Capacity Trade-Offs & Trading Capacity

Nicole Adler and Eran Hanany*

August 11, 2010

Abstract

Accounting for desire for variety within the consumer demand function, a two stage hybrid competitive-cooperative game analyzes differentiated oligopolies under varying market structures from competition through pooling contracts up to anti-trust immune alliances. From a managerial perspective, pooling agreements not only benefit firms by better matching supply and demand but also rank highest in consumer surplus and social welfare, of interest to regulators. Moreover (partial) mergers appear preferable to no agreement on 'thin' markets, defined as relatively low demand with weak initial profit margins. A numerical analysis of the airline industry demonstrates under asymmetric and uncertain demand that codesharing on parallel links is preferable to competitive outcomes for multiple consumer types.

Keywords: agreements among firms, variety, pooling agreements, anti-trust regulation

1 Introduction

Consumers demonstrate a clear preference for variety as this increases the likelihood of purchasing the product closest to their desires. In turn, a firm offering variety increases the probability of matching the producer's product line to the preferences of a set of heterogeneous consumers, providing the potential for customization and flexibility. For example, an airline carrier produces a schedule which sets their frequency in each market per season. A passenger would prefer higher frequency since this reduces potential schedule delay, defined as the difference between the time that the passenger prefers to travel and the closest scheduled alternative. In empirical analyses of aviation markets, both Hansen (1990) and Pels, Nijkamp and Rietveld (2000) demonstrate that passenger utility increases in frequency and Belobaba (2009) shows that higher frequency leads to disproportionately higher market

*Adler: Hebrew University of Jerusalem and Northwestern University, Mount Scopus Jerusalem Israel 91905, msnic@huji.ac.il. Hanany: Tel Aviv University and Northwestern University, Ramat Aviv 69978 Israel, hananye@post.tau.ac.il. We thank Guy Arie, Jan Bruckner, Martin Dresner, Xiaowen Fu, Yigal Gerchak, David Gillen, Igal Hendel, Peter Klibanoff, Benny Mantin and Michael Whinston for helpful comments and discussion. We also thank audiences at Hong Kong Polytechnic University and Northwestern University and at ATRS '08, IFPSA '08, INFORMS '09 and ORSIS '09. This research was partially supported by Grant No. 1029/09 from the Israel Science Foundation (ISF) and by the Recanati Foundation.

share. Copeland, Dunn and Hall (2010) analyze the changes in new car prices over time, demonstrating that consumers prefer variety as defined by levels of inventory in a car showroom. Consequently, the empirical literature indicates that consumers are willing to pay for higher levels of variety offered.

Accounting for product differentiation as well as consumer preference for variety, we provide conditions under which consumer surplus is maximized when firms reach agreement. Firms choose capacities, levels of variety offered, product prices and whether to sign production capacity sharing agreements. We show that overall consumer surplus is always maximized when firms pool capacities, provided capacities and prices are set independently. Moreover, the lowest consumer surplus and overall social welfare outcome occurs when firms do not pool, compete in prices but set capacities jointly i.e. capacity agreements are worse than mergers. Notably in the aviation industry context, anti-trust immune alliances are strictly preferable to bilaterals with respect to consumer surplus, producer profits and overall social welfare. An additional conclusion is that under thin market conditions, agreeing jointly on capacity and pooling increases consumer surplus beyond that of the competitive equilibrium outcome. Thin markets arise when the demand magnitude, a measure of market size, as well as the marginal profit from the first unit of capacity, both fall below a threshold. In light of the literature to date this may be somewhat surprising since it is generally argued that a reduction in the level of direct competition through alliances or mergers will necessarily lead to higher prices that increase firm profits at the expense of consumer surplus (Deneckere and Davidson 1985). This ought to be of interest to Competition Authorities as it may be of note when evaluating the potential effects of a merger apriori. A numerical analysis which also accounts for demand uncertainty in asymmetric markets provides a realistic analysis of a setting in which this argument is highlighted. Furthermore, it becomes clear that heterogeneous consumers who place emphasis to a different extent on variety as compared to price, may disagree with respect to their preferences for pooling versus pure competition in standard size markets, although overall consumer surplus is still higher under competitive pooling agreements.

Markets are analyzed under pure competition and then under various potential agreements between firms in order to understand the impact of (partial) mergers on social welfare. This research analyzes the effect of agreements between two companies with respect to pricing, capacity and pooling. Five agreement types are defined in Table 1.1 and range from no agreement and complete separation in the decision making process up to anti-trust immune alliances or mergers in which firms maximize profits jointly. F refers to capacity, p refers to price and *Pooling* refers to an agreement in which the two companies have access to each other's capacity in order to better match firm output to the desires of the consumer.

Table 1.1: Agreement Types

	Competition	Capacity Agreement	Competitive Pooling Agreement	Capacity Pooling Agreement	Joint Profit Maximization
Compete	*F, p*	*p*	*F, p*	*p*	
Coordinate		*F*		*F*	*F, p*
Pooling	*X*	*X*	*v*	*v*	*v*

In this research we define a firm's decision process within a two-stage setting in which capacities and levels of variety offered are chosen in the first stage and prices in the second stage. Kreps and Scheinkman's (1983) celebrated result shows that a two-stage model of a market with homogeneous products is equivalent to the equilibrium outcome of a one-shot Cournot game. Demand is thus determined via price competition, at which point production takes place at zero cost subject to the capacity constraints generated in the first stage. Davidson and Deneckere (1986) argue that this setup is reasonable since quantities are a medium to long term strategic variable and prices are more easily changed in the short term. Yin and Ng (1997) and Martin (1999) extend the model to the case of differentiated products and show that the same results hold. Martin also explains that under the extended game, there is no issue with regard to the rationing rule because demand is well defined over every price vector since products are imperfect substitutes. In our two-stage setting, capacity impacts the consumer's choice function directly via the levels of variety offered, permitting an analysis of both the consumer and producer surplus in a market with differentiated goods. In addition to capturing desire for variety, our modeling approach accounts for the impact of spillovers which occur when a firm's capacity rises leading to a reduction in demand for the rival's product (Friedman 1988).[1]

Two-stage Nash equilibrium and Nash bargaining solutions are computed where relevant for each of the agreement types, thus studying hybrid competitive and cooperative outcomes in an imperfectly competitive market. The model is solved analytically under the assumptions of a single consumer type, certainty in demand and symmetry in demand and costs. We first prove that a unique pure strategy solution in capacities, levels of variety offered and prices always exists for each agreement type. Then we characterize the ranking of agreement types with respect to consumer and producer surpluses. There are two main trade-offs affecting surplus ranking. On the one hand, a reduction in the level of competition through mergers tends to lead to a reduction in capacity and an increase in prices, which benefit firms at the expense of consumers. On the other hand, as a result of our modeling of demand for variety, both consumers and producers alike prefer the firms to pool which

[1]Brueckner and Flores-Fillol (2007) and Allon and Federgruen (2009) both analyze differentiated oligopolies with firms choosing quality. The substantial difference between these papers and the model presented here lie in the assumption in their papers that all demand is met, irrespective of the quality chosen.

permits each firm access to the other's capacity. Since consumers gain from pooling through increased customization, their higher willingness to pay in turn causes the firms to further increase capacity and levels of variety offered. In other words, our results show that there are no free riding effects whereby pooling causes the firms to reduce capacity by relying on the pooling agreement. The combined impact of these two effects leads to our finding that, under thin market conditions, the second effect outweighs the first whereas in the remaining markets, producers and consumers show directly opposing preferences. Producers demonstrate a strong preference for tighter agreements achieving highest profitability under mergers, whereas consumers prefer pooling agreements with firms competing in capacity and price. Hence competitive pooling agreements maximize overall social welfare and capacity agreements produce the worst market outcome.

This paper first presents the proposed model and then several propositions describing the levels of capacity, levels of variety offered, prices, consumer surplus, producer profits and overall social welfare achieved according to various types of agreements among firms. We then present a numerical analysis of parallel agreements among airlines which permits the introduction of uncertainty over demand outcomes to be analyzed directly. The final section presents conclusions and recommendations for future research. Proofs are collected in an Appendix.

2 Modeling Demand for Variety

In this section we consider markets consisting of two symmetric companies serving a single representative consumer type per market modeled with a demand function linear in price (Singh and Vives 1984). The linear demand function draws from a consumer's quadratic utility function with respect to the consumption of two imperfectly substitutable goods (q_a) offered by the firms,

$$U(q_1, q_2) \equiv \sum_a q_a V_a - \frac{1}{2}(\sum_a q_a^2 + 2\gamma q_1 q_2), \tag{2.1}$$

where $0 < \gamma < 1$ represents the degree of imperfect substitutability between the firms' products. The modeling approach defines the vertical differentiation parameters V_a within the consumers' utility as a function of the levels of variety offered by the firms. Consumers are partially aware of the producers' production capabilities in that they may easily discover the different makes of car produced or the number of colors on offer. Neither producers nor consumers necessarily know apriori the precise consumer's desires. On the one hand, consumers aware of their specific taste are more likely to purchase a product closest to their preference from a company offering higher levels of variety. On the other hand, consumers unsure of their precise preferences are more likely to find an appropriate product when searching at a firm offering greater variety (Kreps 1979). Hence, in either case, firms offering greater choice are better able to match their products to the desires of the consumer. Note that horizontal differentiation traditionally refers to differences in dimensions over which customers may disagree, such as the preferred color of a car, whereas vertical differentiation refers to dimensions of agreement such as quality. Contrary to the standard modeling approach, we assume that the level of variety offered is a decision variable of the firm. As a result, higher levels of variety are equated to vertical differentiation which reflect a quality dimension over which all consumers can agree in principal, despite the fact that they may have different preferences and at the end of the process choose their most preferred product from the menu offered. Thus the model presented encompasses both horizontal differences, captured by the parameter γ in the demand function measuring imperfect substitutability, and simultaneously in a vertical sense captured by the dependence of V_a on the level of variety offered.

Under the assumption that s represents the minimum quantity produced per product type, based on technological constraints, the firm's capacity $F_a = s \cdot f_a$, where f_a represents the level of variety offered by firm a. We assume that s is chosen exogenously, prior to the capacity, levels of variety offered and pricing decisions analyzed in this context. For simplicity and in line with empirical research (Hansen 1990, Pels et al. 2000 and Copeland et al. 2010), we assume that V_a is log linear in f_a to reflect decreasing marginal utility as variety expands, thus $V_a \equiv \alpha_a + \beta \ln(1 + f_a)$ for positive parameters α_a and β. The results presented in this research are not dependent on this assumption and more general concave formulations could be assumed instead but would require a more complicated analysis.

The model analyzes two companies, $a \in \{1, 2\}$, choosing levels of variety offered f_a, resulting in capacity $s \cdot f_a$, and subsequently prices p_{ab} per home market $b \in \{1, 2\}$, where 1 or 2 represents the consumer's home market. For market b, firm a's demand is

$$d_{ab} \equiv m(\theta u_{ab} - \sum_{a'} u_{a'b}), \tag{2.2}$$

where $m > 0$ represents demand magnitude, $\theta \equiv 1 + \frac{1}{\gamma}$, and

$$u_{ab} \equiv \alpha_{ab} + \beta \ln(1 + \sum_{a' \in \rho_a} f_{a'}) - p_{ab} \tag{2.3}$$

represents the combined effect of company a's decisions on consumer utility, where $\beta > 0$ represents the importance of variety to the consumer and $\alpha_{ab} > 0$ represents preferences over firms. Under the assumption of symmetry, the willingness to pay for the home-based product is higher than that offered by the alternative firm, $\alpha_{11} = \alpha_{22} = \overline{\alpha} \geq \underline{\alpha} = \alpha_{12} = \alpha_{21}$. The pooling aspect in the agreement is represented by the set of partners of firm a, denoted by ρ_a, where $\rho_a = \{1, 2\}$ for each a when a pooling agreement has been signed, otherwise $\rho_a = \{a\}$. Thus pooling constitutes a risk mitigating strategy with respect to variability amongst consumers as to the most preferred product (in Section 3 we extend the model to incorporate further uncertainty relating to the demand magnitude m, resulting in additional advantages drawing from pooling agreements).

