm} (3.1.2.2)

$$f^\text{comp}_{l,A} = \frac{72c^T - 3c^O - 3\tau^A - \tau^C}{288(n+1)c^G} = \frac{27(c^T - 3c^O - 3c^S)}{288(n+1)c^G} < \frac{36c^T - 3c^O - 3c^S}{288nc^G} = f^\text{eff}_{l,A}, \forall l$$ \hspace{1cm} (3.1.2.3)

$$f^\text{comp}_{l,B} = \frac{24c^T - 3c^O - 3\tau^B - \tau^C}{288(n+1)c^G} = \frac{27(c^T - 3c^O - 3c^S)}{288(n+1)c^G} < \frac{36c^T - 3c^O - 3c^S}{288nc^G} = f^\text{eff}_{l,B}, \forall l$$ \hspace{1cm} (3.1.2.4)

$$f^\text{comp}_{l,C} = f^\text{comp}_{l,A} + f^\text{comp}_{l,B} = \frac{54c^T - 3c^O - 3c^S}{288(n+1)c^G} < \frac{72c^T - 3c^O - 3c^S}{288nc^G} = f^\text{eff}_{l,C}, \forall l.$$ \hspace{1cm} (3.1.2.5)

In the unique user equilibrium of unregulated competition, the service provider charges $\tau^\text{comp}_s$ and customer flows $f^\text{comp}_{ls}$ in equilibrium are defined according to two cases. In case 1, the demand (from origin 0 to destination 4) is strictly lower than threshold (3.1). In this case, the competing service providers A and B serve the entire demand and given the symmetry of our simplified network, this means that they each serve half the market. Their charge is a function of the service cost and increases with growing demand due to congestion because a service provider benefits from charging for congestion. Service provider C plays a different strategy as it can use its monopoly power to set a high charge, which decreases with growing demand and congestion cost in order to keep the service sufficiently attractive given the outside option cost, $C^T$.

From equation (3.1.1.1) we see that with very low congestion costs, the competing service providers A and B would set their charges according to the differentiated Bertrand equilibria outcome i.e. based on their service costs alone. However, given that congestion does exist in equilibrium, the service provider charges will internalize congestion costs to a degree. The
extent of internalization depends on the assumption as to whether customers exhibit market power or not, i.e. the extent to which they respond to self-inflicted congestion. Under Wardrop or potential game equilibria, in which each infinitesimal flow chooses independently i.e. there are infinitely many small atomistic customers (as \( n \) approaches infinity), the congestion cost coefficient in (3.1.1.1) would be approximately one half. In our oligopolistic, non-atomistic game with customers exhibiting market power, the service providers will always charge more.

The system optimal solution flows are obtained by considering the charge dependent user equilibrium flow expressions presented (in (3.1.1.3) and (3.1.2.3) to (3.1.2.5)) and replacing the term \( n + 1 \) everywhere by the term \( 2n \), representing full internalization of congestion costs. The efficient flows \( f_{s,\text{eff}} \) that minimize total social costs are then derived when setting the charge \( \tau_s = C^S \) for all \( s \). The user equilibrium outcome is efficient in case 1.

In case 2, where condition (3.1) holds, the congestion is such that demand served will remain constant at the threshold level, thus the service provider charges also remain constant. Now, efficiency implies only serving partial demand. Network effects emerge in case 2 through the fact that any route requires service provider C, who therefore sets a relatively high monopolistic charge. This leads service providers A and B, who have increasing reaction functions due to the differentiated Bertrand competition created by congestion, to also set inefficiently high prices. Consequently, total equilibrium flows are strictly lower than the efficient flows and congestion is inefficiently low too.

**Corollary I: Price-cap regulation leads to inefficiency**

If price-cap regulation is imposed such that the price-cap is almost equal to the service provider cost, and does not include a congestion cost element, then the price-cap will always be lower than the unrestrained equilibrium charges. Consequently, the service provider charges in the user equilibrium will equal the price-caps, leading to excessive congestion and inefficient flows from the point of view of minimizing overall social costs. Moreover, the service providers will not be able to cover the set-up cost of technology adoption, leading to the inefficient outcome of no technology adoption.

**Theorem II: Horizontal integration of collaborating providers (C,A) competing with provider B**

**Case 2: excessive demand**

\[
D \geq \frac{64(C^T - 3C^O - 3C^S)}{288(n+1)C^O} \tag{3.2}
\]

\[
\tau_A^{\text{int}, A, C} = C^S + \frac{16}{288}(C^T - 3C^O - 3C^S) < \tau_A^{\text{comp}} \tag{3.2.2.1}
\]

\[
\tau_B^{\text{int}, A, C} = C^S + \frac{32}{288}(C^T - 3C^O - 3C^S) < \tau_B^{\text{comp}} \tag{3.2.2.2}
\]

\[
\tau_C^{\text{int}, A, C} = C^S + \frac{112}{288}(C^T - 3C^O - 3C^S) > \tau_C^{\text{comp}} \tag{3.2.2.3}
\]
\[
\begin{align*}
&f_{\text{int} A,C}^{l_A} = \frac{72(C^T - 3C^O - 3\tau_A + \tau_B - \tau_C)}{288(n+1)C^G} > f_{l_A}^{\text{eff}}, \forall l \quad (3.2.2.4) \\
&f_{\text{int} A,C}^{l_B} = \frac{72(C^T - 3C^O - 3\tau_A + 3\tau_B - \tau_C)}{288(n+1)C^G} < f_{l_B}^{\text{eff}}, \forall l \quad (3.2.2.5) \\
&f_{\text{int} A,C}^{l_C} = f_{\text{int} A,C}^{l_A} + f_{\text{int} A,C}^{l_B} = \frac{64(C^T - 3C^O - 3C^S)}{288(n+1)C^G} < f_{l_C}^{\text{eff}}, \forall l. \quad (3.2.2.6)
\end{align*}
\]

In the unique user equilibrium under horizontal integration of the asymmetrically collaborating service providers A and C, we permit individual charge levels hence there are again two cases. In case 1, when the demand is strictly lower than threshold \(3.2\), the results are the same as in Theorem I. Providers A and C use the monopoly position of C to extract a maximum surplus from the flow. The threshold is slightly higher than that of condition \(3.1\). In case 2, where condition \(3.2\) is met, we have an asymmetric equilibrium with constant service flows at the threshold level and constant charges. Compared to the results of no integration, service provider C increases its charge, consequently the providers A and B reduce their charges, with an even lower charge for A that attracts a higher fraction of the partial demand served. The total charges are lower, hence the total flow is higher. Thus despite remaining inefficient, the total flow is higher compared to no horizontal integration, i.e. closer to the efficient flows.

**Corollary II:** Horizontal integration may coordinate charges and improve efficiency

Integrated collaborating service providers are able to coordinate charges to attract more customers, thus leading to improved efficiency from the point of view of minimizing overall social costs.

Figure 2 illustrates the charges and profits of service providers as the demand increases. The blue lines (solid and dashed) represent the results when all three service providers compete, and the red lines (solid, dashed and dotted) represent the results for the merger of service providers A and C (the joint profits of service providers A and C are shown with dotted-dashed lines).

![Figure 2: Charges and profits vs. demand](image)
Heterogeneous customers: flexible demand from origin 1 to destination 4, and captive demand from 1 to 2 and from 1 to 3

We now show how network effects arising from the combination of captive and flexible demand lead service providers to choose over which demand to compete. Consider the case where the customers demand services from origin 1 to destinations 2 and 3, defined as $D_{\text{cap}}$ per customer per OD pair, and flexible services from origin 1 (rather than 0) to destination 4, using either service provider A or B, defined as $D_{\text{flex}}$ per customer. Note that provider C does not play any role in the service of the two demands. The results depend on the outside option cost, $C^T$, being sufficiently high that all captive demand will choose to be served (specific conditions are stated in the Appendix). The results are presented in the three theorems below.

**Theorem III: Unregulated competition with heterogeneous customers**

\[
D_{\text{flex}} < \sqrt{\left(\frac{D_{\text{cap}} \left(\frac{c^T-c^O-c^S}{(n+1)c^G}\right) - D_{\text{cap}}}{D_{\text{cap}} - D_{\text{cap}} - D_{\text{cap}}}\right)}
\]  

(3.3’)

**Case 1: lowest flexible demand**

\[\tau_A^{\text{comp}} = \tau_B^{\text{comp}} = C^T - C^O - (n + 1)C^G D_{\text{cap}} \]  

(3.3.1.1)

\[f_{l,\text{flex}}^A = \frac{c_{\text{comp}}^A}{f_{l,\text{flex}}^B} = 0 < \frac{1}{2} D_{\text{flex}} = f_{l,\text{flex}}^B = f_{l,\text{flex}}^A, \forall l\]

(3.3.1.2)

\[f_{l,\text{cap}}^A = \frac{e_{\text{comp}}^A}{f_{l,\text{cap}}^B} = D_{\text{cap}} = f_{l,\text{cap}}^B = f_{l,\text{cap}}^A, \forall l\].

(3.3.1.3)

**Case 2: low flexible demand leads to asymmetry**

\[\tau_A^{\text{comp}} = \frac{1}{2} (C^T - 2C^O - (n + 1)C^G (D_{\text{cap}} + 2D_{\text{flex}})) \]  

(3.3.2.1)

\[f_{l,\text{flex}}^A = D_{\text{flex}} and f_{l,\text{flex}}^B = 0 or f_{l,\text{flex}}^A = 0 and f_{l,\text{flex}}^B = D_{\text{flex}}, \forall l\]

(3.3.2.2)

\[f_{l,\text{flex}}^A = f_{l,\text{flex}}^B = D_{\text{flex}} = f_{l,\text{flex}}^A + f_{l,\text{flex}}^B, \forall l\]

(3.3.2.3)

\[f_{l,\text{cap}}^A = f_{l,\text{cap}}^B = D_{\text{cap}} = f_{l,\text{cap}}^A = f_{l,\text{cap}}^B, \forall l\].

(3.3.2.4)

**Case 3: meeting flexible demand threshold**

\[D_{\text{flex}} \leq \frac{C^T - 2C^0 - 2c^S}{3(n+1)c^G} - D_{\text{cap}} \]  

(3.3.3)

\[\tau_A^{\text{comp}} = \tau_B^{\text{comp}} = \frac{C^S}{2n} + \frac{c_{\text{cap}}^A}{2n} \left(2n \left(D_{\text{cap}} + D_{\text{flex}}\right)\right)\]

(3.3.3.1)

\[f_{l,\text{flex}}^A = \frac{1}{2} D_{\text{flex}} + \frac{c_{\text{comp}}^A}{2(n+1)c^G} = \frac{1}{2} D_{\text{flex}} = f_{l,\text{flex}}^A, \forall l\]

(3.3.3.2)

\[f_{l,\text{flex}}^B = D_{\text{flex}} - f_{l,\text{flex}}^A = \frac{1}{2} D_{\text{flex}} = f_{l,\text{flex}}^B, \forall l\]

(3.3.3.3)

\[f_{l,\text{cap}}^A = f_{l,\text{cap}}^B = D_{\text{cap}} = f_{l,\text{cap}}^A = f_{l,\text{cap}}^B, \forall l\].

(3.3.3.4)

**Case 4: increasing flexible demand**

\[\frac{C^T - 2C^0 - 2c^S}{3(n+1)c^G} - D_{\text{cap}} < D_{\text{flex}} < \frac{C^T - 2C^0 - 2c^S}{2(n+1)c^G} - D_{\text{cap}} \]  

(3.3.4)

\[\tau_A^{\text{comp}} = \tau_B^{\text{comp}} = \frac{1}{2} (C^T - 2C^O - (n + 1)C^G (D_{\text{cap}} + D_{\text{flex}})) \]  

(3.3.4.1)

\[f_{l,\text{flex}}^A = \frac{1}{2} D_{\text{flex}} + \frac{c_{\text{comp}}^A}{2(n+1)c^G} = \frac{1}{2} D_{\text{flex}} = f_{l,\text{flex}}^A, \forall l\]

(3.3.4.2)

\[f_{l,\text{flex}}^B = D_{\text{flex}} - f_{l,\text{flex}}^A = \frac{1}{2} D_{\text{flex}} = f_{l,\text{flex}}^B, \forall l\]

(3.3.4.3)

\[f_{l,\text{cap}}^A = f_{l,\text{cap}}^B = D_{\text{cap}} = f_{l,\text{cap}}^A = f_{l,\text{cap}}^B, \forall l\].

(3.3.4.4)
Case 5: excessive flexible demand

\[ D_{\text{flex}} \geq \frac{c^T-2c^O-2c^S}{2(n+1)c^O} - D_{\text{cap}} \geq 0 \]  

(3.3.5)

\[ \tau_{\text{comp}} = \frac{1}{4} (c^T - 2c^O + 2c^S) \]  

(3.3.5.1)

\[ f_{l,\text{flex,A}} = \frac{c^T-2c^O-2\tau_A}{2(n+1)c^O} - \frac{1}{2} D_{\text{cap}} = \frac{c^T-2c^O-2c^S}{4(n+1)c^O} - \frac{1}{2} D_{\text{cap}} < f_{l,\text{flex,A}}^{\text{eff}}, \forall l \]  

(3.3.5.2)

\[ f_{l,\text{flex,B}} = \frac{c^T-2c^O-2\tau_B}{2(n+1)c^O} - \frac{1}{2} D_{\text{cap}} = \frac{c^T-2c^O-2c^S}{4(n+1)c^O} - \frac{1}{2} D_{\text{cap}} < f_{l,\text{flex,B}}^{\text{eff}}, \forall l \]  

(3.3.5.3)

\[ f_{l,\text{cap,A}} = f_{l,\text{cap,B}} = D_{\text{cap}} = f_{l,\text{cap,A}}^{\text{eff}} = f_{l,\text{cap,B}}^{\text{eff}}, \forall l \].  