The firms maximize profits in two stages, first setting capacity and levels of variety offered, and then choosing prices given that both firms know the first stage decision outcomes. After choosing levels of capacity, the firms are limited to selling only up to the capacity to which they have precommitted. Under the first four contracts (see Table 1.1), the firms compete in Nash equilibrium by choosing prices as best responses to the opponent's choices according to

$$\begin{aligned} \max_{(p_{ab})_b} \pi_a \quad &\equiv \quad \sum_b p_{ab} d_{ab} - c f_a \\ & s.t. \sum_b d_{ab} \leq s f_a, \end{aligned} \tag{2.4}$$

where $\frac{c}{s}$ represents the unit cost of capacity. In the second stage, no company will set their prices such that their demand is higher than their capacity since increasing prices in this scenario will simply increase revenues and profits without introducing any additional costs. Consequently, a firm's demand levels will never be higher than their capacity in equilibrium, justifying the constraint in this formulation. Under the fifth, joint profit maximization contract, prices are chosen simultaneously to maximize the sum of profits under the joint capacity constraint,

$$\begin{aligned} &\max_{(p_{ab})_{ab}} \sum_a \sum_b p_{ab} d_{ab} - c \sum_a f_a \\ &s.t. \sum_a \sum_b d_{ab} \leq s \sum_a f_a. \end{aligned} \tag{2.5}$$

We assume $\frac{\overline{\alpha}+\underline{\alpha}}{2} > \frac{c}{s} > \beta$, which means that the average direct willingness to pay for the product, α, is sufficient to ensure profitability from the first unit of capacity and the value of variety offered to the consumer, β, is smaller than the unit cost. This ensures that the constraints in the formulations above are always binding and that the firms choose to produce (as will be shown in propositions 2.1 and 2.2).

With respect to capacity and variety decisions, denote firm a's second stage profit as a function of the first stage choices by $\widetilde{\pi}_a(f_a, f_{-a})$, thus summarizing the effects of the second stage price choices through the first stage decisions. Under no agreement and competitive pooling agreements, the firms compete by choosing a best response to the opponent's choice, i.e.

$$\max_{f_a} \widetilde{\pi}_a(f_a, f_{-a}), \tag{2.6}$$

resulting in a Nash equilibrium. Under capacity and capacity pooling agreements, the firms cooperate in capacity and levels of variety offered and compete in prices leading to the Nash bargaining solution of model (2.7).

$$\begin{aligned} &\max_{(f_a)_a} \prod_a [\widetilde{\pi}_a(f_a, f_{-a}) - \widetilde{\pi}_a(f_a^{comp}, f_{-a}^{comp})] \\ &s.t. \quad \widetilde{\pi}_a(f_a, f_{-a}) \geq \widetilde{\pi}_a(f_a^{comp}, f_{-a}^{comp}), \qquad \forall a, \end{aligned} \tag{2.7}$$

where f_a^{comp} refers to the corresponding competitive contract solution. Consequently the threat point for a capacity agreement is no agreement (competition) and the threat point for a capacity pooling agreement is the competitive pooling agreement. Under joint profit maximization, capacity and levels of variety offered are optimized simultaneously to maximize the sum of profits, i.e.

$$\max_{(f_a)_a} \sum_a \widetilde{\pi}_a(f_a, f_{-a}). \tag{2.8}$$

As a consequence of this formulation, two companies reaching agreement cannot be worse off than had no agreement been signed.

2.1 Effects of Agreements on Capacity and Price

In this section we characterize the second stage prices and profits and then the first stage capacities and levels of variety offered that result from each of the five agreement types. Let

$y = 0$ for the first four contracts and $y = 1$ for the fifth contract, representing competition and cooperation in prices respectively.

Proposition 2.1 *For each agreement type, a unique solution exists in prices as a function of the levels of variety offered under binding constraints, defined by*

$$\begin{aligned}\widetilde{p}_{ab}(f_a, f_{-a}) &= -\frac{s}{2m\theta}[(1-y)f_a + (\frac{1}{\theta-2} + \frac{y}{2})\sum_{a'} f_{a'}] + \beta \ln(1 + \sum_{a' \in \rho_a} f_{a'}) \\ &+ \frac{(\theta + y - 1)\frac{\overline{\alpha}+\underline{\alpha}}{2} + \theta\alpha_{ab}}{2\theta + y - 1}. \end{aligned} \quad (2.9)$$

Given these prices, the profit as a function of the levels of variety offered is

$$\begin{aligned}\widetilde{\pi}_a(f_a, f_{-a}) &= -\frac{s^2}{2m\theta}\left(f_a + \frac{1}{\theta-2}\sum_{a'} f_{a'}\right) f_a + \left(s\beta \ln(1 + \sum_{a' \in \rho_a} f_{a'}) + s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c\right) f_a \\ &+ \frac{m(\overline{\alpha}-\underline{\alpha})^2\theta^2(\theta+y-1)}{2(2\theta+y-1)^2}. \end{aligned} \quad (2.10)$$

The prices increase in capacity and the level of variety offered but beyond a certain level begin to drop, given the rival's chosen capacity i.e. the variety effect is stronger as the market grows but at a certain point, the negative expression in (2.10) due to increasing capacity grows stronger and dominates.

In order to characterize capacity and level of variety offered under the five agreement types, let $l = 1$ represent no pooling and $l = 2$ for pooling, which refers to the number of companies agreeing to pool, and let $r = 1$ for competition and $r = 0$ for cooperation. For example, $(l = 1, r = 1, y = 0)$ represents no agreement, $(l = 2, r = 0, y = 0)$ refers to capacity pooling and $(l = 2, r = 0, y = 1)$ represents profit maximization agreements. After solving for all agreement types, we can summarize the set of solutions as demonstrated in Proposition 2.2.

Proposition 2.2 *For each agreement type, a unique symmetric solution exists in capacities, levels of variety offered and prices as characterized by the first order conditions*

$$H \equiv -\frac{s^2(2\theta - r)f^*}{2m\theta(\theta-2)} + s\beta[\ln(1 + lf^*) + \frac{(r + (1-r)l)f^*}{1 + lf^*}] + s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c = 0, \quad (2.11)$$

where f^* represents the level of variety offered by each firm, and the prices are

$$p^*_{ab} \equiv -\frac{sf^*}{2m(\theta-2)} + \beta\ln(1 + lf^*) + \frac{(\theta + y - 1)\frac{\overline{\alpha}+\underline{\alpha}}{2} + \theta\alpha_{ab}}{2\theta + y - 1}. \quad (2.12)$$

The constraints in these solutions are always binding.

The term $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$ represents the initial profit margin, i.e. marginal profit from the first unit of capacity. Proposition 2.3 shows that this term is important for characterizing the ordering of agreement types with respect to capacity and level of variety offered.

Proposition 2.3 *(1) The level of variety offered f^* and the capacity $s \cdot f^*$ are increasing in β*
(2) $f^*_{competitive\ pooling} > f^*_{no\ agreement} > f^*_{capacity}$ and $f^*_{capacity\ pooling} > f^*_{capacity}$
(3) There exist $\hat{m}(\beta) > 0$ and $\hat{h}(m, \beta) > 0$ such that $f^*_{capacity\ pooling} > f^*_{competitive\ pooling}$ for $\frac{s}{2\beta\theta(\theta-2)} < m < \hat{m}(\beta)$ and $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c < \hat{h}(m, \beta)$. The opposite is true otherwise
(4) There exist $\tilde{m}(\beta) > 0$ and $\tilde{h}(m, \beta) > 0$ such that $f^*_{capacity\ pooling} > f^*_{no\ agreement}$ for $\frac{s}{4\beta\theta(\theta-2)} < m < \tilde{m}(\beta)$ and $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c < \tilde{h}(m, \beta)$. The opposite is true otherwise
(5) f^* is equal under capacity pooling and joint profit maximization agreements.

Proposition 2.3 shows that for sufficiently small demand magnitude and initial profit margin, capacity and level of variety offered are higher when firms reach a capacity pooling or joint profit maximization agreement compared to the competitive scenario.

Proposition 2.4 *(1) Price p^*_{ab} is increasing in β*
(2) $p^*_{capacity} > p^*_{no\ agreement}$
(3) For any $m, \beta > 0$ there exist $\hat{h}(m, \beta) > 0$ such that $p^*_{capacity\ pooling} > p^*_{competitive\ pooling}$, $p^*_{competitive\ pooling} > p^*_{no\ agreement}$ and $p^*_{capacity\ pooling} > p^*_{capacity}$ for $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c > \hat{h}(m, \beta)$
(4) For any $m, \beta > 0$ there exist $\tilde{h}(m, \beta) > 0$ such that $p^*_{competitive\ pooling} > p^*_{capacity}$ for $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c < \tilde{h}(m, \beta)$. The opposite is true otherwise
(5) Under no home bias, $p^*_{capacity\ pooling} = p^*_{joint\ profit}$. With home bias, joint profit maximization results in lower prices for local consumers and higher prices for non-local consumers compared to the capacity pooling agreement.

Proposition 2.4 shows that the greater the consumer values variety, the higher the prices charged. It also shows that prices are lowest when the two firms compete fully and that the type of agreement affects price, which in turn is dependent on the initial profit margin.

2.2 Effects of Agreements on Social Welfare

After computing capacities and prices per agreement type, we now evaluate their impact on consumer and producer surplus, the sum of which equals overall social welfare. Consumer welfare is defined as the utility (Equation 2.1) less prices paid for the product bundle summed over all consumers (see e.g., Hsu and Wang 2005):

$$\sum_b \frac{m}{2}[\theta \sum_{a'} (u_{a'b})^2 - (\sum_{a'} u_{a'b})^2]. \tag{2.13}$$

Proposition 2.5 *For each agreement type, the profit of each firm equals*

$$\pi^* \equiv -\frac{s^2(f^*)^2}{2m(\theta-2)} + \left(s\beta \ln(1 + lf^*) + s\frac{\overline{\alpha} + \underline{\alpha}}{2} - c\right) f^* + \frac{\theta^2(\theta + y - 1)m(\overline{\alpha} - \underline{\alpha})^2}{2(2\theta + y - 1)^2}, \tag{2.14}$$

the consumer utility equals

$$u^*_{ab} \equiv \frac{sf^*}{2m(\theta-2)} - \frac{(\theta + y - 1)(\frac{\overline{\alpha}+\underline{\alpha}}{2} - \alpha_{ab})}{2\theta + y - 1}, \tag{2.15}$$

the consumer welfare equals

$$\frac{s^2(f^*)^2}{2m(\theta-2)}+\frac{\theta(\theta+y-1)^2m(\overline{\alpha}-\underline{\alpha})^2}{2(2\theta+y-1)^2}, \tag{2.16}$$

and the total social welfare equals

$$-\frac{s^2(f^*)^2}{2m(\theta-2)}+2\left(s\beta\ln(1+lf^*)+s\frac{\overline{\alpha}+\underline{\alpha}}{2}-c\right)f^*+\frac{\theta(\theta+y-1)(3\theta+y-1)m(\overline{\alpha}-\underline{\alpha})^2}{2(2\theta+y-1)^2}. \tag{2.17}$$

Proposition 2.5 shows that capacity, level of variety offered and prices change according to agreement types, but their combined effect on consumer welfare (2.16) is captured by changes in capacity, i.e. higher capacity results in higher consumer surplus, despite potential increases in price. However, ranking agreements with respect to producer surplus and social welfare are less clear-cut. In Proposition 2.6 we rank agreement types according to producer profits highlighting the differences in ranking as a function of the size of the relevant market.

Proposition 2.6 *(1) Profit π^* is increasing in β*
(2) $\pi^*_{joint\ profit\ maximization} > \pi^*_{capacity\ pooling} > \pi^*_{capacity} > \pi^*_{no\ agreement}$ and $\pi^*_{capacity\ pooling} > \pi^*_{competitive\ pooling}$
(3) For any $m,\beta>0$ there exists $\hat{h}(m,\beta)>0$ such that $\pi^*_{competitive\ pooling} > \pi^*_{no\ agreement}$ for $s\frac{\overline{\alpha}+\underline{\alpha}}{2}-c\geq\hat{h}(m,\beta)$
(4) For any $m,\beta>0$ there exists $\tilde{h}(m,\beta)>0$ such that $\pi^*_{competitive\ pooling} > \pi^*_{capacity}$ for $s\frac{\overline{\alpha}+\underline{\alpha}}{2}-c<\tilde{h}(m,\beta)$. The opposite is true otherwise.
(5) Under no home bias, $\pi^*_{capacity\ pooling} = \pi^*_{joint\ profit}$. With home bias, joint profit maximization results in higher profit compared to the capacity pooling agreement.

Part (2) of Proposition 2.6 demonstrates that producers strictly prefer mergers in all cases and coordinating capacity is the next best outcome. Part (3) suggests that for a sufficiently high initial profit margin, the firms prefer pooling capacities over no agreement. Part (4) suggests that under thin market conditions it is in the interests of the firms to produce higher capacity under a pooling agreement, whilst still competing in capacity and prices, compared to a capacity agreement that lowers f without pooling.

Proposition 2.7 *(1) The consumer surplus and total social welfare are increasing in β*
(2) Consumer surplus ranking is the same as the capacity ranking for the first four agreement types
(3) Overall social welfare ranking is the same as the capacity ranking for the first four agreement types, except when comparing no agreement and capacity pooling, in which case higher threshold values than those appearing in part (4) of Proposition 2.3 are relevant
(4) Under no home bias, consumer surplus and social welfare are equal under joint profit maximization and capacity pooling agreements. With home bias, joint profit maximization results in higher consumer surplus and social welfare compared to capacity pooling.

Part (2) of Proposition 2.7 shows that capacity choice best predicts consumer surplus rankings hence the comparative statics are similar to that of the Cournot model. Part (3) of Proposition 2.7 demonstrates that overall social welfare is lower under no agreement compared to competitive pooling due to the consumer surplus component. In addition, when consumers demonstrate preferences for local products, part (4) shows that firms are better able to price discriminate under joint profit maximization such that both producers and consumers are better off. In other words, while capacity remains constant, prices for preferred consumers drop, resulting in more local customers served and a smaller price dispersion. The capacity pooling agreement has softened competition sufficiently to ensure higher prices than alternative options however prevents the firms from targeting preferred consumers. Joint profit maximization consequently allows each firm to concentrate on their home market to the advantage of producers and consumers alike. In summation, competitive pooling is the most preferable alternative in terms of consumer surplus and overall social welfare in standard sized markets with relatively low home bias and joint profit maximization is the best outcome in thin markets.

3 Airline Analysis under Demand Uncertainty

In this section, we analyze the potential effect of various aviation agreements in a given market with the aim of identifying the trade-offs between producers (airlines) and consumers (business and leisure passengers) surpluses. Generally country to country agreements will either permit a competitive market, in which airlines are free to set frequencies and prices in an unrestricted manner or a bilateral market, in which two or more airlines are designated as carriers. The resultant capacity agreement generally restricts frequency and/or plane sizes and sometimes limits pricing policies too. Airline-to-airline codeshare agreements are defined according to the following categories; competitive and cooperative block codeshare, competitive and cooperative free sale agreements and anti-trust immune alliances. Codeshare agreements require a marketing carrier to swap or purchase a set of seats on the operating carrier's flight in advance. The marketing carrier is then responsible for selling the seat inventory and receives all revenues from sales, which essentially pools the risk between the two carriers and increases their schedule offering. Codeshare agreements may also include schedule coordination with the aim of minimizing stopover time at a hub. Competitive block codeshares which would arise only in multi-lateral or competitive markets (such as within or across the United States and European Union) do not permit the airlines to set frequencies or prices cooperatively but do permit symmetric seat swaps both of leisure and business seats. Cooperative block codeshares permit airlines to set frequencies jointly but to still compete in prices. An example of this would be a bilateral constrained connection with a block codeshare between the two airlines offering direct service. Free sale competitive agreements assume that the carriers codeshare and pay a pre-agreed price or percentage of ticket revenues, provided the operating carrier can accommodate the request. Free sale cooperative agreements permit all of the aforementioned detail and allow carriers to coordinate frequencies, for example through a bilateral. Under anti-trust immune alliances, we assume that airlines maximize profits jointly by coordinating both frequency and price.

In this section, we first discuss the literature to date on the effects of airline agreements and then extend models (2.4) and (2.5) to consider demand uncertainty. In translating the agreement types specified in Table 1.1 into aviation terminology, bilaterals are the equivalent of capacity agreements, competitive codeshares equal competitive pooling agreements, cooperative codeshares in which airlines codeshare under a bilateral equal capacity pooling agreements and anti-trust immune alliances are equivalent to the joint profit maximization scenario. Finally, we present a numerical analysis of three airline markets in order to ascertain the effects of such agreements on producers, consumers and overall social welfare with the third airline offering an indirect service. The results of the free-sale agreements were identical to those of the codeshare except for the highly asymmetric markets hence are presented for only one of the destinations analyzed.

3.1 Effects of Airline Agreements on Social Welfare

International air service agreements were first established during the Chicago Convention of 1944. The framework developed was based on bilateral negotiations between countries and according to Borenstein and Rose (2007), liberalization of international agreements began in the late 1970s, modeled after the 1978 U.S.-Netherlands agreement which introduced greater flexibility in frequency and pricing but fell short of being competitively determined. Describing the exact behavior of airlines at the route level is subject to debate but appears to point to pricing between Cournot and Bertrand levels. After analyzing route competition between United and American Airlines, Brander and Zhang (1993) argued that some periods were characterized by cooperative behavior with price to marginal cost ratios higher than that implied by Cournot behavior and other periods were characterized by competitive behavior, although ratios were higher than those implied by Bertrand expectations. Airlines react to the level of aggressive competition with fares on average 10 to 20 percent higher when moving from duopoly to monopoly routes (Borenstein 1989, Hurdle et al. 1989, Oum, Zhang and Zhang 1993). Roller and Sickles (2000) argue that a two-stage product differentiated game better describes the European aviation market and arrive at the conclusion that one-stage specifications would result in biased results with regard to market power.

Much work has been undertaken using econometric analysis, largely in the U.S. where data is more accessible, mostly to analyze complementary codeshares including Borenstein (1990), Oum, Park and Zhang (1996), Brueckner and Whalen (2000), Brueckner (2001, 2003), Bamberger, Carlton and Neumann (2004), Armantier and Richard (2006, 2007) and Whalen (2007). Gillen, Harris and Oum (2002) model and measure the economic effects of bilateral agreements in the Japanese-Canadian market. Theoretical literature includes Park (1997), Hassin and Shy (2004), Heimer and Shy (2006) and Bilotkach (2007a, 2007b) amongst others. All the literature appears to agree on one point: passengers on parallel links may be worse off when the carriers enter a codeshare agreement but on complementary links the opposite is true. Brueckner and Whalen (2000) studied airfares and found that overlapping alliance service had a positive, but statistically insignificant, effect on gateway fares. Their point estimates suggested that an alliance between two previously non-allied carriers would raise fares on an overlap route by about 5 percent. The compensation mechanism embedded in the codeshare agreements relaxes airfare competition, thus although passengers have a greater range of frequencies from which to choose, the airlines are free to extract a higher surplus. However, recent literature including Gayle (2007), Czerny (2009), Wan, Zou and

Dresner (2009) and Brueckner and Proost (2010) have begun to question these conclusions. Gayle surmises that if competition between the airlines is weak (based on their elasticities), the equilibrium prices should not change much if they jointly price their products and indeed, he did not find any significant change between collusive and pre-alliance fares. Czerny argues that complementary airline codeshare agreements may not be welfare improving. Wan et al. suggest that the price effects of alliances on parallel routes may be either insignificantly different from the pre-alliance case or may even drop. Brueckner and Proost argue that under standard alliance agreements, consumers are worse off on parallel links due to higher prices but under joint venture alliances, carve outs would result in excessive efficiency losses by preventing the integration of operations and resultant cost savings.

Wen and Hsu (2006) present an interactive airline network design procedure to facilitate bargaining interactions required by international codeshare alliance agreements. Wen and Hsu argue that all airlines are strictly better off working under alliance agreements, however they did not include pricing as a direct issue and consequently were unable to analyze the consumers' perspective. Adler and Smilowitz (2007) present a framework to analyze global alliances and mergers in the airline industry under competition. The pressure on airlines to merge or ally would appear to be very strong, a point that is strengthened by the results of this model. Consequently, parallel airline agreements cannot be presumed either positive or negative and need to be assessed with respect to market conditions on a case-by-case basis. No-one has analyzed the parallel market in terms of consumer surplus, despite the fact that it represents the vast majority of agreements globally, nor evaluated the effect of the agreement type on both capacity and prices, which is the precise aim of this research.

3.2 Modeling Demand for Variety under Uncertainty

We extend the model of Section 2 to incorporate a third producer, two consumer types and demand uncertainty in the first stage of the game. We assume the existence of three airlines, two of which serve a direct link between their respective hubs and a third flies indirectly between the two nodes. The two airlines, $a \in \{1, 2\}$, with hubs at nodes 1 and 2 respectively, choose flight frequency f_a on the link and subsequently prices p_{ab} for a round trip in market $b \in \{1B, 1L, 2B, 2L\}$, where 1 or 2 represents the origin of the passenger's trip and B or L represent business and leisure markets, the former placing greater emphasis on frequency and the latter on price. The third airline, denoted by $a = 0$, offers a fixed indirect service with flight frequency f_a from the passenger's origin to their destination (nodes 1 or 2) but the corresponding prices p_{ab} remain decision variables. For flights serving market b, airline a's demand d_{ab} is assumed multiplicatively random linear $d_{ab}(m_b) = m_b A(\frac{1}{A\theta} + \theta v_{ab} - \sum_{a'} v_{a'b})$, where m_b is a random variable representing demand magnitude, $n = 3$ is the number of service providers, $\theta = n + 1$ distributes demand equally between $n + 1$ travel alternatives (including no travel) when the value resulting from all alternatives is zero, and v_{ab} for $a \in \{0, 1, 2\}$ is the additional value for a passenger, beyond the no travel or alternative mode option, when using airline a (thus $v_{ab} = u_{ab} - \frac{1}{A(n+1)}$) given by

$$v_{ab} = \alpha_{ab} + \beta_b \ln(1 + \sum_{a' \in \rho_a} f_{a'}) - p_{ab}, \tag{3.1}$$

where ρ_a is the set of code share partners of a and α_{ab} takes into account the additional utility due to frequent flyer programs or disutility for $a = 0$ due to indirect itineraries. If a passenger in a particular market has a preference for a specific airline e.g. the home-based airline, α_{ab} will capture their taste. The parameter A in the demand function represents the increase in normalized demand of each airline, $\frac{d_{ab}}{m_b}$, were all airlines to increase the utility of the passenger by one unit. f_0 is the frequency offered on the indirect itinerary and is assumed exogenous and fixed, however the indirect airline's prices remain endogenous.