(3.3.5.4)

In the unique user equilibrium of an unregulated duopoly, flexible demand (from origin 1 to destination 4) will be split between both service providers if and only if it is sufficiently large in comparison to the captive demand, i.e. meets condition (3.3). If (3.3) does not hold, two cases are possible. For sufficiently low flexible demand, case 1, the service provider charges are extremely high such that only captive demand is served. Given a single service charge level, the service providers A and B each prefer to capture the profits on their own high value demand (the captive demand) rather than reduce the charge in order to capture the flexible demand. The charges decrease with growing demand as a function of the cost, \(c^T\), at which the customer is indifferent to using the route or foregoing the service. In this case, since the flexible demand is not completely served, the outcome is inefficient from the point of view of minimizing total social costs. The special case where \(D_{\text{cap}}\) equals zero is equivalent to the existing analysis in the literature without a network (Brueckner, 2002; Perakis and Sun, 2014). We see here that the network matters because the results of the model are qualitatively different. Only when \(D_{\text{cap}}\) is positive do we arrive at Threshold (3.3), below which the service providers set charges sufficiently high that the flexible demand will not be served.

In case 2 however, above a certain flexible demand threshold (specified in the Appendix), one of the service providers reduces their charges so that the entire flexible flow will use their service. The remaining service provider will charge as stated in (3.3.1.1). There is specialization between the two providers, which leads to efficient flows.

When condition (3.3) holds, service provider charges and customer flows in equilibrium are defined according to three cases. In case 3, service provider charges are lower, both serve the flexible demand, and given the symmetry of our simplified network, they each serve half the market. As in Theorem I with low demand, the charge increases with growing demand due to congestion. In case 4, both service providers continue to serve the entire flexible market but flows are sufficient such that the service provider charges decrease as flow increases. Both providers set the charge such that the customers are indifferent to being served or not. In case 5,
the congestion is such that not all flexible demand will be carried and the service provider charge remains constant.

As before, the system optimal solution flows for all cases are obtained by considering the charge dependent user equilibrium flow expressions presented in cases 3 to 5 (where case 3 is applicable also below threshold (3.3)) and by replacing the term \( n + 1 \) everywhere by the term \( 2n \) in order to fully internalize congestion costs. The efficient flows that minimize total social costs are then derived when setting \( \tau_A = \tau_B = C^S \). Efficiency implies serving all flexible demand in cases 1 through 4 and serving it only partially in case 5. Despite the partial internalization of congestion costs by the customers, the service provider charges in case 5 are too high and flows are inefficiently low. Consequently, the network creates relatively weak competitive effects across service providers.

**Theorem IV: Horizontal integration of parallel providers A and B**

\[
\begin{align*}
\tau_{A, B}^{\text{int}} &= \tau_{B, A}^{\text{int}} = \frac{1}{2} \left( C_T - 2C^O - (n + 1)C^G (D_{\text{cap}} + D_{\text{flex}}) \right).
\end{align*}
\] (3.4.4.1)

The unique user equilibrium under horizontal integration between two competing parallel service providers will lead to the same results as Theorem III except in cases 2 and 3 when the monopolist charges are higher than those of the duopoly case. In other words, as before, the monopolist sets the charge such that the customers are indifferent to being served or not. Furthermore, in case 2 an integrated service provider setting a single charge earns lower profits because it cannot replicate the two charge competitive outcome. At threshold (3.3.5), the charge remains constant and a decreasing percentage of the flexible traffic is served, again with flows too low from the point of view of minimizing overall social costs. Figure 3 illustrates the charge and profits of the service providers as the flexible demand increases in comparison to that of captive demand. The blue line represents the results when service providers A and B compete and the dotted red line represents the results for the merger.

![Figure 3: Charges and profits vs. demand ratio](image-url)
We see that for the lowest flexible demand, charges are equivalent for the monopolist and duopolist markets both of which serve only captive demand. In case 2, one service provider continues to set the monopolist charge and the other chooses a charge slightly lower than that of the horizontally integrated case, thus carrying all flexible demand. Above the demand threshold (3.3), the charges of the duopolists are lower than that of the monopolist, and these charges increase with growing congestion. In cases 4 and 5, the charges are the same with or without integration.

**Corollary III: Merger cost savings do not lead to lower charges**

Although the pressure to merge service providers may exist in order to encourage lower service costs due to economies of scale, the service costs of a single server do not impact the charge to the customers (except in case 5 where flexible demand is relatively very high and only served partially). Hence, lower costs will not lead to lower charges without the presence of regulation. Moreover, the merger may not improve the efficiency of the flows from the point of view of minimizing overall social costs.

Finally, in order to examine the impact of partial vertical integration, we assume that customer 1 and service provider A reach an agreement whereby a new, capacity increasing technology is adopted exclusively by both parties. We assume that the new technology increases the service provider’s fixed costs, however reduces variable costs for both the provider and the customer, such that \( C_A^S < C^S \) and \( C_{1A}^O < C^O \).

**Theorem V: Technology adoption by provider A and customer 1**

\[
\begin{align*}
\tau_A^{\text{int} A,1} &= C^S - 2 \frac{C_S - C_A^S}{3} + \frac{1}{3n} (C^O - C_{1A}^O) + \frac{n+1}{2n} C^G \left(2n(D_{\text{cap}} + D_{\text{flex}})\right) \\
\tau_B^{\text{int} A,1} &= C^S - \frac{C_S - C_A^S}{3} - \frac{1}{3n} (C^O - C_{1A}^O) + \frac{n+1}{2n} C^G \left(2n(D_{\text{cap}} + D_{\text{flex}})\right).
\end{align*}
\]

In the unique user equilibrium under duopoly competition in a parallel network with partial vertical integration in which service provider A and customer 1 adopt new technologies jointly, due to the service cost saving for A, the charges to both service providers decrease, with higher impact on the charge of A. Furthermore, due to the operating cost saving of customer 1, the charge of service provider A increases whereas service provider B’s charge decreases. Thus even without any service cost saving, the equilibrium charge (3.5.1) of A increases in comparison to equation (3.3.3.1) by a fraction of customer 1’s operating cost savings, which is one sixth when \( n = 2 \). The service provider purchases new technologies, which increase their fixed costs, but this may be offset by an increase in their charges. In order to compete, service provider B will
adjust their charges downwards accordingly. On the other hand, we should not forget that for as long as service provider charges are capped at their cost levels, no service provider will be interested in signing either horizontal or vertical agreements since cost recovery of the improved service is not possible.

IV. CASE STUDY: AIR TRAFFIC CONTROL IN EUROPE

In Europe, air traffic control (ATC) is provided by 30 national providers. They each have a national monopoly on their territory\(^3\). The current provision of ATC is inefficient. The FAA and Eurocontrol (Performance Review Commission, 2013) compared the US and European ATC systems and argued that the latter are more expensive by 34%. Most agree that the main reason for this cost difference is the large number of service providers in Europe, each procuring their own systems, mostly training their own staff, creating their own operating procedures, providing services in a small airspace and not adopting the most performant technologies. In this case study we analyze the effects of different regulatory and institutional interventions on the service provision equilibrium.

In the remainder of this section we first discuss briefly the literature, next describe the European airspace to be modelled. We then detail both types of ATC service providers and the five airlines analyzed. Finally, we present the set of scenarios tested, including a base-run which reproduces the outcome in 2011.

IV.1 Literature review on air traffic control modeling

Economic based research on the topic of ATC capacity includes Morrison and Winston (2008) and Winston (2013). Both papers acknowledge that the FAA has not used pricing instruments to address congestion issues in airspace, rather relied on capacity expansions. Zou and Hansen (2012) argue that given the substantial investments required to develop new technologies known as the NextGen system, a cost-benefit assessment based on equilibrium outcomes is of critical importance. Through the computation of supply-demand equilibrium, they show that classical cost-benefit analyses distort delay savings estimates and potentially demand estimates too. In Lulli and Odoni (2007), it is pointed out that air traffic flow management of en-route sectors in Europe is highly congested, particularly in the central and western sectors. The authors also demonstrate that issues of efficiency and equity in European airspace are far more complicated as compared to that of the US. Whilst in the US, there is one nationwide FAA which allocates resources across 21 air control centers and most airport tower services in addition to developing

\(^3\) More institutional detail can be found in Baumgartner and Finger (2014)
and adopting NextGen technologies, in Europe the fragmentation into 30 regional monopolies leads to a multi-agent problem.

Economic based research on the topic of ATC regulation to date analyzes individual providers and thus ignores some of the complications of the decentralized system. Castelli et al. (2013) argue that EU regulation\(^4\) removes the requirement that ATC service providers simply cover their full costs thus potentially generating a more commercial approach to the supply of such services. Accordingly, they develop a Stackelberg game in which a single ATC service provider sets a charge in order to maximize profit and subsequently individual flights are routed in order to minimize costs. The authors argue that there is sufficient overflight traffic in the European system, with choice as to airspace preference, that the single unit price is tempered by the interplay between those flights served by a single monopoly and those that may be served by competing ATC companies. Castelli et al. (2011) argue that under cases of congestion, a slot allocation system could be organized by a central planner such that a flight receiving an earlier slot is charged accordingly in order to compensate an alternative, delayed flight. This is one of the first attempts to consider an economic mechanism rather than an administrative approach for handling delays. Jovanović et al. (2014) develop a Stackelberg game with a single network planner and an airline with multiple flights, and argue that a congestion based charge with rebates would help to better balance demand in the European airspace.

We apply our network congestion game with multiple ATC providers and airline customers in order to ascertain whether these conclusions hold and under what conditions could the European ATC system be encouraged to adopt technologies that would increase capacity. As compared to atomistic customers, carriers with market power account for at least self-imposed congestion, which in turn impacts the ATC charges.

\(IV.1\) Network

The set of arcs is partitioned into air traffic control en-route sectors and the airspace above airports. Length of arcs is measured in km (airport arcs are of length 1km in order to count movements, i.e., arrivals/departures). The network analysed is depicted in Figure 4 and includes six ATC providers, represented by the coloured arcs, six major airports in each of the six regions, three regional airports and four additional nodes to aggregate flights to and from the region. Despite this being a clear simplification of reality, the network game should be sufficiently rich as to enable us to understand the two types of network effects described in Section III, namely

\(^4\) Regulation 1191/2010 of the European Union
asymmetric service collaboration and heterogeneous customers, and how the players will react to changes in institutional or regulatory rules.

Figure 4: European air traffic control network case study

IV.2 The ATC service providers

We focus on 6 ATC providers, including NATS (UK), LVNL (Netherlands), DFS (Germany), Belgocontrol (Belgium), DSNA (France), and AENA (Spain). The activities of the Maastricht Upper Airspace Control Centre (MUAC) has been added to the services provided in the individual countries, namely the Netherlands, Belgium and Germany, in order to reduce the number of players. In 2011, according to the ACE 2011 Benchmarking Report, these ATC providers were responsible for 48.9% of European traffic (in terms of flight hours controlled) and 52.3% of total en-route ATM/CNS costs. Eurocontrol's performance review unit also publishes the en-route ATFM delay minutes per ATC provider and their costs which are based on the Cook and Tanner study (2011). Out of the total European ATM system, 62.3% of the delay minutes were attributed to the ATC providers in this case study. Consequently, the total costs to the airlines flying in the relevant airspace as a result of these delays amounted to €933 million per year which mostly draws from additional fuel burn and crew costs. Real delay costs may be substantially higher were passenger loss and schedule delay to be considered within this analysis. The parameters applied in the network congestion model concerning the ATC providers were drawn from the ACE report and are summarized in tables 3 and 4. Staff and other operating costs constitute the variable costs whereas depreciation, capital and exceptional items were classified as fixed costs.

5 Published in 2013, https://www.eurocontrol.int/ACE/
Table 3: 2011 En-route ATC Service Provision Data

<table>
<thead>
<tr>
<th>Country</th>
<th>Revenues (000 €)</th>
<th>Variable Costs (000 €)</th>
<th>Fixed Costs (000 €)</th>
<th>Total Distance (km)</th>
<th>Average Charge per km (€)</th>
<th>Variable Cost per km (€²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NATS</td>
<td>651,366</td>
<td>368,015</td>
<td>153,001</td>
<td>707,474,135</td>
<td>0.921</td>
<td>0.520</td>
</tr>
<tr>
<td>LVNL</td>
<td>169,365</td>
<td>131,399</td>
<td>14,302</td>
<td>191,563,198</td>
<td>0.884</td>
<td>0.686</td>
</tr>
<tr>
<td>DFS</td>
<td>739,112</td>
<td>638,401</td>
<td>167,398</td>
<td>1,007,485,777</td>
<td>0.734</td>
<td>0.654</td>
</tr>
<tr>
<td>Belgocontrol</td>
<td>155,805</td>
<td>111,422</td>
<td>17,331</td>
<td>166,751,138</td>
<td>0.934</td>
<td>0.668</td>
</tr>
<tr>
<td>DSNA</td>
<td>1,167,138</td>
<td>804,653</td>
<td>113,876</td>
<td>1,463,618,011</td>
<td>0.797</td>
<td>0.550</td>
</tr>
<tr>
<td>Aena</td>
<td>794,710</td>
<td>498,756</td>
<td>135,599</td>
<td>859,175,623</td>
<td>0.925</td>
<td>0.581</td>
</tr>
</tbody>
</table>

The ATC terminal providers cover the nine airports included in Figure 3, however the data available from the ACE report is based on country level data as shown in Table 4. The fixed costs for countries in the case study were split based on the airports’ relevant proportions of activities.