Airlines 1 and 2 maximize their expected profits π_a in two stages by first choosing frequency f_a before demand realization, then setting prices $p_{ab}(m)$ as a function of the realization of the demand magnitude vector $m = (m_b)$ for all markets b. To model the free sale agreements we allow airline a to sell or buy excess seat capacity to/from their codeshare partner for a price τ paid to the operating carrier, set at the same stage as the frequencies. Denote by $t_{aa'b}(m)$ the number of type b passengers of airline a flying on the seats of airline a', to be determined after the realization of demand magnitude, m.

$$\begin{aligned}
\max_{(p_{ab})_b,(t_{aa'b})_{a'b}} \pi_a &= \sum_b \mathbb{E}_m[p_{ab}(m)\sum_{a'\in\rho_a} t_{aa'b}(m)] - c_a f_a \\
&+ \sum_{a'\in\rho_a\setminus\{a\}} \sum_b \mathbb{E}_m[t_{a'ab}(m) - t_{aa'b}(m)]\tau \\
s.t. \quad & \sum_{a'\in\rho_a} t_{aa'b}(m) \le d_{ab}(m_b),\ \forall b, m \\
& \sum_b t_{aab}(m) \le s_a f_a,\ \forall m \\
& \sum_{a'\in\rho_a\setminus\{a\}} \sum_b t_{aa'b}(m) \le \sum_{a'\in\rho_a\setminus\{a\}} \max\{0, s_{a'}f_{a'} - \sum_b t_{a'a'b}(m)\},\ \forall m,
\end{aligned} \tag{3.2}$$

where s_a is the exogenous plane size in terms of number of seats. The first constraint restricts airline a to selling no more that its own demand and the second constraint prevents airline a from selling more than its capacity. The third constraint restricts airline a to selling no more than the codeshare residual capacity (Hanany and Gerchak 2008). When code sharing has been agreed, demand depends on the total frequency of the code shared partners (the airlines other than 0) instead of individual frequency. Consumer surplus is computed by

$$\frac{1}{2}\sum_b \mathbb{E}_{m^b} m^b A \left(\theta \sum_a (u_a^b(m^b))^2 - (\sum_a u_a^b(m^b))^2\right). \tag{3.3}$$

3.3 Numerical Analysis of Airline Markets

Three markets are analyzed in this section; Tel Aviv to London, Brussels and Bangkok respectively. Based on direct demand levels for March 2007, as reported by the Israeli Ministry of Transport (IMOT), the Israeli-English market was relatively symmetric in size with 6,300 passengers carried per week. The Israeli-Belgian market was asymmetric to the extent that one third of travellers fly Brussels-Tel Aviv-Brussels and two thirds vice versa, in a 1,500 passenger per week market. Israel-Thailand is the most extreme example with over

95 percent of passengers travelling Tel Aviv-Bangkok-Tel Aviv in an 1,800 passenger per week market. It is public knowledge that there is a multilateral agreement on the Tel-Aviv-London link and bilateral agreements on the other two. No codeshare agreement exists on the London link and British Airways (BA) flew twice daily in each direction and El AL (LY) flew 11 flights in each direction per week in March 2007. El Al had codeshare agreements with both S.N. Brussels (SN) and Thai (TG) airlines, the former resulting in both airlines serving the link approximately 5 times a week and the latter with only El Al operating a service 3 times a week (according to El Al's website in March 2007). A third airline offering indirect service has also been included in the numerical analysis, in this case Lufthansa (LH), who were in a position to serve all three markets via their Frankfurt hub. We assume that consumers prefer direct connections, present a home carrier bias and that business travelers place a greater value on frequency and less on price than their leisure counterparts. We have assumed that approximately one seventh of the market is willing to purchase business class tickets based on aircraft configurations and relevant load factors. The parameters of the model have been calibrated such that the relevant equilibria outcome approximately reflects the results of the market for this season in terms of frequency and prices, permitting a what-if analysis of the potential market outcomes were different agreements to be signed among the relevant parties. Demand information has been collected from IMOT and the Israeli Central Bureau of Statistics (ICBS) websites and posted prices were collected from Orbitz for the relevant time frame.

Aside from adding a third player, the numerical analysis also includes demand uncertainty in the first stage, which then becomes known prior to pricing in the second stage. In order to develop five separate demand scenarios, we collected data on the total number of passengers carried to represent one scenario and then increased or decreased each market by 50 percent such that the correlation of demand across the four markets were close to zero (Swan 1993). In order to compute airline costs, Swan and Adler (2006) found that great circle distance, GCD, and the number of seats on an aircraft, s_a, are the two main factors affecting aircraft trip costs. Two market-based equations were developed based on average length of haul and aircraft size. Equation (3.4) gives the cost function for medium to short haul markets (i.e. less than 5,000 kilometers). Equation (3.5) provides the cost function for long haul markets (more than 5,000 kilometers).

$$c_a^{\text{short}} = \$0.038(GCD + 722)(s_a + 104) \tag{3.4}$$

$$c_a^{\text{long}} = \$0.023(GCD + 2200)(s_a + 211) \tag{3.5}$$

The two airlines offering direct service are assumed to be symmetric in costs.

Table 3.1: London-Tel Aviv Airline Market (per week)

(Surplus in $000s):	Competitive			Bilateral			Competitive Codeshare			Cooperative Codeshare			Alliance		
UK-Israel business surplus	3,366			3,058			3,579			3,451			3,028		
UK-Israel leisure surplus	1,125			772			1,039			899			959		
Israel-UK business surplus	3,190			2,881			3,418			3,288			2,831		
Israel-UK leisure surplus	1,032	8,714		680	7,391		946	8,982		806	8,444		851	7,669	
producer surplus (partial)		5,335			5,518			6,689			6,718			6,993	
social welfare ($000s)		14,048			12,909			15,671			15,162			14,663	
Demand-weighted average prices	LH	BA	LY	LH	BA	LY	LH	BA	LY	LH	BA	LY	LH	BA	LY
UK-Israel business	891	2,584	1,590	969	2,681	1,670	840	2,776	1,864	870	2,813	1,898	982	3,095	2,297
UK-Israel leisure	481	1,030	773	540	1,154	887	492	1,144	916	516	1,192	962	512	1,167	968
Israel-UK business	756	2,009	2,177	834	2,105	2,258	704	2,201	2,451	735	2,237	2,486	847	2,593	2,813
Israel-UK leisure	447	887	920	506	1,010	1,034	458	1,000	1,062	482	1,048	1,109	478	1,042	1,097
frequency	12.00	27.32	21.95	12.00	22.79	17.81	12.00	26.53	21.53	12.00	24.74	19.88	12.00	22.44	22.44

Table 3.2: Effect of Potential BA-El Al Agreements as compared to the Competitive Scenario

	Bilateral	Competitive Codeshare	Cooperative Codeshare	Alliance
business surplus	-9.40%	**6.73%**	2.79%	-10.62%
leisure surplus	-32.71%	-8.00%	-20.97%	-16.11%
consumer surplus	-15.17%	**3.08%**	-3.09%	-11.98%
producer surplus	3.43%	25.39%	25.93%	**31.09%**
social welfare	-8.11%	**11.55%**	7.93%	4.37%
prices business	3.14%	10.45%	11.62%	23.90%
prices leisure	8.18%	10.23%	13.13%	13.70%
frequency	-17.61%	-2.45%	-9.42%	-8.92%

Table 3.3: Sensitivity Analysis on London-Tel Aviv Market with respect to Frequency

Lufthansa frequency halved	Competitive	Bilateral	Competitive Codeshare	Cooperative Codeshare	Alliance
business surplus	-7.96%	-11.35%	-7.50%	-10.36%	-11.94%
leisure surplus	-32.16%	-61.74%	-34.93%	-52.67%	-50.56%
consumer surplus	-13.95%	-21.25%	-13.57%	-18.91%	-21.05%
producer surplus	15.64%	16.51%	13.00%	13.62%	13.25%
social welfare	-2.72%	-5.11%	-2.22%	-4.49%	-4.69%
prices business	9.52%	12.09%	9.76%	11.47%	11.06%
prices leisure	16.95%	24.09%	17.32%	22.14%	21.90%
frequency	1.23%	-0.29%	1.58%	-1.82%	-1.90%
Lufthansa frequency doubled					
business surplus	5.82%	7.46%	5.42%	6.55%	7.96%
leisure surplus	19.33%	34.30%	21.12%	28.78%	27.22%
consumer surplus	9.16%	12.73%	8.89%	11.04%	12.51%
producer surplus	-9.55%	-9.69%	-8.37%	-8.50%	-8.35%
social welfare	2.06%	3.15%	1.52%	2.38%	2.56%
prices business	-4.82%	-5.31%	-5.29%	-5.61%	-5.56%
prices leisure	-6.80%	-8.37%	-7.16%	-8.06%	-8.02%
frequency	-3.44%	-3.16%	-3.64%	-2.38%	-2.34%
Reduced importance of frequency (20%)					
business surplus	-16.06%	-15.46%	-16.81%	-16.96%	-16.90%
leisure surplus	-25.14%	-27.12%	-25.05%	-27.25%	-27.18%
consumer surplus	-18.31%	-17.75%	-18.63%	-19.04%	-19.33%
producer surplus	-13.80%	-14.10%	-16.73%	-16.67%	-16.59%
social welfare	-16.60%	-16.19%	-17.82%	-17.99%	-18.02%
prices business	-4.97%	-5.01%	-6.46%	-6.33%	-6.66%
prices leisure	-2.54%	-3.72%	-4.50%	-4.59%	-4.40%
frequency	-11.17%	-10.86%	-11.08%	-11.78%	-11.83%

Table 3.4: Brussels-Tel Aviv Airline Market (per week)

	Competitive			Bilateral			Competitive Codeshare			Cooperative Codeshare			Alliance		
Surplus (in $000s):															
Belgium-Israel business	1,102			907			1,162			1,134			1,012		
Belgium-Israel leisure	405			247			369			346			378		
Israel-Belgium business	2,137			1,757			2,258			2,206			1,937		
Israel-Belgium leisure	711	4,355		440	3,351		643	4,431		604	4,290		654	3,980	
producer surplus (partial)		1,920			1,920			2,579			2,579			2,714	
social welfare (in $000s)		6,275			5,270			7,010			6,869			6,694	
Demand-weighted average prices:	LH	SN	LY	LH	SN	LY	LH	SN	LY	LH	SN	LY	LH	SN	LY
Belgium-Israel business	771	2,169	1,349	898	2,275	1,447	730	2,396	1,579	746	2,411	1,594	839	2,662	1,947
Belgium-Israel leisure	306	769	561	388	948	737	321	902	697	332	926	721	321	885	707
Israel-Belgium business	681	1,575	1,950	808	1,682	2,048	640	1,803	2,180	655	1,818	2,195	749	2,144	2,474
Israel-Belgium leisure	283	621	711	365	799	887	298	753	848	309	777	871	298	756	839
frequency	12.00	11.54	11.61	12.00	6.98	6.84	12.00	10.95	10.66	12.00	10.35	10.08	12.00	10.61	10.61

Table 3.5: Bangkok-Tel Aviv Airline Market (per week)

	Competitive			Bilateral			Competitive Codeshare			Cooperative Codeshare			Free sale (τ=$310)			Alliance		
Surplus (in $000s):																		
Thailand-Israel business	74			70			77			78			78			66		
Thailand -Israel leisure	20			17			21			21			21			19		
Israel - Thailand business	4,252			4,058			4,403			4,440			4,421			4,024		
Israel - Thailand leisure	1,549	5,895		1,356	5,501		1,549	6,050		1,590	6,129		1,595	6,114		1,554	5,664	
producer surplus (partial)		2,375			2,376			2,444			2,447			2,474			2,586	
social welfare ($000s)		8,271			7,877			8,494			8,576			8,588			8,250	
Demand-weighted average prices	LH	TG	LY	LH	TG	LY	LH	TG	LY	LH	TG	LY	LH	TG	LY	LH	TG	LY
Thailand-Israel business	1,007	1,417	5,065	1,076	1,501	5,183	919	1,936	5,019	910	1,948	4,994	922	1,988	4,982	1,149	2,760	5,466
Thailand -Israel leisure	753	956	1,387	805	1,069	1,532	745	1,146	1,401	739	1,153	1,370	748	1,198	1,364	767	1,211	1,442
Israel - Thailand business	3,054	1,391	3,984	3,123	1,475	4,102	2,966	1,910	3,937	2,957	1,922	3,912	2,969	1,962	3,900	3,145	2,518	4,300
Israel - Thailand leisure	1,136	644	1,603	1,188	757	1,749	1,128	834	1,617	1,123	841	1,586	1,131	886	1,580	1,146	919	1,612
frequency	12.00	0.37	5.23	12.00	0.15	4.24	12.00	0.67	5.07	12.00	0.63	5.34	12.00	0.00	5.73	12.00	2.59	2.59

Table 3.6: Effect of Potential Thai-El Al Agreements as compared to the Competitive Scenario

	Bilateral	Competitive Code-share	Cooperative Code-share	Free sale	Alliance
business surplus	-4.58%	3.57%	4.44%	3.98%	-5.44%
leisure surplus	-12.53%	0.01%	2.66%	2.97%	0.23%
consumer surplus	-6.69%	2.62%	3.97%	3.71%	-3.93%
producer surplus	0.02%	2.91%	3.03%	4.17%	8.88%
social welfare	-4.76%	2.70%	3.70%	3.84%	-0.25%
prices business	2.86%	-2.36%	-2.55%	-1.82%	9.13%
prices leisure	5.49%	-0.81%	-1.18%	0.76%	2.80%
frequency	-21.79%	2.51%	6.50%	2.18%	-7.70%

The results, presented in Tables 3.1 to 3.6, first present the consumer surpluses per directional market and producer surpluses for the two direct carriers only. The indirect airline has an exogenously set frequency since we assume that it carries passengers to a substantial number of destinations not considered within the current framework but prices remain endogenous. Subsequently, the tables present demand-weighted average prices and frequencies. In the London-Tel Aviv market, presented in Table 3.1, it becomes clear that business travelers prefer the competitive codeshare agreement in which airlines compete for frequency and price but agree to codeshare via seat swap. Leisure travelers, however, strictly prefer the pure competitive situation and the airlines (British Airways and El Al) maximize profits under an alliance. It should be noted that under an anti-trust immune alliance, the two carriers coordinate both frequency and price in order to maximize joint profits and subsequently could split the frequencies such that only one airline serves the market up to sharing the flights evenly, which is the solution presented in Table 3.1. It is also possible to track the changes in prices and frequencies across the different agreement types and Table 3.2 presents the changes in these values with respect to the competitive scenario. Social welfare is maximized under the competitive codeshare scenario and the trade-off between the different agreement types becomes clear. Bilaterals severely dampen frequency (17 percent reduction) and increase prices (on average by 5 percent), such that the consumer surplus is reduced by 15 percent. The airlines prefer the stronger agreements (namely codeshare or alliance), since this permits higher prices and higher profits (up to 31 percent beyond those achieved in the competitive solution).

In the solution presented in Table 3.1, ranking of the agreement types based on the average capacity across firms leads to the following:

$$\begin{aligned} f^*_{\text{comp=no agreement}} &> f^*_{\text{comp codeshare=comp pooling}} > f^*_{\text{alliance=profit max}} \\ &> f^*_{\text{coop codeshare=capacity pooling}} > f^*_{\text{bilat=capacity}} \end{aligned} \tag{3.6}$$

Comparing this ranking with the analytical results of Proposition 2.3 shows that the only difference lies in the swapping of no agreement and competitive pooling. The reason for the difference lies in the fact that uncertainty in demand caused the load factor to drop below one, in which case the capacity constraint is no longer binding. As a result of uncertainty, competitive pooling allows airlines to reduce excess capacity further than would be optimal in the no agreement setting. The capacity pooling (cooperative codeshare) and capacity (bilateral) frequencies do not swap under uncertainty because the frequencies are already relatively low due to the cooperative setting and so do not contain the buffer available in the competitive setting. The prices are almost entirely consistent with the results of Proposition 2.4.

Table 3.3 presents a sensitivity analysis of the Tel-Aviv-London market with respect to the indirect airline's scheduled capacity and reduced consumer preference for capacity. If the indirect service was reduced by half i.e. Lufthansa's frequency was halved to one flight daily, consumer surplus drops by 14 to 21 percent whereas the two direct airlines enjoy increases in profit of 13 to 16 percent, due to a substantial rise in airfares and insignificant change in frequency. Alternatively, were the indirect carrier permitted to double frequencies, consumers would enjoy a 5 to 30 percent increase in surplus due to a reduction in prices despite a small decrease in direct service, whereas the direct carriers would lose approximately 9 percent in profits. The last part of Table 3.3 analyses the importance of frequency in the consumer's utility function by reducing β by 20 percent. The result (in comparison to the original base run) shows a decrease in frequency of around 11 percent and a reduction in prices of around 5 percent, independent of agreement type.

Table 3.4 analyses the Brussels-Tel Aviv market in which the two current carriers, S.N. Brussels and El Al, have a codeshare agreement. The market is strictly limited by the bilateral that each country signed in which a single designated carrier per country is defined limiting frequency between the two cities per season. The general pattern of results is similar to that of the Tel Aviv-London market. The codeshare agreement reduces frequencies by 11 percent compared to the competitive situation and increases prices by approximately 10 percent. The bilateral, on the other hand, reduces frequencies by 40 percent and increases prices by approximately 5 percent over the competitive solution. Consequently, if the transportation authorities were forced to choose between pure bilaterals or codeshares, the latter would be strictly preferable. In this market, the competitive codeshare agreement maximizes social welfare, whilst the anti-trust immune alliance maximizes the two direct airlines profits. Again, the different consumer types show disagreement, with leisure passengers preferring a competitive market outcome and business passengers maximizing surplus through the competitive codeshare agreement.

Finally, in Table 3.5 and 3.6, we analyze the Bangkok-Tel Aviv market, in which the demand is rather weak and one sided, with a market of around 100,000 passengers annually of which over 95 percent originates from Israel. This is a reasonably standard picture for a tourist destination. Social welfare is maximized in this market under the free sale agreement although consumers are fairly ambivalent to the codeshare or free sale agreement and the direct carriers would still prefer to ally. Both consumer and producer surplus are higher under free sale or codeshare agreements as compared to the competitive outcome (Table 3.6). It should be noted that the Israel-Belgian market is slightly smaller hence low demand is not a sufficient reason for permitting airlines to ally, rather the costs of serving the market also need to be considered. The costs to fly Tel Aviv-Bangkok are twice as high due to the greater distances involved, although once seat capacity is taken into account, the costs to fly to Bangkok are only 40 percent higher per seat. In this case, it could be argued that regulatory authorities or departments of transport should permit the airlines to sign an agreement on this gateway-to-gateway link since it is in the interests of both consumers and producers alike.

4 Conclusions and Future Directions

The modeling framework developed in this research accounts for the value of variety and price in the consumer's demand function. The impact of this change leads to new insights into the effects of mergers and alliances on consumer surplus, suggesting that not all mergers are necessarily detrimental to consumers. In particular, benefits to consumers from pooling agreements may outweigh disutilities arising from capacity reduction by colluding firms. Conditions under which these effects occur were characterized analytically, alongside ranking of various agreement types with respect to capacity, level of variety offered, prices and surpluses of both producers and consumers.

In a numerical section, we then analyzed the impacts of country and airline agreements on capacity and prices on gateway-to-gateway routes in the aviation industry. The hybrid competitive and cooperative model permits analysis under asymmetric and uncertain demand of business and leisure travellers' surplus, airline profits and overall social welfare. It is clear that there is a trade-off in the travelers' utility function between frequency/capacity (the higher the better) and airfare (the lower the better). The traveler may also have carrier preferences which can be accounted for as vertical differentiation variables in the utility function. Competition authorities interested in understanding which agreement is most appropriate on parallel links, can observe that the bilateral agreements are consistently the worst of all possibilities, drastically reducing frequencies and increasing prices. If a competitive pooling/codeshare is not an alternative (this choice is only relevant on domestic flights or under 'open-skies' policies), then in most cases the competitive outcome is the most appropriate with respect to consumer surplus, however this is not always true. On a thin route, it is possible to identify cases in which cooperative block codes-shares or free sale agreements are the most preferable market outcomes. Furthermore, whilst leisure passengers in reasonable demand markets prefer the no agreement outcome, business passengers prefer competitive or cooperative codeshares and overall consumer surplus is also maximized under codeshares. Airlines strictly prefer alliances and in terms of overall social welfare, this outcome may still be preferable to that of competition, dependent on market parameter values.

In conclusion, there is a need to analyze each codeshare request individually and the modeling approach presented here is relatively simple and could be applied when necessary. Up until recently, the literature argued that complementary codeshares are considered positive whereas parallel agreements are always onerous on consumers. This modeling approach shows that such results are over-simplistic. Indeed frequencies may be lower than the competitive scenario (anywhere between 2 and 10 percent lower) and prices might increase (from 10 percent for codeshares up to 20 percent for anti-trust immune alliances), yet the trade-off between opposing interests of consumer types and the producers suggests that competitive codeshares are not always a bad alternative on the gateway-to-gateway links, dependent on amongst other things the size of the market. It may also be worthwhile pushing departments of transport towards the relaxation of bilateral agreements which consistently proved to be the worst market outcome, suggesting that multi-laterals or opening the skies are strictly preferable for all consumers.

In considering future directions, clearly this model is very simplistic and requires assumptions which it may be interesting to reduce or remove. For example, on the theoretical level, the assumption of linear demand which could be adapted to include a logit style function, the log of the level of variety offered which could be replaced with a concave increasing function, and the consideration of asymmetric information reflecting uncertainty about the type of consumer purchasing the ticket may enrich the analysis in new directions. We also have not considered the potential cost advantages firms may gain when reaching an agreement that would further strengthen the results arrived at here. It would also be interesting to widen the analysis to consider networks, which would permit a more in-depth evaluation of the effects of agreements within oligopolies.

References

Adler, N., K. Smilowitz. 2007. "Hub-and-spoke air network alliances and mergers: Price-location competition in the airline industry." *Transportation Research part B*. 41(4): 394-407.

Allon, G., A. Federgruen. 2009. "Competition in service industries with segmented markets." *Management Science*, 55(4): 619-634.