Table 4: 2011 Terminal Air Traffic Control Data

<table>
<thead>
<tr>
<th>Country</th>
<th>Fixed Costs (000 €)</th>
<th>IFR Airport Movements</th>
<th>Income From Charges per Movement (€)</th>
<th>Variable Cost per Movement (€²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>9,863,000</td>
<td>1,746,362</td>
<td>115</td>
<td>87</td>
</tr>
<tr>
<td>Netherlands</td>
<td>5,313,000</td>
<td>485,525</td>
<td>113</td>
<td>99</td>
</tr>
<tr>
<td>Germany</td>
<td>41,208,000</td>
<td>2,059,372</td>
<td>101</td>
<td>86</td>
</tr>
<tr>
<td>Belgium</td>
<td>9,208,000</td>
<td>380,572</td>
<td>69</td>
<td>130</td>
</tr>
<tr>
<td>France</td>
<td>31,704,000</td>
<td>1,892,868</td>
<td>122</td>
<td>127</td>
</tr>
<tr>
<td>Spain</td>
<td>49,253,000</td>
<td>1,854,896</td>
<td>116</td>
<td>117</td>
</tr>
</tbody>
</table>

The data presented in tables 3 and 4 suggests that in certain countries some cross-subsidizing may occur where the service provider produces both en-route and airport terminal services, for example in Belgium. This is likely to impact the choice of investments to be implemented.

IV.3 The airlines as customers of the ATC service providers

When applying the general model to the case study, we note that the airline customers are limited players in this game because of the assumption that their schedule has already been set. As a result, the airlines are defined as cost minimizers rather than profit maximizers, which avoids the issue of modeling passengers. However, increases in congestion and/or air traffic control charges may encourage airlines to move to the off-peak, less congested times or to cancel flights. The reduced form model attempts to approximately capture this behavior in order to understand the market at the strategic level. Consequently, a day has been separated into two timeframes, and we include in the generalized cost function a revenue loss to airlines moving flights from the peak to off-peak. This formulation allows the correct balancing of the desire to avoid congestion and reduce costs yet meet consumer demand. The peak capacity has been limited according to the airport infrastructure. This model could account for airline and passenger preferences in greater detail, but this is not necessary in order to understand the
implications of intervention policies such as price-caps or changes in ownership form on the behavior of ATC providers.

Hundreds of airlines fly over European airspace providing both scheduled and charter services. For the sake of simplicity, we aggregate the airlines into three groups which best represent the structure of commercial aviation today. The groups include airline alliances, low cost carriers and non-aligned carriers. The aligned group is represented by three airlines: Lufthansa-Brussels (LH), British Airways-Iberia (BA) and Air France-KLM (AF), the main European airlines in the three airlines alliances that exist today. Each aligned airline is modelled with a two-hub system. LH utilizes Frankfurt and Brussels, BA utilizes London and Madrid whilst AF utilizes Paris and Amsterdam. For the purposes of this case, the low cost carrier group is represented by Easyjet (EJ) because the airline was ranked fourth in terms of seat capacity in Western Europe in 2013. Emirates airline was chosen as the representative carrier for the non-aligned carrier group (Rest). The Dubai based airline was ranked first among world airlines in terms of available seat kilometers in 2013 and Europe was their largest market by seat capacity.

We next describe our assumptions with respect to the airline related parameters when applying the model to this case study. First, the maximal demand $D_{o,d}$ for flights between each OD pair is set per airline based on their scheduled timetable in 2011, and an airline can choose to fly in the peak, to fly in the off peak or not to fly. Since ATC charges are price-capped, and represent approximately 5-8% of the airline’s total operating cost, we assume that demand elasticity with respect to ATC charges is relatively low. Therefore we set the cost of not flying, the outside option $C_{o,d}^T$ per flight, at 20 times the sum of the ATC charges in 2011 for the least costly flight path from origin $o$ to destination $d$. This is sufficiently high to ensure that no flights are cancelled in the base case scenario. Sensitivity analyses of the impact of this value are tested, as described in Section V.

The airline groups achieve different cost levels which are mostly a direct function of the level of service they provide, output, network, average stage length and the employment costs of the airlines' country of registration. The cost per available seat kilometer (CASK) in 2011 for BA, LH, AF, LC and Rest was 7.3, 8.8, 6.9, 5.0 and 5.2 euro cents, respectively, of which 85% are assumed to be operating costs with the remainder drawing from congestion and ATC charges. For purposes of simplification, we assume that all airlines are flying 150 seat, short-haul aircraft and estimate the operating cost per km $C_{o,t}^O$ for each airline accordingly.

Congestion impacts the cost categories to varying degrees. To be specific, the more indirect the flight path, the higher the fuel and staff operating costs for the airline. Based on
conversations with airline management, and in order to approximately fit the reported CASK in 2011, we assume that the congestion cost parameter $C_{ls}^G$ is 0.1 euro cent per flight per km en-route and 0.07 euro cent per movement above airports. Therefore, the total congestion cost increases in the square of frequencies according to our model. Indeed, the greater the delay in airspace, the higher the congestion costs for the airlines, which are frequently more substantial than the ATC service charges (Ball et al. 2010, Cook and Tanner, 2011). Congestion in air transport is caused in part by limited airport capacity, due to runway and terminal handling restrictions, and limited air traffic control capacity in the air. We assume that airport capacity is allocated efficiently across airlines by grandfathered slots. This better represents air traffic control congestion everywhere outside the US$^6$ where aircraft are served for the most part on a first come, first served basis which creates higher demand for air traffic control in the peak period.

There is little to no published information on the difference in airfares between peak and off-peak periods. As a first test, we set a revenue loss parameter $C_{la2}^R$ of 6 euros per km in the off-peak, which amounts to 50 euros per flight per OD ticket, for all airlines except the low cost carrier, which charges lower airfares hence receives 50% lower revenue losses (Swan and Adler, 2006). This ensures a preference to fly in the peak in the base case scenario. Sensitivity analyses of the impact of this value are also tested and described in Section V. Airlines are restricted on the airport arcs to a maximal flow in the peak of $K_{la1}$, which is taken to be 80% of the airline’s total movements in 2011 at the relevant airport. This represents slot allocation regulation at the large airports.

Finally, direct ATC user charges in 2011 add an additional 5 to 8% to the airline operating costs$^7$. It is standard practice for airline dispatchers to choose the flight path approximately four hours prior to the flight by balancing all the costs and accounting for potential weather disruptions. The flight path is then filed with Eurocontrol which, acting as the network manager, passes on the information to the relevant ATC providers and bills the airlines accordingly.

IV.4 Scenarios

In order to analyse the potential impact of changes in institutional or regulatory arrangements, we study four groups of scenarios. The first group is referred to as the base case, in which we reproduce the 2011 equilibria outcome for the case study depicted in Figure 4. In scenario group 2, we highlight the potential impact of horizontal integration of neighboring service providers.

$^6$ In the US currently only three airports are slot controlled (Adler and Yazhensky, 2018).

$^7$ Normally, the shorter the average stage length, the relatively higher the percentage of air traffic control charges as a function of a carrier’s direct operating costs (Swan and Adler, 2006).
We assume that there will be no savings in labor costs or reduction in air control centers due to the power of the labor unions and the politics of sovereign protection but savings of up to 30% are possible in the sum of the fixed costs due to joint purchasing power and standardization of processes. In addition, we test the assumption that variable cost savings are possible due to a one-third reduction in the joint support staff as suggested in the 2012 Eurocontrol Masterplan\(^8\). In scenario group 3 we analyze the potential impact of technology on the equilibrium outcomes by modeling the expected costs and benefits of new technologies to both the ATC providers and the airlines. We note that all parameters in these scenarios draw from the Eurocontrol Masterplan. In scenario group 4, under vertical integration, an ATC provider and its relevant hub airline are assumed to adopt new technology and via the best-equipped best-served scheduling rule are able to achieve the benefits of the technology locally.

Within each group of scenarios, we analyze four sub-cases including the system optimum with cost recovery constraint, user equilibrium cost recovery constraint, and user equilibrium profit maximization with and without upper bounds on charges. We recall that system optimal sub-cases assume that a central planner organizes flight paths by minimizing the costs of all airlines. User equilibrium refers to sub-cases in which each airline chooses flight paths taking as given the flows of other airlines. Under cost recovery constraints, we assume that the ATC charges are equal to the current price-caps. Current price-caps are set such that the ATC providers earn a small level of profit, which is expected to be invested in capacity, and ensure that the providers can continue should there be exogenous shocks in the future.

We note that all results presented here were also analyzed using Eurocontrol’s demand forecast for the year 2020 and 2030. Because the results were qualitatively the same, and for reasons of brevity, we do not show most of these results here. For those interested in greater detail, additional results can be found in Adler et al. (2015)\(^9\).

\textbf{IV.5 Numerical solution method}

In the numerical analysis used for finding a sub-game perfect equilibrium solution, the second stage quadratic, convex problem is solved using CPLEX (version 12-6-2), and the first stage is solved using a grid search algorithm. From a starting solution, we solve for the first service provider and then all customers in an iterative process until no customer finds it worthwhile to deviate from the current values of their decision variables. The algorithm then moves to the second provider and repeats the process. An entire cycle is completed when all

\(^8\) https://www.atmmasterplan.eu/

service providers have been analyzed. The process continues until convergence, where an entire cycle is completed with no service provider or customer changing their decision variables (which occurred in reasonable computation time, typically within 10 cycles). To ensure a unique solution, we repeated the entire process from several starting solutions.

V. CASE STUDY RESULTS

In this section, we first discuss the base run (scenario 1), which replicates the results of 2011. We then discuss horizontal integration through the concept of functional airspace blocks (scenario 2) and the adoption of two levels of technology provision (scenario 3). Finally, we analyze the regional forerunner concept in which a geographically aligned airline, ATC provider and airports reach agreement to adopt new technology jointly (scenario 4).

Scenario Group 1: Base Case

In the base run scenario, the solution closest to the 2011 real world outcome is the user equilibrium with cost recovery constraint. As shown in the Scenario 1 Table, the ATC provider’s output (km served) and their revenues and profits from the airlines covering the 6 countries included in the analysis represent a close approximation to the outcome for 2011, as detailed in tables 3 and 4. Additionally, each airline’s schedule (maximal demand) was fully met, with around 80% of the flights occurring during the peak window, and the costs per available seat kilometer (CASK) per airline also matched those that occurred in 2011. We note that all service providers covered their costs in 2011 except for DFS. However, in the ATC terminal sector, charges do not fully recover their costs for half the airports hence the losses are either covered through alternative airport revenues (for example on the commercial side) or from the provider’s profits, depending on the institutional arrangements that differ across countries.

In the system optimum with cost recovery constraint, Eurocontrol chooses the airline flight paths in order to minimize overall airline costs and manages to save a moderate 0.08% which is due to the relatively low congestion levels suffered in 2011 in the en-route sectors following the financial crisis of 2008. This is in line with Theorem 1 (case 1) of Section III, which predicts that with sufficiently low demand, the user equilibrium flows are equal to the efficient system optimal flows. However, whilst the three aligned carriers, BA, LH and AF achieve lower costs, the low cost carriers and international carriers are worse off. The flows change slightly such that more flights are funneled through Belgian and German airspace at the expense of the French and Spanish airspace, resulting in overall increased profits in the ATC sector. In summation, although airlines achieve lower costs overall throughout the network, it is unlikely that they
would prefer to leave the choice of flight paths to a central planner because the system optimal approach may result in some airlines gaining at the expense of others.