Armantier, O., O. Richard. 2006. "Evidence on pricing from the Continental Airlines and Northwest Airlines codeshare agreement." In *Advances in Airline Economics 1*, ed. D. Lee, 91-108. Amsterdam: Elsevier.

Armantier, O., O. Richard. 2007. "Domestic airline alliances and consumer welfare." SSRN Discussion Paper 869240.

Bamberger, G. E., D. W. Carlton, L. R. Neumann. 2004. "An empirical investigation of the competitive effects of domestic airline alliances." *Journal of Law and Economics*, 47(1): 195-222.

Belobaba, P. 2009. "Overview of airline economic, markets and demand." In *The global airline industry*, ed. P. Belobaba, 47-72. Wiley.

Bilotkach, V. 2007a. "Complementary vs. semi-complementary airline partnerships." *Transportation Research part B*, 41(4): 381-393.

Bilotkach, V. 2007b. "Airline partnerships and schedule coordination." *Journal of Transport Economics and Policy*, 41(3): 413-425.

Borenstein, S. 1989. "Hubs and high fares: dominance and market power in the U.S. airline industry." *RAND Journal of Economics*, 20(3): 344-365.

Borenstein, S. 1990. "Airline mergers, airport dominance and market power." *The American Economic Review*, 80(2): 400-404.

Borenstein, S., N. L. Rose. 2007. "How airline markets work... or do they? Regulatory reform in the airline industry." NBER Working Paper 13452.

Brander, J. A., A. Zhang. 1993. "Dynamic oligopoly in the airline industry." *International Journal of Industrial Organization*, 11(3): 407-35.

Brueckner, J. K. 2001. "The economics of international code sharing: An analysis of airline alliances." *International Journal of Industrial Organization*, 19(10): 1475-1498.

Brueckner, J. K. 2003. "International airfares in the age of alliances: the effects of code sharing and antitrust immunity." *Review of Economic Statistics*, 85(1): 105-118.

Brueckner, J. K., S. Proost. Forthcoming. "Carve-outs under airline antitrust immunity." *Journal of Urban Economics*.

Brueckner, J. K., R. Flores-Fillol. 2007. "Airline schedule competition." *Review of Industrial Organization*, 30(3): 161-177.

Brueckner, J. K., W. T. Whalen. 2000. "The price effects of international airline alliances." *Journal of Law & Economics*, 43(2): 503 – 545.

Copeland, A., W. Dunn, G. Hall. 2010. "Inventories and the automobile market." Brandeis University Working Paper.

Czerny, A. I. 2009. "Codesharing, price discrimination and welfare losses." *Journal of Transport Economics and Policy*, 43(2): 193-210.

Davidson, C., R. Deneckere. 1986. "Long-run competition in capacity, short-run competition in price, and the Cournot model." *Rand Journal of Economics*, 17(3): 404-415.

Deneckere, R., C. Davidson. 1985. "Incentives to form coalitions with Bertrand competition." *Rand Journal of Economics*, 16(4): 473-486.

Friedman, J. W. 1988. "On the strategic importance of prices versus quantities." *Rand Journal of Economics*, 19(4): 607-622.

Gayle, P. G. 2007. "Airline codeshare alliances and their competitive effects." *Journal of Law and Economics*, 50(4): 781-819.

Gillen, D., R. Harris, T. H. Oum. 2002. “Measuring the economic effects of bilateral liberalization in air transport.” *Transportation Research part E*, 38(3-4): 155-174.

Hanany, E., Y. Gerchak. 2008. “Nash Bargaining over Allocations in Inventory Pooling Contracts.” *Naval Research Logistics*, 55(6): 541-550.

Hansen, M. 1990. “Airline competition in a hub-dominated environment: An application of non-cooperative game theory.” *Transportation Research part B*, 24(1): 27-43.

Hassin, O., O. Shy. 2004. “Codesharing agreements and interconnections in markets for international flights.” *Review of International Economics*, 12(3): 337 – 352.

Heimer, O., O. Shy. 2006. “Codesharing agreements, frequency of flights and profits under parallel operation.” In *Competition Policy and Antitrust*, ed. D. Lee, 163–182. Oxford: Elsevier.

Hsu, J., X. H. Wang. 2005. “On welfare under Cournot and Bertrand competition in differentiated oligopolies.” *Review of Industrial Organization*, 27(2): 185-191.

Hurdle, G..J., R. L. Johnson, A. S. Joskow, G. J. Warden, M. A. Williams. 1989. “Concentration, potential entry and performance in the airline industry.” *Journal of Industrial Economics*, 38(2): 119-139.

Kreps, D. M. 1979. “A representation theorem for ‘preference for flexibility’”. *Econometrica*, 47(3): 565–577.

Kreps, D. M., J. A. Scheinkman. 1983. “Quantity precommitment and Bertrand competition yield Cournot outcomes.” *The Bell Journal of Economics*, 14(2): 326-337.

Martin, S. 1999. “Kreps and Scheinkman with product differentiation: an expository note.” University of Copenhagen, Centre for Industrial Economics Discussion Paper 1999-11.

Oum, T. H., J-H Park, A. Zhang. 1996. “The effects of airline codesharing agreements on firm conduct and international air fares.” *Journal of Transportation Economics and Policy*, 30(2): 187-202.

Oum, T. H., A. Zhang, Y. Zhang. 1993. “Inter-firm rivalry and firm-specific price elasticities in deregulated airline markets.” *Journal of Transport Economics and Policy*, 27(2): 171-192.

Park, J-H. 1997. “The effect of airline alliances on markets and economic welfare.” *Transportation Research part E*, 33(3): 181-195.

Pels, E., P. Nijkamp, P. Rietveld. 2000. “Airport and airline competition for passengers departing from a large metropolitan area.” *Journal of Urban Economics*, 48(1): 29-45.

Roller, L-H, R. C. Sickles. 2000. “Capacity and product market competition: measuring market power in a ’puppy-dog’ industry.” *International Journal of Industrial Organization*, 18(6): 845-865.

Singh, N., X. Vives. 1984. "Price and quantity competition in a differentiated duopoly." *Rand Journal of Economics*, 15(4): 546-554.

Swan, W. 1993. "Modeling variance for yield management." Boeing Commercial Aircraft, Seattle, WA, working Paper. (www.seaburyapg.com/swan)

Swan, W, N. Adler. 2006. "Aircraft trip cost parameters: A function of stage length and seat capacity." *Transportation Research part E*, 42(2): 105-115.

Wan, X., L. Zou, M. Dresner. 2009. "Assessing the price effects of airline alliances on parallel routes." *Transportation Research part E*, 45(4): 627-641.

Wen, Y-H, C-I Hsu. 2006. "Interactive multiobjective programming in airline network design for international airline codeshare alliance." *European Journal of Operational Research*, 174(1): 404-426.

Whalen, W. T. 2007. "A panel data analysis of codesharing, antitrust immunity and open skies treaties in international aviation markets." *Review of Industrial Organization*, 30(1): 39-61.

Yin, X., Y-K Ng. 1997. "Quantity precommitment and Bertrand competition yield Cournot outcomes: a case with product differentiation." *Australian Economic Papers*, 36: 14-22.

A Appendix

Proof of Proposition 1. Under the first four agreements, each firm's profit function can be formulated with the capacity constrained Lagrange multiplier λ_a, representing the value of having an additional unit of capacity, as

$$\sum_b m(k_{ab} - \theta p_{ab} + \sum_{a'} p_{a'b})p_{ab} - cf_a - \lambda_a(\sum_b d_{ab} - sf_a),$$

where $k_{ab} \equiv \theta[\alpha_{ab} + \beta \ln(1 + \sum_{a' \in \rho_a} f_{a'})] - \sum_{a'}[\alpha_{a'b} + \beta \ln(1 + \sum_{a'' \in \rho_{a'}} f_{a''})]$ depends on the first stage choices, but not on the second stage prices. The first order conditions for airline a and market b are

$$0 = m(k_{ab} - (2\theta - 1)p_{ab} + \sum_{a'} p_{a'b} + (\theta - 1)\lambda_a).$$

Solving this system of linear equations for a and b, each equation can be rewritten as $(2\theta - 1)p_{ab} - \sum_{a'} p_{a'b} = k_{ab} + (\theta - 1)\lambda_a$. Summing for a, $\sum_{a'} p_{a'b} = \frac{1}{2\theta-3} \sum_{a'} k_{a'b} + \frac{\theta-1}{2\theta-3} \sum_{a'} \lambda_{a'}$. Substituting and rearranging terms we find

$$p_{ab} = \frac{1}{2\theta - 1}[k_{ab} + \frac{1}{2\theta - 3} \sum_{a'} k_{a'b} + (\theta - 1)\lambda_a + \frac{\theta - 1}{2\theta - 3} \sum_{a'} \lambda_{a'}].$$

We assume here that the capacity constraint is binding for each a (this is verified in the proof of Proposition 2.2). Substituting p_{ab} into the binding constraint $\sum_b d_{ab} = sf_a$, we have

$\lambda_a\theta - \frac{\theta-1}{2\theta-3}\sum_{a'}\lambda_{a'} = \frac{1}{2m}\gamma_a$, where $\gamma_a \equiv m\sum_b(k_{ab} + \frac{1}{2\theta-3}\sum_{a'}k_{a'b}) - \frac{2\theta-1}{\theta-1}sf_a$. Summing for a, $\sum_{a'}\lambda_{a'} = \frac{1}{2m}\frac{2\theta-3}{(2\theta-1)(\theta-2)}\sum_{a'}\gamma_{a'}$. Substituting and rearranging terms,

$$\lambda_a = \frac{1}{2\theta m}[\gamma_a + \frac{\theta-1}{(2\theta-1)(\theta-2)}\sum_{a'}\gamma_{a'}].$$

Since $\sum_{a'}\lambda_{a'} = \frac{1}{2m}\frac{2\theta-3}{(2\theta-1)(\theta-2)}\sum_{a'}\gamma_{a'}$, we can substitute to find

$$p_{ab} = \frac{1}{2\theta-1}[k_{ab} + \frac{1}{2\theta-3}\sum_{a'}k_{a'b} + \frac{1}{2m}\frac{\theta-1}{\theta}\gamma_a + \frac{1}{2m}\frac{\theta-1}{\theta(\theta-2)}\sum_{a'}\gamma_{a'}].$$

Substituting $\sum_{a'}\gamma_{a'} = m\frac{2\theta-1}{2\theta-3}\sum_b\sum_{a'}k_{a'b} - \frac{2\theta-1}{\theta-1}s\sum_{a'}f_{a'}$ and $\sum_{a'}k_{a'b} = (\theta-2)\sum_{a'}[\alpha_{a'b} + \beta\ln(1+\sum_{a''\in\rho_{a'}}f_{a''})]$ and rearranging terms, the expression for $\widetilde{p}_{ab}(f_a, f_{-a})$ is proved for $y=0$. A similar derivation proves the expression for $y=1$. Substituting these prices into the profit function, we arrive at the expression for $\widetilde{\pi}_a(f_a, f_{-a})$. ■

Proof of Proposition 2. In a symmetric solution, the levels of variety offered f_a are equal for both firms. Under no agreement and the competitive pooling agreement, the levels of variety offered are determined in Nash equilibrium. Differentiating $\widetilde{\pi}_a(f_a, f_{-a})$, since $\frac{\partial\widetilde{\pi}_a}{\partial f_a}(0,0) = s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c > 0$, $f_a = 0$ for both a cannot be in equilibrium. Since

$$\frac{\partial\widetilde{\pi}_a}{\partial f_a} = -\frac{s^2[(2\theta-3)f_a + \sum_{a'}f_{a'}]}{2m\theta(\theta-2)} + s\beta[\ln(1+\sum_{a'\in\rho_a}f_{a'}) + \frac{f_a}{1+\sum_{a'\in\rho_a}f_{a'}}] + s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$$

is concave in f_a and attains negative values for sufficiently high f_a, for any opponent's level of variety offered, f_{-a}, there exists a unique level of variety offered $f_a(f_{-a}) > 0$ such that $\frac{\partial\widetilde{\pi}_a}{\partial f_a} = 0$ and $\frac{\partial^2\widetilde{\pi}_a}{\partial(f_a)^2} < 0$, thus forming the best response function of firm a to f_{-a}. Since this function is continuously decreasing in f_{-a}, it has a unique fixed point f^*, forming a Nash equilibrium and satisfying $\frac{\partial\widetilde{\pi}_a}{\partial f_a}(f^*, f^*) = 0$. This proves that the first order condition $H = 0$ characterizes the equilibrium for $r=1$, $y=0$ and $l=1$ or 2.
Under capacity and capacity pooling agreements, levels of variety offered are determined according to the Nash bargaining solution. The derivative of $\prod_a[\widetilde{\pi}_a(f_a, f_{-a}) - \widetilde{\pi}_a(f_a^{comp}, f_{-a}^{comp})]$ at a symmetric point equals $\sum_{a'}\frac{\partial\widetilde{\pi}_{a'}}{\partial f_a}(f,f)[\widetilde{\pi}_{-a'}(f,f) - \widetilde{\pi}_{-a'}(f^{comp}, f^{comp})]$. Since symmetry implies that $\widetilde{\pi}_1(f_1,f_2) = \widetilde{\pi}_2(f_2,f_1)$, $\sum_{a'}\frac{\partial\widetilde{\pi}_{a'}}{\partial f_a}(f,f) = \frac{\partial[\widetilde{\pi}_a(f,f)]}{\partial f}$. Since

$$\frac{\partial[\widetilde{\pi}_a(f,f)]}{\partial f} = -\frac{s^2(2\theta)f}{2m\theta(\theta-2)} + s\beta[\ln(1+lf) + \frac{lf}{1+lf}] + s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$$

is strictly concave in f, there exist f' close to f^{comp} such that $\widetilde{\pi}_a(f', f') > \widetilde{\pi}_a(f^{comp}, f^{comp})$ for both a, thus f^{comp} cannot be a solution. Adding that $\frac{\partial[\widetilde{\pi}_a(f,f)]}{\partial f}(0) > 0$, a symmetric solution must satisfy $\frac{\partial[\widetilde{\pi}_a(f,f)]}{\partial f} = 0$. Concavity implies that there exists a unique $f^* > 0$ satisfying this condition as well as $\frac{\partial^2[\widetilde{\pi}_a(f,f)]}{\partial f^2}(f^*) < 0$, thus proving that the first order condition $H = 0$ characterizes the solution for $r=0$, $y=0$ and $l=1$ or 2.
It remains to be proved that the first order condition under joint profit maximization, i.e.

for $l=2$,$r=0$ and $y=1$, characterizes the solution. An argument similar to the one for the Nash bargaining solution can be used to show that since the derivatives of $\sum_a \widetilde{\pi}_a(f_a, f_{-a})$ are positive at $(0,0)$, symmetry implies that a symmetric solution must satisfy $\frac{\partial[\widetilde{\pi}_a(f,f)]}{\partial f}=0$, the existence of which was already established. Note that this derivative does not depend on y, verifying the condition in the proposition.

We now show that the capacity constraint is binding for each a under $H=0$, as assumed in Proposition 2.1. We first show that given the solution characterized above, the case where the constraint is not binding for a but is binding for its opponent leads to a contradiction. In this case the prices are

$$\begin{aligned} p_{ab} &= \frac{1}{2\theta-2}(k_{ab}+p_{(-a)b}) \\ &= -\frac{sf}{4m(\theta-1)(\theta-2)}+\frac{1}{2}[\frac{2\theta\alpha_a^b-\frac{\overline{\alpha}+\underline{\alpha}}{2}}{2\theta-1}+\beta\ln(1+lf)], \end{aligned}$$

thus $\sum_b d_{ab}=-\frac{s(2\theta-3)f}{2(\theta-2)}+m(\theta-1)[\frac{\overline{\alpha}+\underline{\alpha}}{2}+\beta\ln(1+lf)]$. Since $H=0$,

$$\begin{aligned} \sum_b d_{ab}-sf &= \sum_b d_{ab}-sf+\frac{m}{s}(\theta-1)H \\ &= \frac{(2\theta-r)(\theta-1)-\theta(2\theta-3)}{2\theta(\theta-2)}sf+m(\theta-1)[\frac{c}{s}-\beta\frac{(r+(1-r)l)f}{1+fl}] \\ &> 0 \end{aligned}$$

because $\frac{c}{s}>\beta$, violating the capacity constraint. Thus the constraint must be binding for both a.

Now we show that there is no equilibrium where the constraint is not binding for both a. In this case, the prices are $p_{ab}=\frac{1}{2\theta-1}[k_{ab}+\frac{1}{2\theta-3}\sum_{a'}k_{a'b}]$ (see proof of Proposition 2.1). With these prices, a similar analysis to the one given above for binding constraints leads to the following first order condition for the level of variety offered:

$$\begin{aligned} &\frac{(\theta-2)(\theta-1)[(2\theta-1)(\theta-2)+(\theta-1)r(2-l)][r+(1-r)l]}{(1+lf)(2\theta-3)^2(2\theta-1)} \\ &\qquad \cdot 4m\beta[\frac{\overline{\alpha}+\underline{\alpha}}{2}+\beta\ln(1+lf)]-c = 0. \end{aligned}$$

Moreover, total demand for firm a equals

$$\begin{aligned} \sum_b d_{ab} &= \sum_b m(k_{ab}-\theta p_{ab}+\sum_{a'}p_{a'b}) \\ &= \frac{2m(\theta-2)(\theta-1)}{2\theta-3}[\frac{\overline{\alpha}+\underline{\alpha}}{2}+\beta\ln(1+lf)]. \end{aligned}$$

Substituting the first order conditions we have $\sum_b d_{ab}=\frac{c(1+f)(2\theta-3)(2\theta-1)}{2\beta[(2\theta-1)(\theta-2)+r(\theta-1)]}$ for $l=1$ and $\frac{c(1+2f)(2\theta-3)}{2\beta(\theta-2)(2-r)}$ for $l=2$, where $r=0$ or 1. Both expressions are greater than $\frac{cf}{\beta}$. Since $\frac{c}{s}>\beta$, in both cases the demand is higher than sf, violating the constraint. Therefore the constraint

must be binding for both a.
The expression for p^*_{ab} follows immediately from $\widetilde{p}_a(f_a, f_{-a})$ in Proposition 2.1 under equal levels of variety offered by both firms. ■

Proof of Proposition 3. To show (1), note that total differentiation of H from Proposition 2.2 implies that $\frac{\partial f^*}{\partial \beta} = -\frac{\partial H/\partial \beta}{\partial H/\partial f^*}$. Since H is positive at $f^* = 0$ and strictly concave in f^*, $H = 0$ implies $\frac{\partial H}{\partial f^*} < 0$. Since $\frac{\partial H}{\partial \beta} > 0$, $\frac{\partial f^*}{\partial \beta} > 0$.
To show (2), note that

$$\frac{1}{s\beta}(H_{\text{competitive pooling}} - H_{\text{no agreement}}) = \ln(\frac{1+2f}{1+f}) + \frac{f}{1+2f} - \frac{f}{1+f}$$

equals 0 at $f = 0$, increases for $0 < f < \frac{1+\sqrt{5}}{2}$, decreases for $f > \frac{1+\sqrt{5}}{2}$ and converges to $\ln 2 - \frac{1}{2} > 0$ as $f \to \infty$, thus is positive for $f > 0$, implying that $f^*_{\text{competitive pooling}} > f^*_{\text{no agreement}}$. Similarly,

$$\frac{1}{s\beta}(H_{\text{capacity pooling}} - H_{\text{capacity}}) = \ln(\frac{1+2f}{1+f}) + \frac{2f}{1+2f} - \frac{f}{1+f}$$

equals 0 at $f = 0$ and increases for $f > 0$, thus is also positive for $f > 0$ implying that $f^*_{\text{capacity pooling}} > f^*_{\text{capacity}}$. Since $H_{\text{no agreement}} - H_{\text{capacity}} = \frac{s^2}{2m\theta(\theta-2)}f > 0$ for $f > 0$, $f^*_{\text{no agreement}} > f^*_{\text{capacity}}$.
To prove (3) consider the difference

$$\delta(f) \equiv H_{\text{capacity pooling}} - H_{\text{competitive pooling}} = -\frac{s^2 f}{2m\theta(\theta-2)} + \frac{s\beta f}{1+2f}.$$

Since $\delta(0) = 0$ and $\delta(f)$ is strictly concave for $f > 0$, a necessary condition for $\delta(f) > 0$ is $\frac{\partial \delta}{\partial f}(f = 0) = s\beta - \frac{s^2}{2m\theta(\theta-2)} > 0$, i.e. $m > \frac{s}{2\beta\theta(\theta-2)}$. Under this necessary condition, since $\frac{\partial \delta}{\partial f} < 0$ for f sufficiently large, there exists $\bar{f} > 0$, increasing in m and β, such that $\delta = 0$ at $f = \bar{f}(m, \beta)$, and $\delta(f) > 0$ if and only if $0 < f < \bar{f}(m, \beta)$. Thus

$$f^*_{\text{capacity pooling}} > f^*_{\text{competitive pooling}} \text{ if and only if } H(f = \bar{f}(m, \beta)) < 0.$$

Let $\bar{H} = H - (s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c)$. Since $\frac{\partial \bar{H}_{\text{capacity pooling}}}{\partial f}(f = 0) = 4s\beta - \frac{s^2}{m(\theta-2)}$, it equals $-2\beta s(\theta-2) < 0$ for $m = \frac{s}{2\beta\theta(\theta-2)}$. Since $\bar{H}_{\text{capacity pooling}}$ is concave in f and continuous in m, $\bar{H}_{\text{capacity pooling}}(f = \bar{f}(m, \beta)) < 0$ for sufficiently small $m > \frac{s}{2\beta\theta(\theta-2)}$. Since $\bar{H}_{\text{capacity pooling}}(f = \bar{f}(m, \beta))$ increases in m above some point where it is still negative, there exists $\hat{m} > 0$, that depends on β, such that $\bar{H}(f = \bar{f}(m, \beta)) < 0$ if and only if $\frac{s}{2\beta\theta(\theta-2)} < m < \hat{m}(\beta)$. For m in this range, letting $\hat{a}(m, \beta) = -\bar{H}(f = \bar{f}(m, \beta))$, we have

$$s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c < \hat{a}(m, \beta) \text{ if and only if } H(f = \bar{f}(m, \beta)) < 0,$$

thus proving (3).

The proof of (4) is similar to (3), with the changes:

$$\begin{aligned}\delta(f) &\equiv H_{\text{capacity pooling}} - H_{\text{no agreement}} \\ &= -\frac{s^2 f}{2m\theta(\theta-2)} + s\beta[\ln(\frac{1+2f}{1+f}) + \frac{2f}{1+2f} - \frac{f}{1+f}],\end{aligned}$$

$\frac{\partial\delta}{\partial f}(f=0) = 2s\beta - \frac{s^2}{2m\theta(\theta-2)} > 0$, $m > \frac{s}{4\beta\theta(\theta-2)}$, and $\frac{\partial \bar{H}_{\text{capacity pooling}}}{\partial f}(f=0) = -4\beta s(\theta-1) < 0$ for $m = \frac{s}{4\beta\theta(\theta-2)}$.