### Scenario 1 Table: User equilibrium cost recovery outcome

<table>
<thead>
<tr>
<th>Airline</th>
<th>CASK</th>
<th>Annual Costs (000 €)</th>
<th>Revenue Losses (000 €)</th>
<th>ATC en-route Prices per km</th>
<th>Annual Revenues (000 €)</th>
<th>Annual Profits (000 €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA</td>
<td>0.073</td>
<td>6,884,000</td>
<td>669,168</td>
<td>NATS 0.921</td>
<td>786,052</td>
<td>189,243</td>
</tr>
<tr>
<td>LH</td>
<td>0.088</td>
<td>7,614,643</td>
<td>631,977</td>
<td>LVNL 0.884</td>
<td>263,163</td>
<td>44,642</td>
</tr>
<tr>
<td>AF</td>
<td>0.073</td>
<td>4,299,149</td>
<td>424,181</td>
<td>DFS 0.734</td>
<td>516,551</td>
<td>-111,098</td>
</tr>
<tr>
<td>LC</td>
<td>0.054</td>
<td>11,364,837</td>
<td>440,049</td>
<td>Belgocontrol 0.934</td>
<td>128,523</td>
<td>19,272</td>
</tr>
<tr>
<td>Rest</td>
<td>0.053</td>
<td>7,801,272</td>
<td>271,849</td>
<td>DSNA 0.797</td>
<td>1,225,682</td>
<td>265,978</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>AENA 0.925</td>
<td>417,589</td>
<td>19,699</td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td>37,963,901</td>
<td>2,437,224</td>
<td></td>
<td>3,337,560</td>
<td>427,735</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ATC terminal</th>
<th>Prices per movement</th>
<th>Annual Revenues (000 €)</th>
<th>Annual Profits (000 €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHR</td>
<td>115</td>
<td>76,898</td>
<td>13,043</td>
</tr>
<tr>
<td>AMS</td>
<td>113</td>
<td>53,783</td>
<td>2,064</td>
</tr>
<tr>
<td>FRA</td>
<td>101</td>
<td>199,440</td>
<td>19,872</td>
</tr>
<tr>
<td>BRU</td>
<td>69</td>
<td>37,324</td>
<td>-38,653</td>
</tr>
<tr>
<td>CDG</td>
<td>122</td>
<td>172,509</td>
<td>-19,397</td>
</tr>
<tr>
<td>MAD</td>
<td>116</td>
<td>79,599</td>
<td>-12,088</td>
</tr>
<tr>
<td>MAN</td>
<td>115</td>
<td>97,718</td>
<td>22,898</td>
</tr>
<tr>
<td>BER</td>
<td>101</td>
<td>36,054</td>
<td>493</td>
</tr>
<tr>
<td>PMI</td>
<td>116</td>
<td>30,146</td>
<td>-5,044</td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td>783,472</td>
<td>-16,812</td>
</tr>
</tbody>
</table>

In the user equilibrium price-cap approach, charges were limited to up to 20% higher values than those charged in 2011. Although the ATC providers could opt to charge less and acquire a larger share of en-route traffic, in the sub-game perfect equilibrium they charge according to their upper limit, leading to profits approximately three times higher than those achieved in 2011. These results are in line with Corollary I of Section III. Therefore, the ATC providers could collect additional revenues to fund new technology were this deemed necessary without impacting demand dramatically. The results show that the airlines would continue to fly despite a slight increase in their cost per available seat km. The other pricing alternative tested was to require the providers to set two separate charges, one for the peak and one for the off-peak, so as to internalize congestion but keeping revenue neutrality from the ATC perspective. We note that the providers will only set two separate charges were the government(s) to require them to do so by, for example, setting two individual price-caps. Under a setting in which prices could increase by up to 20% in the peak but were reduced to at most 80% in the off-peak, airline costs, including charges, increase by a range of 1 to 2%. We also note that the slot allocations prevent airlines from increasing movements in the peak and the higher peak charge does not induce airlines to move to the off-peak. In other words, the 20% higher ATC charges are counter
balanced by the lower airfares in the off-peak, leading airlines to continue to serve the peak market where possible.

In the user equilibrium profit maximization without price-caps, the ATC providers are free to set charges such that they maximize their profits. The results show that the charges would increase tenfold and profits accordingly. Interestingly, this is the only case in which the ATC providers distinguish between peak and off-peak pricing endogenously which is in accordance with Theorem I where the ATC charges of a duopoly partly internalize the external congestion costs. However, the airlines’ costs per available seat kilometer double despite the fact that 50 to 60% of demand is pushed to the off-peak and LH, the most costly airline, reduces its schedule by one half. Consequently, we arrive at the conclusion that there is insufficient competition across ATC providers in order to justify the removal of price regulation as has occurred in the airline industry globally and in the airport industry in the UK and Australia.

For all cases, peak demand is limited to 80% of the flows (constraint (3)) based on an analysis of data drawn from the CODA database\(^\text{10}\). The airlines may also choose to fly off-peak, which would induce revenue losses from lower airfares. For sensitivity analysis, we tested all cases with 50% lower revenue losses with no notable change in airline flight patterns. The results suggest that all airlines prefer to fly in the peak given current ATC charge levels because the cost savings from flying off-peak are insufficient to counter balance the likely revenue losses. With the available data we were able to model a simple peak/off peak differentiation of ATC charges however we note that air traffic congestion is probably more of the bottleneck congestion type\(^\text{11}\).

Finally, we tested the effect of reducing the outside option cost \(C_{od}^T\) by 25%. The results show that efficiency in terms of minimizing total social costs, i.e. the system optimal outcome when the service provider charge equals the service cost, implies cancelling some of the flights. Consequently, due to ATC charges that are too low, the user equilibria with cost recovery and with price-caps generate very high congestion as compared to the efficient flows. In contrast, under user equilibrium without price-caps, the high charges generate too low congestion as compared to the efficient flows. This is in line with Theorems I and III, which show that competition may fail to promote efficiency in the presence of the two types of network effects.

\(^{10}\) https://www.eurocontrol.int/articles/coda-publications

\(^{11}\) In a pure bottleneck, where all users want to use the facility at the same moment and have the same values of time and delay, pricing by the minute is much more powerful than simple peak/off peak price differentiation. In theory, this would convert all queuing into additional revenues, while the costs for the airlines would not increase as they would merely see their queuing costs converted to ATC charges. If this is a correct representation of airspace congestion, fine-tuned pricing of capacity would become more interesting than in our scenario.
described in Section III, namely asymmetric service collaboration and heterogeneous customers. In summation, the network creates relatively weak competitive effects across service providers.

**Scenario Group 2: Horizontal integration of service providers**

In 2004, the European Union passed a law creating 9 functional airspace blocks (FABs) through cross-border merges that were meant to be in place by 2012. Scenario group 2 analyses the possibility that providers’ horizontal integration may lead to technology adoption and a reduction in costs. We assume that there will be no changes in labor costs and any cost savings will draw from the ability to purchase equipment jointly, resulting in a 30% saving in fixed costs through co-operation. An important question to be answered is the merger charge level as compared to the individual providers. Setting a single rate per unit operation could force harmonization, and lead to the use of more direct flight paths. This was the view of most ATC regulators when conceiving the idea of FABs. We set the price-cap on charges per km to the weighted average of the 2011 prices according to the level of activity of each provider. According to Corollary III of Section III, in the user equilibrium price-cap outcome, the ATC provider will have no incentives to decrease charges when costs decrease hence they will continue to charge according to the price-caps.

In the case of scenario 2a, we assume that Belgocontrol and DSNA cooperate and the weighted average price becomes 0.811 cents per km, which increases the cost of French airspace whereas Belgian airspace becomes cheaper to the airlines. As a result, most airlines are worse off and only the low cost carrier manages to reduce their costs by rebalancing flight paths. In case 2b, we assume that the Dutch and German providers cooperate, resulting in a weighted average charge of 0.758 which increases the costs of flying through German airspace but substantially reduces the price to fly over the Netherlands. The result is an increase in costs for Lufthansa and for the low cost carriers but lower costs for the other carriers. Finally, in case 2c we assume that two large service providers cooperate, namely DFS and DSNA, with a weighted average charge of 0.771, which increases costs in Germany and lowers costs in France. As a result, all airlines are worse off with their costs increasing by 0.17% to 0.58%. Consequently, unless some of the cost savings are passed on to the airlines through lower or differentiated ATC charges, at least one or more airlines are worse off as a result of such cooperation, which may explain why the airline industry has not pushed harder for the implementation of the single European skies approach. This is an illustration of Corollary III, and this result is also in line with the findings of Castelli et al. (2005).
Scenario 2 Table: Horizontal integration

<table>
<thead>
<tr>
<th>Airlines</th>
<th>Base Case</th>
<th>Case 2a</th>
<th>Case 2b</th>
<th>Case 2c</th>
<th>Case 2d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Annual Costs (000 €)</td>
<td>Annual Costs (000 €)</td>
<td>% Change</td>
<td>Annual Costs (000 €)</td>
<td>% Change</td>
</tr>
<tr>
<td>BA</td>
<td>6,884,000</td>
<td>6,884,779</td>
<td>0.01%</td>
<td>6,882,309</td>
<td>-0.02%</td>
</tr>
<tr>
<td>LH</td>
<td>7,614,643</td>
<td>7,617,811</td>
<td>0.04%</td>
<td>7,616,839</td>
<td>0.03%</td>
</tr>
<tr>
<td>AF</td>
<td>4,299,149</td>
<td>4,301,215</td>
<td>0.05%</td>
<td>4,294,085</td>
<td>-0.12%</td>
</tr>
<tr>
<td>LC</td>
<td>11,364,837</td>
<td>11,359,781</td>
<td>-0.04%</td>
<td>11,370,129</td>
<td>0.05%</td>
</tr>
<tr>
<td>Rest</td>
<td>7,801,272</td>
<td>7,804,871</td>
<td>0.05%</td>
<td>7,779,867</td>
<td>-0.27%</td>
</tr>
<tr>
<td>Total/Avg</td>
<td>37,963,901</td>
<td>37,968,457</td>
<td>0.01%</td>
<td>37,943,229</td>
<td>-0.05%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ATC</th>
<th>Charge</th>
<th>Annual Revenues (00 €)</th>
<th>Annual Profit</th>
<th>ATC</th>
<th>Charge</th>
<th>Annual Revenues (000 €)</th>
<th>% Change</th>
<th>Annual Profits (000 €)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgocontrol</td>
<td>0.934</td>
<td>128,523</td>
<td>19,272</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DSNA France</td>
<td>0.797</td>
<td>1,225,682</td>
<td>265,978</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1,354,205</td>
<td>285,250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LVNL</td>
<td>0.884</td>
<td>263,163</td>
<td>44,642</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFS Germany</td>
<td>0.734</td>
<td>516,551</td>
<td>-111,098</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>779,714</td>
<td>-66,456</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Case 2c</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DSNA France</td>
<td>0.797</td>
<td>1,225,682</td>
<td>265,978</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFS Germany</td>
<td>0.734</td>
<td>516,551</td>
<td>-111,098</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1,742,233</td>
<td>154,880</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Horizontal integration is only likely to occur if the costs to the merged ATC provider are reduced sufficiently that the savings outweigh the reduction in revenues, which would require a minimum reduction in fixed costs of 40% in this case study. Alternatively, FABs should be permitted to differentiate charges on flight legs according to the relevant cost base. This would require a Boiteux-Ramsey mark-up on top of the marginal service costs such that providers’ revenues cover their total costs. This merits further exploration but would be a revolution in an industry where average cost pricing is the rule and cooperation among service providers has proven difficult.

Finally, when reducing again the outside option cost $C_{od}^T$ by 25%, the charge set by the horizontally integrated provider under user equilibrium without price-caps becomes lower than before the integration; consequently congestion is higher and closer to the efficient flows. This is in line with Corollary II, which shows that in the presence of network effects generated by asymmetric service collaboration, integrated collaborating service providers are able to coordinate charges and improve efficiency.
Scenario Group 3: Technology adoption

We analyze the potential impact of technology implementation based on two technology packages: the pilot common project (PCP) and the first step of SESAR as defined in the 2012 Eurocontrol Masterplan. The PCP consists of technology adoption approximately equivalent to 10% of the full Step 1 process\textsuperscript{12}. We note that all parameters in these scenarios draw from the Eurocontrol Masterplan. The PCP is expected to cost approximately €2.5 billion of which the service providers are expected to cover 65%, the airlines 16% and the airports the remainder. Congestion en-route is estimated to be reduced by 8.7% and the operational costs to the airlines drop by a relatively minor 0.633%, after accounting for the trade-off between the costs of the PCP and the savings from more direct flights which reduce fuel usage. The providers are expected to achieve a reduction in variable operating costs due to improved productivity per ATCO of 8.4% but fixed costs increase by 22% due to the investment in PCP technology. The Masterplan estimates assume that the providers will not change their charge levels.

The results are presented in the Scenario 3 Table and show that the overall savings to the airlines outweigh the investment costs and all airlines are slightly better off, with average cost savings of less than 1%. Most providers are better off, in particular the smaller providers, but AENA is worse off hence would be unlikely to willingly participate. Based on a sensitivity analysis, allowing the providers to increase their charges by 10% would incentivize participation in the PCP such that the airlines and providers all gain from this effort.

Step 1 of SESAR is expected to cost approximately €30 billion by 2030, of which the providers are expected to cover 16% and the airlines 50% according to the Masterplan. We assume that congestion en-route is reduced by approximately 27% and the operational costs to the airlines increase by a relatively small 0.1%, after accounting for the technology investments less the savings from reductions in fuel usage. The providers are expected to achieve a reduction in variable operating costs due to improved productivity per ATCO of 8.4% but fixed costs increase by 53% due to the estimated technology investments. The Eurocontrol Masterplan also requires the providers to reduce their charge levels to the airlines by 6.1%. The user equilibrium price-cap results show that the airlines’ costs will decrease by approximately 2.5% overall, hence the airlines should be willing to invest. The advantages of the technology and procedure adoption are likely to be slightly lower as demand increases, given an estimated increase in demand of 38.7% from 2011 to 2030. This suggests that the reduction in congestion afforded by the new technology is necessary if additional demand is to be accommodated.

\textsuperscript{12} We note that Steps 2 and 3 of the 2012 Eurocontrol Masterplan, leading to trajectory based ATC, were defined but not expected to be in place before 2030.
The ATC service providers, whether en-route or terminal, are all worse off after investing in SESAR step 1 projects, although this would be somewhat tempered were demand to increase as expected.

**Scenario 3 Table: Adopting Single European Sky Technology**

<table>
<thead>
<tr>
<th>Airlines</th>
<th>CASK Base</th>
<th>Annual Costs (000 €)</th>
<th>% Change</th>
<th>SESAR Step 1</th>
<th>% Change</th>
<th>Annual Costs (000 €)</th>
<th>% Change</th>
<th>SESAR Step 1</th>
<th>% Change</th>
</tr>
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<td>BA</td>
<td>0.073</td>
<td>6,884,000</td>
<td>-6.64%</td>
<td>6,749,438</td>
<td>-1.95%</td>
<td>9,569,588</td>
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<td>10,546,679</td>
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<td>4,157,722</td>
<td>-3.29%</td>
<td>5,945,737</td>
<td>-2.41%</td>
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<td>LC</td>
<td>0.054</td>
<td>11,364,837</td>
<td>-0.55%</td>
<td>11,011,597</td>
<td>-3.11%</td>
<td>15,870,438</td>
<td>-2.60%</td>
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</tr>
<tr>
<td>Rest</td>
<td>0.053</td>
<td>7,801,272</td>
<td>-0.89%</td>
<td>7,627,594</td>
<td>-2.23%</td>
<td>10,910,281</td>
<td>-1.89%</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>37,963,901</td>
<td>-0.88%</td>
<td>36,980,070</td>
<td>-2.59%</td>
<td>52,842,723</td>
<td>-4.60%</td>
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<table>
<thead>
<tr>
<th>ATC en-route</th>
<th>Price</th>
<th>Annual Profits (000 €)</th>
<th>% Change</th>
<th>SESAR Step 1</th>
<th>% Change</th>
<th>Annual Profits (000 €)</th>
<th>% Change</th>
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<td>0.921</td>
<td>189,243</td>
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<tr>
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<td>44,642</td>
<td>31%</td>
<td>36,590</td>
<td>-18%</td>
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<td>59,771</td>
</tr>
<tr>
<td>DFS</td>
<td>0.734</td>
<td>-111,098</td>
<td>2%</td>
<td>-211,205</td>
<td>-90%</td>
<td>-89,375</td>
<td>-187,390</td>
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<tr>
<td>Belgocontrol</td>
<td>0.934</td>
<td>19,272</td>
<td>61%</td>
<td>8,061</td>
<td>-58%</td>
<td>33,366</td>
<td>22,110</td>
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<tr>
<td>DSNA</td>
<td>0.797</td>
<td>265,978</td>
<td>16%</td>
<td>188,928</td>
<td>-29%</td>
<td>413,127</td>
<td>321,401</td>
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<tr>
<td>AENA</td>
<td>0.925</td>
<td>19,699</td>
<td>-39%</td>
<td>-70,355</td>
<td>-45%</td>
<td>79,838</td>
<td>-18,770</td>
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<tr>
<td><strong>Total</strong></td>
<td></td>
<td>427,736</td>
<td>15%</td>
<td>32,672</td>
<td>-92%</td>
<td>825,801</td>
<td>405,857</td>
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<table>
<thead>
<tr>
<th>ATC terminal</th>
<th>Price</th>
<th>Annual Profits (000 €)</th>
<th>% Change</th>
<th>SESAR Step 1</th>
<th>% Change</th>
<th>Annual Profits (000 €)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHR</td>
<td>115</td>
<td>13,043</td>
<td>-8%</td>
<td>-20,671</td>
<td>-258%</td>
<td>20,258</td>
<td>-13,380</td>
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<tr>
<td>AMS</td>
<td>113</td>
<td>2,064</td>
<td>-40%</td>
<td>-24,714</td>
<td>-129%</td>
<td>4,609</td>
<td>-21,911</td>
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<tr>
<td>FRA</td>
<td>101</td>
<td>19,872</td>
<td>21%</td>
<td>-36,225</td>
<td>-282%</td>
<td>31,293</td>
<td>-24,939</td>
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<tr>
<td>BRU</td>
<td>69</td>
<td>-38,653</td>
<td>0%</td>
<td>-68,789</td>
<td>-78%</td>
<td>-51,388</td>
<td>-80,123</td>
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<tr>
<td>CDG</td>
<td>122</td>
<td>-19,397</td>
<td>12%</td>
<td>-88,428</td>
<td>-356%</td>
<td>-22,127</td>
<td>-89,396</td>
</tr>
<tr>
<td>MAD</td>
<td>116</td>
<td>-12,088</td>
<td>-42%</td>
<td>-78,270</td>
<td>-548%</td>
<td>-12,354</td>
<td>-77,907</td>
</tr>
<tr>
<td>MAN</td>
<td>115</td>
<td>22,898</td>
<td>23%</td>
<td>17,810</td>
<td>-22%</td>
<td>32,117</td>
<td>27,125</td>
</tr>
<tr>
<td>BER</td>
<td>101</td>
<td>493</td>
<td>-503%</td>
<td>-28,154</td>
<td>-581%</td>
<td>2,573</td>
<td>-25,926</td>
</tr>
<tr>
<td>PMI</td>
<td>116</td>
<td>-5,044</td>
<td>-48%</td>
<td>-32,889</td>
<td>552%</td>
<td>-5,145</td>
<td>-32,713</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>-16,812</td>
<td>-1%</td>
<td>-360,330</td>
<td>-164%</td>
<td>-339,170</td>
<td>536%</td>
</tr>
</tbody>
</table>

A sensitivity analysis suggests that were the providers permitted to increase their charges by an upper limit of 22%, both the airlines and the providers would be in a position to gain from the new technologies, although the impact on the airlines would now be rather marginal. This increase could be justified as ATC would be providing an improved service that reduces costs for the airlines. This means relaxing the price-caps is justified but the price-caps cannot be abolished as long as there is insufficient competition as suggested in Corollary I of section III.
Scenario Group 4: Regional forerunner

In scenario group 4, we test whether the vertical cooperation between a service provider, a local airline and an airport enables the adoption of the PCP program in a “regional forerunner” approach. We assume that the provider invests in the PCP technology and achieves higher levels of output per controller and that the participating airline achieves slightly lower operating costs and congestion levels, but only on the flight paths associated with the relevant airspace. A useful example of this type of cooperation would be FRAMaK, a Free Route Airspace Project run by a consortium of airspace users and providers (MUAC, the Karlsruhe Upper Area Control Centre and Lufthansa). 298 new direct routes were implemented in 2012, increasing the number of direct, flight-planable, cross border routes in the area to a total of 656. The development of cross border routes by FRAMaK created an advantage for Lufthansa, which is the largest airspace user in the Maastricht-Karlsruhe area, although all airlines can use the same direct routes and enjoy the benefits. This has led to additional user preferred, cross border routes, under pressure from European airlines and Eurocontrol. By 2014, at least 16 of the 64 European ACCs implemented various new Free Route Operations and savings have been estimated in the range of 150,000 tons of CO₂ equivalent to 37 million Euros\(^{13}\).

In scenario 5a we analyze a potential German regional forerunner such that DFS, LH, FRA and a secondary German airport cooperate. In scenario 5b we analyse a potential French regional forerunner with DSNA, AF and CDG cooperating and in scenario 5c we analyze a Spanish regional forerunner such that BA-Iberia, AENA, Madrid and a secondary Spanish airport cooperate. From the airline perspective, the Scenario 4 Table shows that in the user equilibrium price-cap outcome, all airline carriers should be willing to cooperate as their costs are expected to decrease in the region of 1 to 2%. Indeed, the incentive is likely to be underestimated because through the best-equipped best-served rule, which reduces congestion for the relevant airline, the airline’s market share is likely to increase which is not accounted for within the current modeling approach. DFS and DSNA are also likely to enjoy incentives from such cooperation, with DFS gaining 1.7% higher profits and DSNA gaining a 17% advantage. Indeed the AF-DSNA-CDG vertical integration would appear to be particularly positive. These results are in line with Theorem V of section III. For the German co-operation to occur, the smaller airports would need to be compensated for their investments. However, the Spanish regional forerunner is less likely since the ATC providers, both en-route and terminal, are likely to lose from such cooperation.

\(^{13}\) Safety of Air Navigation, Eurocontrol, Belgium, July 8, 2013.
### Scenario 4 Table: Vertical integration

<table>
<thead>
<tr>
<th>Airlines</th>
<th>CASK</th>
<th>Base Case</th>
<th>Annual Costs (000 €)</th>
<th>Case 5a</th>
<th>Case 5b</th>
<th>Case 5c</th>
</tr>
</thead>
<tbody>
<tr>
<td>LH</td>
<td>0.088</td>
<td>7,614,643</td>
<td>7,531,803</td>
<td>-1.09%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AF</td>
<td>0.073</td>
<td>4,299,149</td>
<td>4,225,217</td>
<td>-1.72%</td>
<td></td>
<td>6,833,164</td>
</tr>
<tr>
<td>BA</td>
<td>0.073</td>
<td>6,884,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ATC en-route</th>
<th>Base Case</th>
<th>Annual Profits (000 €)</th>
<th>Case 5a</th>
<th>Case 5b</th>
<th>Case 5c</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFS</td>
<td>-111,098</td>
<td>-109,251</td>
<td>1.66%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DSNA</td>
<td>265,978</td>
<td></td>
<td></td>
<td>311,927</td>
<td>17.28%</td>
</tr>
<tr>
<td>AENA</td>
<td>19,699</td>
<td></td>
<td></td>
<td></td>
<td>11,878</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ATC terminal</th>
<th>Base Case</th>
<th>Annual Profits (000 €)</th>
<th>Case 5a</th>
<th>Case 5b</th>
<th>Case 5c</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRA</td>
<td>19,872</td>
<td>23,999</td>
<td>20.77%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BER</td>
<td>493</td>
<td>-1,985</td>
<td>-502.67%</td>
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</tr>
<tr>
<td>CDG</td>
<td>-19,397</td>
<td>-17,132</td>
<td>11.67%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAD</td>
<td>-12,088</td>
<td>-17,202</td>
<td>-42.31%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMI</td>
<td>-5,044</td>
<td>-7,465</td>
<td>-48.00%</td>
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</tr>
</tbody>
</table>

### Summary and conclusions from the case study

An initial lesson learnt is that there is insufficient competition across flight paths in different ATC regions to permit the removal of price regulation since most zones demonstrate strong spatial monopoly power. Air traffic control charges could increase by a magnitude of ten beyond current prices were price regulation to be dropped. Horizontal integration improves efficiency when congestion is high. In the airport industry, the UK removed price regulation from all but one of their airports, arguing that there is sufficient competition for catchment areas and across hubs. This could occur in the ATC sector if and only if there are sufficient alternative flight paths between origin and destination. Consequently, ATC competition is only likely to arise when ATC providers are in a position to compete for services over the same set of flight paths as a function of new trajectory-based technologies.

Second, although there is an obvious gain from merging operations of neighboring service zones, this horizontal integration is unlikely to happen as long as the current practice of standard service rates based on average costs is applied. The current price-cap system, combined with the incomplete financial integration, will probably imply that one of the service units is likely to lose. Furthermore, the cost of standardizing equipment in the shorter term will likely require subsidies or higher prices, which is in direct opposition to current price-cap policy.

Third, there are important cost-efficiency gains if new technologies and more standardized equipment are introduced. This will mainly benefit the airlines that will receive improved
service. However, it is the ATC providers that are required to finance the additional equipment costs for the most part. Consequently, there are almost no incentives to introduce these new technologies as long as the service providers are bound to the current price-cap policies.

Fourth, a regional forerunner approach, where a large airline combines equipment efforts with its major hub airport and en-route ATC supplier in order to improve the efficiency of its operations, could benefit both the ATC and the local airline which would trigger competition among major hubs and airlines. It would appear that regional forerunners involving a service provider and their largest airline customer may be more successful in achieving the ultimate goal of a single European sky than a top-down regulated approach.

VI. CONCLUSIONS

In this research we analyze a congested network served by providers with monopolistic control over a specific set of arcs. The customers of the service are also non-atomistic hence are likely to internalize at least a part of the congestion in the network. This modeling framework represents a first attempt at applying game theoretic principles to analyze a complex and congested market with multiple actors in each of the two echelons of the supply chain.

The model is complex because it involves networks with a double oligopolistic structure. Nevertheless, we are able to draw several theoretical conclusions that depart from known results due to the presence of network effects. Horizontal integration may improve efficiency, and may even lower prices as compared to the competitive outcome. The adoption of new technologies may cut costs but sufficient incentives to invest in new technologies only exist if price-cap regulation is relaxed. The latter is difficult in a sector where sovereignty of the network creates market power and where demand is rather inelastic because only a small share of the market is subject to competition. The results of the model suggest that although the largest benefits are potentially attainable only through a central service provider, this ignores the problem of incentives that exist in such a public monopoly supply chain. Consequently, alternative market design scenarios should be considered such as partial vertical integration, which creates the benefit of avoiding double marginalization, together with an appropriate regulatory policy.

Through the development of a case study of Western Europe, this research has shown that international transport operations can be seriously hampered by local service monopolies that have no incentives to adopt better technologies. These monopolies are particularly strong in scheduled services like air and rail that are mainly controlled by public agencies. Privatization of monopolies can only improve overall efficiency if there is a smart regulatory system in place that controls prices and capacities.
ACKNOWLEDGMENT

This research was part of the result of a WP-E Research and Long Term Innovation project entitled ACCHANGE. The authors would like to thank the consortium partners Transport & Mobility Leuven (specifically Thomas Blondiau and Eef Delhaye), CORE-INVEST and Moving Dot for their insights and Hadar Israeli, Avigail Lithwick and Oren Petraru for their research assistance. We would also like to thank participants at the following conferences for constructive criticism: ATRS (2014), INFORMS (2014), OPTION (2015), the 3rd European Aviation Conference and the 11th USA/Europe ATM Seminar.

REFERENCES
A Appendix

We provide the proofs of Theorems I through V. Since these results do not involve price-cap regulation, we may assume throughout that the price-cap is sufficiently high so that the corresponding constraint may be ignored. For Theorems I and II we also assume that the outside option cost, $C^T$, is sufficiently high so that all demand, $D$, will choose to be served when the service provider charges are sufficiently low, specifically

$$C^T \geq 3(C^O + C^S).$$

Proof of Theorem I. The equilibrium is constructed using a folding back procedure. Under the simplified network, in the second stage of the game customer $l$ takes as given the choices of other customers and solves the problem

$$\min \left( C^O + C^G[f_{l,A} + f_{l,B} + \sum_{l' \neq l} (f_{l',A} + f_{l',B}) + \tau_C] (f_{l,A} + f_{l,B}) \right)$$

$$+ 2 \left( C^O + C^G[f_{l,A} + \sum_{l' \neq l} f_{l',A} + \tau_A] f_{l,A} + 2 \left( C^O + C^G[f_{l,B} + \sum_{l' \neq l} f_{l',B} + \tau_B] f_{l,B} \right) \right)$$

$$+ C^T(D - f_{l,A} - f_{l,B})$$

s.t.: $f_{l,A} + f_{l,B} \leq D$

$f_{l,A}, f_{l,B} \geq 0.$

Since there exists a symmetric equilibrium by the assumed symmetry, any index $l$ may be omitted. As the objective function is convex and the constraints are linear, the solution is obtained using the first order conditions

$$0 = 3C^O + (n + 1)C^G(3f_A + f_B) + 2\tau_A + \tau_C - C^T - \lambda_A + \mu$$

$$= 3C^O + (n + 1)C^G(f_A + 3f_B) + 2\tau_B + \tau_C - C^T - \lambda_B + \mu,$$

where the Lagrange multipliers are $\mu$ for the first constraint and $\lambda_A, \lambda_B$ for the non-negativity constraints.
Considering all possible cases for the KKT conditions, we find the solution

\[
f_B^* = \begin{cases} 
D & \text{if } 0 \leq \tau_B \leq \tau_A - (n+1)C^G D \text{ and } \tau_B \leq \frac{1}{2} \left[ C^T - 3C^O - \tau_C - 3(n+1)C^G D \right], \\
\frac{1}{2} D + \frac{\tau_A - \tau_B}{2(n+1)C^G} & \text{if } \tau_A - (n+1)C^G D \leq \tau_B \leq \tau_A + (n+1)C^G D \text{ and } \tau_B \leq \frac{1}{2} \left[ C^T - 3C^O - \tau_C - 3(n+1)C^G D \right], \\
\frac{C^T - 3C^O - 2\tau_B - \tau_C}{3(n+1)C^G} & \text{if } \tau_B \leq \frac{1}{2} \left[ C^T - 3C^O - \tau_C - 3(n+1)C^G D \right] \text{ and } \tau_B \leq \frac{1}{2} \left( C^T - 3C^O - \tau_C \right) \text{ and } \tau_B \leq 3\tau_A - (C^T - 3C^O - \tau_C), \\
\frac{C^T - 3C^O + \tau_A - 3\tau_B - \tau_C}{4(n+1)C^G} & \text{if } \tau_B \leq \frac{1}{3} \left( C^T - 3C^O + \tau_A - \tau_C \right) \text{ and } \tau_B \leq 3\tau_A - (C^T - 3C^O - \tau_C), \\
0 & \text{if } \tau_B \geq \frac{1}{2} \left( C^T - 3C^O - \tau_C \right) \text{ or } \tau_B \geq \tau_A + (n+1)C^G D \text{ or } \tau_B \geq \frac{1}{3} \left( C^T - 3C^O + \tau_A - \tau_C \right),
\end{cases}
\]

where \( f_A^* \) have the same expressions except for swapping everywhere the indices \( A \) and \( B \).

Next we analyze the first stage of the game. Each service provider \( s \), taking the charge of all other service provider as given, sets the charge \( \tau_s \) to maximize their own profit, i.e.,

\[
(\tau_A - C^S)2nf_A^*; (\tau_B - C^S)2nf_B^* \text{ and } (\tau_C - C^S)n(f_A^* + f_B^*)
\]

for \( s = A, B, C \), respectively. We analyze the best response maximal profit depending on the charge regions for \( f_s^* \). Since there exists a symmetric equilibrium by the assumed symmetry of service providers \( A, B \), we may consider only cases where \( \tau_A = \tau_B \). For \( \tau_A = \tau_B \geq \frac{1}{2} \left( C^T - 3C^O - \tau_C \right) \), \( f_s^* = 0 \), so the profit is 0 for all \( s \). When \( f_s^* > 0 \), there are two potential solutions:

(a) Given that \( \tau_A, \tau_B \) are in the range for which \( f_A^* = \frac{C^T - 3C^O + \tau_A - 3\tau_B - \tau_C}{4(n+1)C^G} \) and \( f_B^* = \frac{C^T - 3C^O + \tau_A - 3\tau_B - \tau_C}{4(n+1)C^G} \), maximizing simultaneously the concave profit functions \( (\tau_A - C^S)2n(\frac{C^T - 3C^O + \tau_B - 3\tau_B - \tau_C}{4(n+1)C^G}) \) and \( (\tau_C - C^S)2n(\frac{C^T - 3C^O - \tau_B - \tau_C}{4(n+1)C^G}) \) with respect to the relevant charge, the first order conditions imply \( \tau_A^* = \tau_B^* = \frac{C^S + 3(C^T - 3C^O - 3C^S)}{288} \), and \( \tau_C^* = \frac{C^S + 108(C^T - 3C^O - 3C^S)}{288} \), consequently \( f_A^* = f_B^* = \frac{27(C^T - 3C^O - 3C^S)}{288} \). This is possible if and only if \( D \geq \tau_A - \tau_B \geq 0 \), in order to have \( 0 \leq f_A^* + f_B^* \leq D \).

(b) Given that \( \tau_A, \tau_B \) are in the range for which \( f_A^* = \frac{1}{2} D + \frac{\tau_A - \tau_B}{2(n+1)C^G} \) and \( f_B^* = \frac{1}{2} D + \frac{\tau_A - \tau_B}{2(n+1)C^G} \), the profit function of service provider \( C \) is increasing in \( \tau_C \) because \( f_A^* + f_B^* = D \), thus \( \tau_C \) attains its maximal value \( C^T - 3C^O - 2(n+1)C^G D - \tau_A - \tau_B \) in this region. Under this restriction, maximizing simultaneously the service providers’ concave profit functions \( (\tau_A - C^S)n(\frac{D + \tau_B - \tau_B}{n+1}C^G) \) and \( (\tau_B - C^S)n(\frac{D + \tau_A - \tau_B}{n+1}C^G) \), the first order conditions imply \( \tau_A^* = \tau_B^* = C^S + \frac{n+1}{4}C^G(2nD) \) and \( \tau_C^* = C^S + (C^T - 3C^O - 3C^S) - 4(n+1)C^G D \), consequently \( f_A^* = f_B^* = \frac{1}{2} D \).

Thus we consider two cases depending on the value of \( D \):
(Case 2) $D \geq \frac{54(C^T-3C^O-3C^S)}{288} \geq 0$. In this case, by concavity, the only relevant solution is (a). The profits are $\frac{n}{n+1} \frac{486(C^T-3C^O-3C^S)^2}{20736 C^G}$ for service providers $A, B$ and $\frac{n}{n+1} \frac{1458(C^T-3C^O-3C^S)^2}{20736 C^G}$ for $C$.

(Case 1) $0 \leq D < \frac{54(C^T-3C^O-3C^S)}{288}$. In this case, the only possible solution is (b). The profits are $\frac{C^G}{2} \frac{n+1}{2n}(2nD)^2$ for service providers $A, B$ and $[C^T-3C^O-3C^S-4(n+1)C^G D]nD$. When $D = \frac{54(C^T-3C^O-3C^S)}{288}$, the profit of service provider $C$ in solution (a) is strictly higher than that of solution (b), so $C$ dictates solution (a) by setting the charge accordingly. 

**Proof of Theorem II.** The equilibrium is constructed using a folding back procedure. Under the simplified network, the analysis of the second stage of the game is the same as in Theorem 1. In the first stage, the integrated service provider $A, C$ and service provider $B$, taking the charge of the opponent as given, each set their charge to maximize their own profit, i.e.,

$$(\tau_A - C^S)2nf_A^* + (\tau_C - C^S)n(f_A^* + f_B^*) \text{ and } (\tau_B - C^S)2nf_B^*,$$

respectively. As in Theorem 1, when $\tau_A$ or $\tau_B$ are weakly higher than $\frac{1}{2}(C^T - 3C^O - \tau_C)$, $f_s^* = 0$ and the profit is 0. When $f_s^* > 0$, there are two potential solutions:

(a') Given that $\tau_A, \tau_B$ are in the range for which $f_A^* = \frac{2(C^T-3C^O+\tau_B-3\tau_A-\tau_C)}{4(n+1)C^G}$ and $f_B^* = \frac{2(C^T-3C^O+\tau_A-3\tau_B-\tau_C)}{4(n+1)C^G}$, maximizing simultaneously the concave profit functions $(\tau_A - C^S)2nf_A^* + (\tau_B - C^S)2nf_B^*$ with respect to the relevant charge, the first order conditions imply $\tau_A^* = C^S + \frac{16(C^T-3C^O-3C^S)}{288}$, $\tau_B^* = C^S + \frac{32(C^T-3C^O-3C^S)}{288}$ and $\tau_C^* = C^S + \frac{112(C^T-3C^O-3C^S)}{288}$, consequently $f_A^* = \frac{40(C^T-3C^O-3C^S)}{288}$ and $f_B^* = \frac{24(C^T-3C^O-3C^S)}{288}$. This is possible if and only if $D \geq \frac{64(C^T-3C^O-3C^S)}{288} \geq 0$, in order to have $0 \leq f_A^* + f_B^* \leq D$.

(b') Given that $\tau_A, \tau_B$ are in the range for which $f_A^* = \frac{2D+\tau_B-\tau_A}{2(n+1)C^G}$ and $f_B^* = \frac{2D+\tau_A-\tau_B}{2(n+1)C^G}$, the profit function of service provider $C$ is increasing in $\tau_C$ because $f_A^* + f_B^* = D$, thus $\tau_C$ attains its maximal value $C^T - 3C^O - 2(n+1)C^G D - \tau_A - \tau_B$ in this region. Under this restriction, maximizing simultaneously the service providers’ concave profit functions $(\tau_A - C^S)nD + \frac{\tau_B - \tau_A}{2(n+1)C^G} + (\tau_C - C^S)nD$ and $(\tau_B - C^S)nD + \frac{\tau_A - \tau_B}{2(n+1)C^G}$, the first order conditions imply $\tau_A^* = \tau_B^* = C^S + \frac{n+1}{2n} C^G (2nD)$ and $\tau_C^* = C^S + (C^T - 3C^O - 3C^S) - 4(n+1)C^G D$, consequently $f_A^* = f_B^* = \frac{1}{2}D$.

Thus we consider several cases depending on the values of $D$:

(Case 2') $D \geq \frac{64(C^T-3C^O-3C^S)}{288} \geq 0$. In this case, by concavity, the only relevant solution is solution (a'). The profits are $\frac{n}{n+1} \frac{2112(C^T-3C^O-3C^S)^2}{20736 C^G}$ for the integrated service provider $A, C$ and $\frac{n}{n+1} \frac{384(C^T-3C^O-3C^S)^2}{20736 C^G}$ for $B$.

(Case 1') $0 \leq D < \frac{64(C^T-3C^O-3C^S)}{288}$. In this case, the only possible solution is (b'). The profits are $\frac{C^G}{2} \frac{n+1}{2n}(2nD)^2 + [C^T-3C^O-3C^S-4(n+1)C^G D]nD$ for the integrated service provider $A, C$ and $\frac{C^G}{2} \frac{n+1}{2n}(2nD)^2$ for $B$. When $D = \frac{54(C^T-3C^O-3C^S)}{288}$, the profit of $A, C$ in solution (a') is strictly higher than that of solution (b'), so $C$ dictates solution (a) by setting the charge accordingly.
For the last three theorems we assume that the captive demand, \( D_{cap} \), is sufficiently small with respect to the outside option cost, \( C^T \), so that service providers always prefer to set the charges to serve all captive demand, specifically

\[
0 \leq D_{cap} \leq \frac{C^T - C^O - C^S}{2(n+1)C^G}.
\] (A.1)

**Proof of Theorem III.** The equilibrium is constructed using a folding back procedure. Under the simplified network, in the second stage of the game customer \( l \) takes as given the choices of other customers and solves the problem

\[
\begin{align*}
\min & \quad \left( C^O + C^G[f_{l,\text{cap},A} + f_{l,\text{flex},A} + \sum_{l' \neq l} (f_{l',\text{cap},A} + f_{l',\text{flex},A})]\right)(f_{l,\text{cap},A} + f_{l,\text{flex},A}) \\
& + \left( C^O + C^G[f_{l,\text{flex},A} + \sum_{l' \neq l} f_{l',\text{flex},A}] + \tau_A \right)f_{l,\text{flex},A} \\
& + \left( C^O + C^G[f_{l,\text{cap},B} + f_{l,\text{flex},B} + \sum_{l' \neq l} (f_{l',\text{cap},B} + f_{l',\text{flex},B})]\right)(f_{l,\text{cap},B} + f_{l,\text{flex},B}) \\
& + \left( C^O + C^G[f_{l,\text{flex},B} + \sum_{l' \neq l} f_{l',\text{flex},B}] + \tau_B \right)f_{l,\text{flex},B} \\
& + C^T[(D_{cap} - f_{l,\text{cap},A}) + (D_{cap} - f_{l,\text{cap},B}) + (D_{\text{flex}} - f_{l,\text{flex},A} - f_{l,\text{flex},B})]
\end{align*}
\]

s.t.:  
\[
\begin{align*}
& f_{l,\text{cap},A} \leq D_{cap} \\
& f_{l,\text{cap},B} \leq D_{cap} \\
& f_{l,\text{flex},A} + f_{l,\text{flex},B} \leq D_{\text{flex}} \\
& f_{l,\text{cap},A} + f_{l,\text{cap},B} + f_{l,\text{flex},A} + f_{l,\text{flex},B} \geq 0.
\end{align*}
\]

Since there exists a symmetric equilibrium by the assumed symmetry, any index \( l \) may be omitted. As the objective function is convex and the constraints are linear, the solution is obtained using the first order conditions

\[
\begin{align*}
0 &= C^O + (n+1)C^G(f_{\text{cap},A} + f_{\text{flex},A}) + \tau_A - C^T - \lambda_{\text{cap},A} + \mu_{\text{cap},A} \\
& = C^O + (n+1)C^G(f_{\text{cap},B} + f_{\text{flex},B}) + \tau_B - C^T - \lambda_{\text{cap},B} + \mu_{\text{cap},B} \\
& = 2C^O + (n+1)C^G(f_{\text{cap},A} + 2f_{\text{flex},A}) + 2\tau_A - C^T - \lambda_{\text{flex},A} + \mu_{\text{flex}} \\
& = 2C^O + (n+1)C^G(f_{\text{cap},B} + 2f_{\text{flex},B}) + 2\tau_B - C^T - \lambda_{\text{flex},B} + \mu_{\text{flex}},
\end{align*}
\]

where the Lagrange multipliers are \( \mu_{\text{cap},A}, \mu_{\text{cap},B}, \mu_{\text{flex}} \) for the first three constraints and \( \lambda_{\text{cap},A}, \lambda_{\text{cap},B}, \lambda_{\text{flex},A}, \lambda_{\text{flex},B} \) for the non-negativity constraints. Considering all possible cases for the KKT
conditions, we find the solution

$$f_{\text{cap},B}^* = \begin{cases} 
\frac{D_{\text{cap}}}{C^T - C^O - (n+1)C^G D_{\text{cap}}} & \text{if } 0 \leq \tau_B \leq C^T - C^O - (n+1)C^G D_{\text{cap}}, \\
\frac{\tau_B - \tau_B}{(n+1)C^G} & \text{if } C^T - C^O - (n+1)C^G D_{\text{cap}} \leq \tau_B \leq C^T - C^O, \\
0 & \text{if } \tau_B \geq C^T - C^O 
\end{cases}$$

and

$$f_{\text{flex},B}^* = \begin{cases} 
D_{\text{flex}} & \text{if } 0 \leq \tau_B \leq \tau_A - (n+1)C^G D_{\text{flex}} \text{ and } \\
\frac{\tau_A - \tau_B}{2(n+1)C^G} & \text{if } \tau_A - (n+1)C^G D_{\text{flex}} \leq \tau_B \leq \tau_A + (n+1)C^G D_{\text{flex}} \text{ and } \\
\frac{C^T - 2C^O - 2\tau_B}{2(n+1)C^G} - \frac{1}{2} D_{\text{cap}} & \text{if } \tau_B \leq \frac{1}{2}(C^T - 2C^O - (n+1)C^G D_{\text{cap}}) \text{ and } \\
0 & \text{if } \tau_B \geq \frac{1}{2}[C^T - 2C^O - (n+1)C^G D_{\text{cap}}] 
\end{cases}$$

where $f_{\text{cap},A}^*$ and $f_{\text{flex},A}^*$ have the same expressions except for swapping everywhere the indices $A$ and $B$.

Next we analyze the first stage of the game. Each service provider $s$, taking the charge of the other service provider as given, sets the charge $\tau_s$ to maximize the profit

$$(\tau_s - C^S)n(f_{\text{cap},s}^* + 2f_{\text{flex},s}^*).$$

We analyze the best response maximal profit depending on the charge regions for $f_{\text{cap},s}^*$ and $f_{\text{flex},s}^*$. For $\tau_s \geq C^T - C^O$, $\tau_s \geq \frac{1}{2}[C^T - 2C^O - (n+1)C^G D_{\text{cap}}]$ and $f_{\text{cap},s}^* = f_{\text{flex},s}^* = 0$, so the profit is 0. For $C^T - C^O - (n+1)C^G D_{\text{cap}} \leq \tau_s \leq C^T - C^O$, since $D_{\text{cap}} \leq \frac{C^T}{(n+1)C^G}$ by (A.1), $C^T - C^O - (n+1)C^G D_{\text{cap}} \geq \frac{1}{2}[C^T - 2C^O - (n+1)C^G D_{\text{cap}}]$. Thus $f_{s}^\text{flex} = 0$ because $\tau_s \geq \frac{1}{2}[C^T - 2C^O - (n+1)C^G D_{\text{cap}}]$. Therefore maximizing the concave profit function

$$(\tau_s - C^S)n\frac{C^T - C^O - \tau_s}{(n+1)C^G},$$

first order condition implies the optimal charge $\tau_s^* = \frac{C^T - C^O + C^S}{2}$ and the optimal frequency $f_{\text{cap},s}^* = \frac{C^T - C^O - C^S}{2(n+1)C^G} \geq D_{\text{cap}}$. The constraint $f_{\text{cap},s}^* \leq D_{\text{cap}}$ then implies $\tau_s^* = C^T - C^O - (n+1)C^G D_{\text{cap}}$ and $f_{\text{cap},s}^* = D_{\text{cap}}$ with positive profit. For any $\frac{1}{2}[C^T - 2C^O - (n+1)C^G D_{\text{cap}}] \leq \tau_s \leq C^T - C^O - (n+1)C^G D_{\text{cap}}$, we still have $f_{\text{flex},s}^* = 0$, thus the profit is maximized again at the upper bound of this interval. We conclude that in equilibrium, for both $s$, $f_{\text{cap},s}^* = D_{\text{cap}}$. Moreover, either $\tau_s^* \leq \frac{1}{2}[C^T - 2C^O - (n+1)C^G D_{\text{cap}}]$, or we have the following solution:

(a) $\tau_s^* = C^T - C^O - (n+1)C^G D_{\text{cap}}$, $f_{\text{flex},s}^* = 0$ and $f_{\text{cap},s}^* = D_{\text{cap}}$ for both $s$, with profit $(C^T - C^O - C^S - (n+1)C^G D_{\text{cap}})nD_{\text{cap}}$.

For the flexible flow, when it is positive there are two potential solutions:
(b) Given that for each service provider $s$, $\tau_s$ is in the range for which $f^*_{s,\text{flex},s} = \frac{CT - 2CO - 2s}{2(n+1)C^G} - \frac{1}{2}D_{\text{cap}}$ and $f^*_{s,\text{cap},s} = D_{\text{cap}}$, maximizing the concave profit function $(\tau_s - C^G)n\left(\frac{CT - 2CO - 2s}{2(n+1)C^G} - D_{\text{cap}}\right)$, the first order condition implies $\tau^*_s = \frac{CT - 2CO + 2s}{4C^G}$, consequently $f^*_{s,\text{flex},s} = \frac{1}{2}\left(\frac{CT - 2CO - 2s}{2(n+1)C^G} - D_{\text{cap}}\right)$ and $f^*_{s,\text{cap},s} = D_{\text{cap}}$ for both $s$. This is possible if and only if $D_{\text{flex}} \geq \frac{CT - 2CO - 2s}{2(n+1)C^G} - D_{\text{cap}} \geq 0$, in order to have $0 \leq f^*_{s,\text{flex},A} + f^*_{s,\text{flex},B} \leq D_{\text{flex}}$.

(c) Given that for both $s$, $\tau_s$ is in the range for which $f^*_{s,\text{flex},A} = \frac{1}{2}D_{\text{flex}} + \frac{\tau_B - \tau_A}{2(n+1)C^G}$, $f^*_{s,\text{flex},B} = \frac{1}{2}D_{\text{flex}} + \frac{\tau_A - \tau_B}{2(n+1)C^G}$ and $f^*_{s,\text{cap},s} = D_{\text{cap}}$ for both $s$, maximizing simultaneously the service providers’ concave profit functions $(\tau_A - C^G)n(D_{\text{cap}} + D_{\text{flex}} + \frac{\tau_B - \tau_A}{(n+1)C^G})$ and $(\tau_B - C^G)n(D_{\text{cap}} + D_{\text{flex}} + \frac{\tau_A - \tau_B}{(n+1)C^G})$, the first order conditions under symmetry imply $\tau^*_s = C^S + \frac{n+1}{2n}C^G[D_{\text{cap}} + D_{\text{flex}}]$, consequently $f^*_{s,\text{flex},s} = \frac{1}{2}D_{\text{flex}}$ and $f^*_{s,\text{cap},s} = D_{\text{cap}}$ for both $s$. This is possible if and only if $0 \leq D_{\text{flex}} \leq \frac{CT - 2CO - 2s}{3(n+1)C^G} - D_{\text{cap}}$ in order to have $\tau^*_A + \tau^*_B \leq CT - 2CO - (n+1)C^G(D_{\text{cap}} + D_{\text{flex}})$.

Thus we consider several cases depending on the values of $D_{\text{cap}}$ and $D_{\text{flex}}$:

(Case 5) $D_{\text{flex}} \geq \frac{CT - 2CO - 2s}{2(n+1)C^G} - D_{\text{cap}} \geq 0$ and $0 \leq D_{\text{cap}} \leq D_{\text{cap}}$, where

$$\bar{D}_{\text{cap}} \equiv \frac{CT - CO - CS - \sqrt{\frac{1}{2}(CT)^2 - (CO + CS)^2}}{2(n+1)C^G},$$

and we note that $\bar{D}_{\text{cap}} \leq \frac{CT - 2CO - 2s}{3(n+1)C^G} \leq \frac{CT - 2CO - 2s}{2(n+1)C^G}$. In this case, by concavity, the only relevant solution is solution (b). Both service provider’s profit is $\frac{n+1}{n+1}(\frac{CT - 2CO - 2s}{8C^G})$. For this profit to be weakly higher than the profit of solution (a), it is necessary and sufficient that $0 \leq D_{\text{cap}} \leq \bar{D}_{\text{cap}}$.

(Case 4) $\frac{CT - 2CO - 2s}{3(n+1)C^G} - D_{\text{cap}} < D_{\text{flex}} < \frac{CT - 2CO - 2s}{2(n+1)C^G} - D_{\text{cap}}$ and $0 \leq D_{\text{cap}} \leq \bar{D}_{\text{cap}}$. In this case, solution (a) is not relevant because $\frac{CT - 2CO - 2s}{2(n+1)C^G} - D_{\text{cap}} > 0$ implies that each service provider prefers to serve some flexible demand. Moreover, solutions (b) and (c) are outside their respective regions, and by concavity, the only relevant solution is the one on the boundary of both regions, i.e. $\tau^*_s = \frac{CT - 2CO - (n+1)C^G(D_{\text{cap}} + D_{\text{flex}})}{2}$. By symmetry $f^*_{s,\text{flex},s} = \frac{1}{2}D_{\text{flex}}$ and $f^*_{s,\text{cap},s} = D_{\text{cap}}$ for both $s$. Each service provider’s profit is $(\frac{CT - 2CO - 2s}{(n+1)C^G}(D_{\text{cap}} + D_{\text{flex}}))n(D_{\text{cap}} + D_{\text{flex}})$. As in Case 5, this profit is weakly higher than the profit of solution (a) if and only if $0 \leq D_{\text{cap}} \leq \bar{D}_{\text{cap}}$.

(Case 3) $D_{\text{flex}} \leq D_{\text{flex}} \leq \frac{CT - 2CO - 2s}{3(n+1)C^G} - D_{\text{cap}}$ and $0 \leq D_{\text{cap}} \leq \bar{D}_{\text{cap}}$, where

$$\bar{D}_{\text{flex}} \equiv \sqrt{D_{\text{cap}}[\frac{CT - CO - CS}{(n+1)C^G} - D_{\text{cap}}] - D_{\text{cap}}}$$

and

$$\bar{D}_{\text{cap}} \equiv \frac{CT - CO - CS - \frac{1}{3}\sqrt{(CT + CO + CS)(5CT - 7CO - 7CS)}}{2(n+1)C^G},$$

and we note that $\bar{D}_{\text{cap}} \leq \bar{D}_{\text{flex}}$, and that $\bar{D}_{\text{flex}} \leq \frac{CT - 2CO - 2s}{3(n+1)C^G} - D_{\text{cap}}$ if and only if $D_{\text{flex}} \leq \bar{D}_{\text{cap}}$.

When $D_{\text{flex}} \leq \frac{CT - 2CO - 2s}{3(n+1)C^G} - D_{\text{cap}}$, the relevant solution is solution (c). Each service provider’s profit is $\frac{n+1}{4n}C^G[2n(D_{\text{cap}} + D_{\text{flex}})]^2$. This profit is higher than the profit of solution (a) if and only
if $\hat{D}_\text{flex} \leq D_\text{flex}$ and $D_\text{cap} \leq \hat{D}_\text{cap}$.

(Case 2) $\hat{D}_\text{flex} < D_\text{flex} < \min\{D_\text{flex}, \frac{C^T - 2C^O - 2CS}{3(n+1)CG} - D_\text{cap}\}$ and $\hat{D}_\text{cap} \leq D_\text{cap} \leq \hat{D}_\text{cap}$, where

$$\hat{D}_\text{flex} = \frac{C^T - 2C^O - 2CS}{4(n+1)CG} - \frac{D_\text{cap}}{2} - \sqrt{\left(\frac{C^T - 2C^O - 2CS}{4(n+1)CG}\right)^2 - \frac{D_\text{cap}}{2} \left(\frac{C^T - C^O - CS}{(n+1)CG} - D_\text{cap}\right)}$$

and

$$\hat{D}_\text{cap} = \frac{7C^T - 6C^O - 6CS - 4\sqrt{2(C^T)^2 - 2(C^O + CS)^2 - CT(C^O + CS)}}{17(n+1)CG}.$$ 

In this case, solution (b) is not relevant because $D_\text{flex} < \frac{C^T - 2C^O - 2CS}{2(n+1)CG} - D_\text{cap}$. Solution (c) is not relevant because each service provider’s profit is strictly lower than the profit of solution (a) (as explained in Case 3, $\hat{D}_\text{flex}$ is exactly the cut-off point for this comparison). Moreover, solution (a) is not relevant because each service provider’s profit when $f^*_\text{flex,s} = D_\text{flex}$ and $f^*_\text{cap,s} = D_\text{cap}$ is strictly higher (the lower bound of the interval, $\hat{D}_\text{flex}$, is exactly the cut-off point for this comparison).

Thus we have an asymmetric equilibrium in which one service provider, say $A$, sets the charge $\tau^*_A = C^T - C^O - (n+1)C^G D_\text{cap}$ consequently $f^*_{\text{flex},A} = 0$, while the other service provider best responds by setting the charge $\tau^*_B = \frac{C^T - 2C^O - (n+1)C^G(D_\text{cap} + 2D_\text{flex})}{2}$. Consequently, $f^*_{\text{flex},B} = D_\text{flex}$ (or the analogous equilibrium where $A$ and $B$ interchange).

(Case 1) $[0 \leq D_\text{flex} \leq \hat{D}_\text{flex}$ and $0 \leq D_\text{cap} \leq \hat{D}_\text{cap}]$ or $D_\text{cap} > \hat{D}_\text{cap}$. In this case, the only relevant solution is solution (a).

**Proof of Theorem IV.** The equilibrium is constructed using a folding back procedure. Under the simplified network, the analysis of the second stage of the game is the same as in Theorem 1, and even simplifies further because the single service provider must set a single charge $\tau = \tau_A = \tau_B$.

Thus $f^*_\text{cap} = f^*_{\text{cap},A} = f^*_{\text{cap},B}$ and $f^*_\text{flex} = f^*_{\text{flex},A} = f^*_{\text{flex},B}$ are:

$$f^*_\text{cap} = \begin{cases} 
\frac{D_\text{cap}}{C^T - C^O - (n+1)C^G D_\text{cap}} & \text{if } 0 \leq \tau \leq C^T - C^O - (n+1)C^G D_\text{cap}, \\
\frac{C^T - 2C^O - 2CS}{2(n+1)CG} - \frac{D_\text{cap}}{2} & \text{if } C^T - C^O - (n+1)C^G D_\text{cap} \leq \tau \leq C^T - C^O, \\
0 & \text{if } \tau \geq C^T - C^O
\end{cases}$$

and

$$f^*_\text{flex} = \begin{cases} 
\frac{1}{2}D_\text{flex} & \text{if } \tau \leq \frac{1}{2}\left[C^T - 2C^O - (n+1)C^G(D_\text{cap} + D_\text{flex})\right], \\
\frac{C^T - 2C^O - 2\tau}{2(n+1)CG} - \frac{1}{2}D_\text{cap} & \text{if } \frac{1}{2}\left[C^T - 2C^O - (n+1)C^G(D_\text{cap} + D_\text{flex})\right] \leq \tau \leq \frac{1}{2}\left[C^T - 2C^O - (n+1)C^G D_\text{cap}\right], \\
0 & \text{if } \tau \geq \frac{1}{2}\left[C^T - 2C^O - (n+1)C^G D_\text{cap}\right].
\end{cases}$$
In the first stage, the single service provider sets \( \tau \) to maximize the per route profit

\[
(\tau - C^S)n(f_{cap}^* + 2f_{flex}^*).
\]

As in Theorem 1, (A.1) implies that \( f_{cap}^* = D_{cap} \). Moreover, either \( \tau^* \leq \frac{1}{2}(C^T - 2C^O - (n+1)C^G D_{cap}) \), or we have the following solution:

(a') \( \tau^* = C^T - C^O - (n+1)C^G D_{cap} \), \( f_{flex}^* = 0 \) and \( f_{cap}^* = D_{cap} \), with per route profit \( (C^T - C^O - C^S - (n+1)C^G D_{cap}) n D_{cap} \).

For the flexible flow, when it is positive there are two potential solutions:

(b') Given that \( \tau \) is in the range for which \( f_{flex}^* = \frac{C^T - 2C^O - 2\tau}{2(n+1)C^G} - \frac{1}{2}D_{cap} \) and \( f_{cap}^* = D_{cap} \), maximizing the concave profit function \( (\tau - C^S)n(\frac{C^T - 2C^O - 2\tau}{(n+1)C^G}) \), we have \( \tau^* = \frac{C^T - 2C^O + 2C^S}{4} \), \( f_{flex}^* = \frac{1}{2}[\frac{C^T - 2C^O - 2C^S}{2(n+1)C^G} - D_{cap}] \) and \( f_{cap}^* = D_{cap} \).

(c') Given that \( \tau \) is in the range for which \( f_{flex}^* = \frac{1}{2}D_{flex} \) and \( f_{cap}^* = D_{cap} \), the profit is maximized at the upper bound of this range with \( \tau^* = \frac{1}{2}(C^T - 2C^O - (n+1)C^G(D_{cap} + D_{flex})) \).

Thus we consider several cases depending on the values of \( D_{cap} \) and \( D_{flex} \):

(Case 5') \( D_{flex} \geq \frac{C^T - 2C^O - 2C^S}{2(n+1)C^G} - D_{cap} \geq 0 \) and \( 0 \leq D_{cap} \leq \tilde{D}_{cap} \), where \( \tilde{D}_{cap} \) is defined in the proof of Theorem 1. In this case, by concavity, the only relevant solution is solution (b'), with per route profit \( \frac{n}{n+1}(\frac{C^T - 2C^O - 2C^S}{2C^G})^2 \). For this profit to be weakly higher than the profit of solution (a'), it is necessary and sufficient that \( 0 \leq D_{cap} \leq \tilde{D}_{cap} \).

(Case 23') \( 2\tilde{D}_{flex} < D_{flex} < \frac{C^T - 2C^O - 2C^S}{2(n+1)C^G} - D_{cap} \) and \( 0 \leq D_{cap} \leq \tilde{D}_{cap} \), where \( \tilde{D}_{flex} \) is defined in the proof of Theorem 1. In this case, the only relevant solution is solution (c'), with per route profit \( \frac{C^T - 2C^O - 2C^S - (n+1)C^G(D_{cap} + D_{flex})}{2} \cdot n(D_{cap} + D_{flex}) \). As in case 5', this profit is weakly higher than the profit of solution (a') if and only if \( 0 \leq D_{cap} \leq \tilde{D}_{cap} \).

(Case 1') \( 0 \leq D_{flex} \leq 2\tilde{D}_{flex} \) and \( 0 \leq D_{cap} \leq \tilde{D}_{cap} \) or \( D_{cap} > \tilde{D}_{cap} \). In this case, the only relevant solution is the above solution (a').

**Proof of Theorem V.** We concentrate on the case of competitive pricing (as in Case 3 of Theorem 1) in which all demand is served, i.e. \( f_{l,cap,A} = f_{l,cap,B} = D_{cap} \) and \( f_{l,flex,A} + f_{l,flex,B} = D_{flex} \) for each customer \( l \) under the simplified network. The equilibrium is constructed using a folding back procedure. Customer \( l \) takes as given the choices of other customers and sets \( f_{l,flex,A} \) to solve
the problem

\[
\begin{align*}
\min & \quad \left( C_{l,A}^O + C^G[nD_{cap} + f_{l,flex,A} + \sum_{l' \neq l} f_{l',flex,A}] + \tau_A \right) (D_{cap} + f_{l,flex,A}) \\
& + \left( C_{l,A}^O + C^G[f_{l,flex,A} + \sum_{l' \neq l} f_{l',flex,A}] + \tau_A \right) f_{l,flex,A} \\
& + \left( C^O + C^G[nD_{cap} + nD_{flex} - f_{l,flex,A} - \sum_{l' \neq l} f_{l',flex,A}] + \tau_B \right) (D_{cap} + D_{flex} - f_{l,flex,A}) \\
& + \left( C^O + C^G[nD_{flex} - f_{l,flex,A} - \sum_{l' \neq l} f_{l',flex,A}] + \tau_B \right) (D_{flex} - f_{l,flex,A}),
\end{align*}
\]

where we assume that \( C_{l,A}^O \leq C^O \) and \( C_{l',A}^O = C^O \) for all \( l' \neq 1 \). By the assumed symmetry, there exists a symmetric equilibrium such that for each service provider \( s \), \( f_{l',flex,s} \) is equal for all \( l' \neq 1 \), and is denoted by \( f_{o,flex,s} \). The first order conditions are

\[
\begin{align*}
&2C_{1,A}^O + C^G[(n + 1)D_{cap} + 4f_{1,flex,A} + 2(n - 1)f_{o,flex,A}] + 2\tau_A \\
&= 2C^O + C^G[(n + 1)D_{cap} + 2(n + 1)D_{flex} - 4f_{1,flex,A} - 2(n - 1)f_{o,flex,A}] + 2\tau_B
\end{align*}
\]

for customer 1, and

\[
\begin{align*}
&2C^O + C^G[(n + 1)D_{cap} + 2f_{1,flex,A} + 2nf_{o,flex,A}] + 2\tau_A \\
&= 2C^O + C^G[(n + 1)D_{cap} + 2(n + 1)D_{flex} - 2f_{1,flex,A} - 2nf_{o,flex,A}] + 2\tau_B
\end{align*}
\]

for each customer \( l' \neq 1 \). Thus the second stage solution is

\[
\begin{align*}
f_{1,flex,A}^* &= \frac{D_{flex}}{2} + \frac{C^O - C_{1A}^O}{2(n + 1)C^G} + \frac{\tau_B - \tau_A}{2(n + 1)C^G} \\
f_{o,flex,A}^* &= \frac{D_{flex}}{2} - \frac{C^O - C_{1A}^O}{2(n + 1)C^G} + \frac{\tau_B - \tau_A}{2(n + 1)C^G} \\
f_{1,flex,B}^* &= \frac{D_{flex}}{2} - \frac{C^O - C_{1A}^O}{2(n + 1)C^G} + \frac{\tau_A - \tau_B}{2(n + 1)C^G} \\
f_{o,flex,B}^* &= \frac{D_{flex}}{2} + \frac{C^O - C_{1A}^O}{2(n + 1)C^G} + \frac{\tau_A - \tau_B}{2(n + 1)C^G}.
\end{align*}
\]

In the first stage each service provider \( s \), taking the charge of the other service provider as given, sets the charge \( \tau_s \) to maximize the profit

\[
(\tau_s - C_s^O)[nD_{cap} + 2(f_{1,flex,s}^* + (n - 1)f_{o,flex,s}^*)],
\]

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and we denote the service cost of service providers $B$ by $C^S$. Solving the first order conditions

\begin{align*}
2\tau_A - \tau_B &= C_A^S + (n + 1)C^G(D_{\text{cap}} + D_{\text{flex}}) + \frac{C^O - C_{1A}^O}{n} \quad \text{and} \\
-\tau_A + 2\tau_B &= C^S + (n + 1)C^G(D_{\text{cap}} + D_{\text{flex}}) - \frac{C^O - C_{1A}^O}{n},
\end{align*}

the solution is

\begin{align*}
\tau^*_A &= C^S - \frac{2C^S - C_A^S}{3} + \frac{C^O - C_{1A}^O}{3n} + \frac{n + 1}{2n}C^G[2n(D_{\text{cap}} + D_{\text{flex}})] \\
\tau^*_B &= C^S - \frac{C^S - C_A^S}{3} - \frac{C^O - C_{1A}^O}{3n} + \frac{n + 1}{2n}C^G[2n(D_{\text{cap}} + D_{\text{flex}})].
\end{align*}

Consequently,

\begin{align*}
f^*_{1,\text{flex},A} &= \frac{D_{\text{flex}}}{2} + \frac{C^S - C_A^S}{6(n + 1)C^G} + \left(\frac{n}{2} - \frac{1}{3n}\right)\frac{C^O - C_{1A}^O}{(n + 1)C^G} \\
f^*_{o,\text{flex},A} &= \frac{D_{\text{flex}}}{2} + \frac{C^S - C_A^S}{6(n + 1)C^G} - \left(\frac{1}{2} + \frac{1}{3n}\right)\frac{C^O - C_{1A}^O}{(n + 1)C^G} \\
f^*_{1,\text{flex},B} &= \frac{D_{\text{flex}}}{2} - \frac{C^S - C_A^S}{6(n + 1)C^G} - \left(\frac{n}{2} - \frac{1}{3n}\right)\frac{C^O - C_{1A}^O}{(n + 1)C^G} \\
f^*_{o,\text{flex},B} &= \frac{D_{\text{flex}}}{2} - \frac{C^S - C_A^S}{6(n + 1)C^G} + \left(\frac{1}{2} + \frac{1}{3n}\right)\frac{C^O - C_{1A}^O}{(n + 1)C^G}.
\end{align*}