(5) follows immediately because H does not depend on y. ■

Proof of Proposition 4. (1) follows from total differentiation of price with respect to β since $\frac{df^*}{d\beta} > 0$ by Proposition 2.3.

To prove (2) first note that

$$\begin{aligned}p^*_{ab} &= \frac{1}{sf^*}[\pi^* - \frac{\theta^2(\theta+y-1)m(\overline{\alpha}-\underline{\alpha})^2}{2(2\theta+y-1)^2}] \\ &\quad -(\frac{\overline{\alpha}+\underline{\alpha}}{2} - \frac{c}{s}) + \frac{(\theta+y-1)\frac{\overline{\alpha}+\underline{\alpha}}{2} + \theta\alpha_{ab}}{2\theta+y-1}.\end{aligned}$$

Thus (2) follows because $\pi^*_{\text{capacity}} > \pi^*_{\text{no agreement}}$ and $f^*_{\text{capacity}} < f^*_{\text{no agreement}}$ by propositions 2.6 and 2.3. A similar argument shows that $p^*_{\text{capacity pooling}} > p^*_{\text{competitive pooling}}$ for sufficiently large values of $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$.

To prove the remainder of (3) note that since $H = 0$,

$$\begin{aligned}p^*_{ab} &= p^*_{ab} - \frac{1}{s}H \\ &= \frac{s(\theta-r)f^*}{2m\theta(\theta-2)} - \beta\frac{(r+(1-r)l)f^*}{1+lf^*} - (\frac{\overline{\alpha}+\underline{\alpha}}{2} - \frac{c}{s}) + \frac{(\theta+y-1)\frac{\overline{\alpha}+\underline{\alpha}}{2} + \theta\alpha_{ab}}{2\theta+y-1}.\end{aligned}$$

Then

$$\begin{aligned}p^*_{\text{competitive pooling}} - p^*_{\text{no agreement}} &= (\frac{s(\theta-1)}{2m\theta(\theta-2)} - \frac{\beta}{1+2f^*_{\text{competitive pooling}}})f^*_{\text{competitive pooling}} \\ &\quad -(\frac{s(\theta-1)}{2m\theta(\theta-2)} - \frac{\beta}{1+f^*_{\text{no agreement}}})f^*_{\text{no agreement}}.\end{aligned}$$

Since $f^*_{\text{competitive pooling}} > f^*_{\text{no agreement}}$ by Proposition 2.3, and f^* is increasing in the initial profit margin $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$, the price difference is positive for sufficiently large $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$. This proves the existence of a threshold $\hat{h}(m,\beta)$. A similar argument applies to the comparison

$$\begin{aligned}p^*_{\text{capacity pooling}} - p^*_{\text{capacity}} &= (\frac{s}{2m(\theta-2)} - \frac{2\beta}{1+2f^*_{\text{capacity pooling}}})f^*_{\text{capacity pooling}} \\ &\quad -(\frac{s}{2m(\theta-2)} - \frac{\beta}{1+f^*_{\text{capacity}}})f^*_{\text{capacity}}.\end{aligned}$$

To prove (4), note that since $f^*_{\text{no agreement}} - f^*_{\text{capacity}} > 0$ by Proposition 2.3,

$$-\frac{s^2}{2m\theta(\theta-2)}[(2\theta-1)f^*_{\text{no agreement}} - (2\theta)f^*_{\text{capacity}}] < H_{\text{no agreement}} - H_{\text{capacity}} = 0,$$

thus $\frac{2\theta}{2\theta-1}f^*_{\text{capacity}} < f^*_{\text{no agreement}} < f^*_{\text{competitive pooling}}$. Therefore

$$\begin{aligned} p^*_{\text{competitive pooling}} - p^*_{\text{capacity}} &> [(\frac{s(\theta-1)}{2m\theta(\theta-2)} - \frac{\beta}{1+2\frac{2\theta}{2\theta-1}f^*_{\text{capacity}}}) \\ &\quad (\frac{2\theta}{2\theta-1})^2 - (\frac{s\theta}{2m\theta(\theta-2)} - \frac{\beta}{1+f^*_{\text{capacity}}})]f^*_{\text{capacity}}. \end{aligned}$$

The right hand side is positive when f^*_{capacity} is sufficiently low, which holds for sufficiently small $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$. For large values of $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$, $f^*_{\text{competitive pooling}} - \frac{2\theta}{2\theta-1}f^*_{\text{capacity}}$ approaches a positive constant and $p^*_{\text{competitive pooling}} - p^*_{\text{capacity}}$ approaches $\frac{s}{2m\theta(\theta-2)}[(\theta-1)(f^*_{\text{competitive pooling}})^2 - \theta(f^*_{\text{capacity}})^2]$. Since $(\theta-1)(\frac{2\theta}{2\theta-1})^2 - \theta = -\frac{\theta}{(2\theta-1)^2} < 0$, we have $p^*_{\text{competitive pooling}} < p^*_{\text{capacity}}$. This proves the existence of a threshold $\tilde{h}(m,\beta)$ that determines where the price ranking reverses.

Finally, (5) is verified because $\overline{\alpha} > \underline{\alpha}$ implies that as y changes from 0 to 1, $\frac{(\theta+y-1)\frac{\overline{\alpha}+\underline{\alpha}}{2}+\theta\alpha_{ab}}{2\theta+y-1}$ decreases for $\alpha_{ab} = \overline{\alpha}$ and increases for $\alpha_{ab} = \underline{\alpha}$. ■

Proof of Proposition 5. The expression for π^* follows immediately from $\widetilde{\pi}_a(f_a, f_{-a})$ in Proposition 2.1 under equal levels of variety offered by both firms. Similarly, the expression for u^*_{ab} follows from p^*_{ab} in Proposition 2.2 and the definition of u_{ab}. The consumer surplus is then

$$\begin{aligned} \sum_b \frac{m}{2}[\theta\sum_{a'}(u^*_{a'b})^2 - (\sum_{a'}u^*_{a'b})^2] &= \frac{m}{2}[\theta[4(\frac{sf^*}{2m(\theta-2)})^2 + (\frac{\theta+y-1}{2\theta+y-1})^2(4(\frac{\overline{\alpha}+\underline{\alpha}}{2})^2 \\ &\quad -4\frac{(\overline{\alpha}+\underline{\alpha})^2}{2} + 2(\overline{\alpha}^2+\underline{\alpha}^2))] - 2(\frac{2sf^*}{2m(\theta-2)})^2] \\ &= \frac{s^2(f^*)^2}{2m(\theta-2)} + \frac{\theta(\theta+y-1)^2 m(\overline{\alpha}-\underline{\alpha})^2}{2(2\theta+y-1)^2}. \end{aligned}$$

The total social welfare then follows from summing the profits of both firms and the consumer surplus. ■

Proof of Proposition 6. (1) follows from total differentiation of profit with respect to β since $\frac{df^*}{d\beta} > 0$ by Proposition 2.3.

To prove (2) note that by Proposition 2.3, $f_a = f^*_{\text{capacity pooling}}$ for both a maximizes the sum of profits $\sum_a \widetilde{\pi}_a(f_a, f_{-a})$ under the capacity pooling agreement. Thus $\pi^*_{\text{capacity pooling}} > \pi^*_{\text{capacity}}$, because for any $f \geq 0$, in particular for $f = f^*_{\text{capacity}}$, the difference between $\sum_a \widetilde{\pi}_a(f,f)$ under the capacity pooling and capacity agreements equals $2s\beta f^*_{\text{capacity}} \ln(\frac{1+2f}{1+f}) > 0$. According to the Nash bargaining solution definition, $\pi^*_{\text{capacity}} \geq \pi^*_{\text{no agreement}}$ and $\pi^*_{\text{capacity pooling}} \geq \pi^*_{\text{competitive pooling}}$. The proof of Proposition 2.2 shows that the inequalities are strict because there exist f' close to f^{comp} such that $\widetilde{\pi}_a(f', f') > \widetilde{\pi}_a(f^{comp}, f^{comp})$ for both a. Since

$f^*_{\text{joint profit maximization}} = f^*_{\text{capacity pooling}}$,

$$\begin{aligned}\pi^*_{\text{joint profit maximization}} - \pi^*_{\text{capacity pooling}} &= \frac{\theta m(\overline{\alpha}-\underline{\alpha})^2}{2}\left(\frac{\theta}{2\theta} - \frac{\theta-1}{2\theta-1}\right) \\ &= \frac{\theta m(\overline{\alpha}-\underline{\alpha})^2}{4(2\theta-1)} > 0.\end{aligned}$$

It remains to prove (3) and (4). Since $H = 0$,

$$\begin{aligned}\pi^* &= \pi^* - f^* H \\ &= \left[\frac{s^2(\theta-r)}{2m\theta(\theta-2)} - s\beta\frac{(r+(1-r)l)}{1+lf^*}\right](f^*)^2 + \frac{\theta^2(\theta+y-1)m(\overline{\alpha}-\underline{\alpha})^2}{2(2\theta+y-1)^2}.\end{aligned}$$

To prove (3), note that

$$\begin{aligned}\pi^*_{\text{competitive pooling}} - \pi^*_{\text{no agreement}} &= \left(\frac{s^2(\theta-1)}{2m\theta(\theta-2)} - \frac{s\beta}{1+2f^*_{\text{competitive pooling}}}\right)(f^*_{\text{competitive pooling}})^2 \\ &\quad -\left(\frac{s^2(\theta-1)}{2m\theta(\theta-2)} - \frac{s\beta}{1+f^*_{\text{no agreement}}}\right)(f^*_{\text{no agreement}})^2.\end{aligned}$$

Since $f^*_{\text{competitive pooling}} > f^*_{\text{no agreement}}$ by Proposition 2.3, and f^* is increasing in the initial profit margin $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$, the profit difference is positive for sufficiently large $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$, thus proving the existence of a threshold $\hat{h}(m,\beta)$.
To prove (4), note that since $f^*_{\text{no agreement}} - f^*_{\text{capacity}} > 0$ by Proposition 2.3,

$$-\frac{s^2}{2m\theta(\theta-2)}[(2\theta-1)f^*_{\text{no agreement}} - (2\theta)f^*_{\text{capacity}}] < H_{\text{no agreement}} - H_{\text{capacity}} = 0,$$

thus $\frac{2\theta}{2\theta-1}f^*_{\text{capacity}} < f^*_{\text{no agreement}} < f^*_{\text{competitive pooling}}$. Therefore

$$\begin{aligned}\pi^*_{\text{competitive pooling}} - \pi^*_{\text{capacity}} &> \left[\left(\frac{s^2(\theta-1)}{2m\theta(\theta-2)} - \frac{s\beta}{1+2\frac{2\theta}{2\theta-1}f^*_{\text{capacity}}}\right)\left(\frac{2\theta}{2\theta-1}\right)^2\right. \\ &\quad \left.-\left(\frac{s^2\theta}{2m\theta(\theta-2)} - \frac{s\beta}{1+f^*_{\text{capacity}}}\right)\right](f^*_{\text{capacity}})^2.\end{aligned}$$

The right hand side is positive when f^*_{capacity} is sufficiently low, which holds for sufficiently small $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$. For large values of $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$, $f^*_{\text{competitive pooling}} - \frac{2\theta}{2\theta-1}f^*_{\text{capacity}}$ approaches a positive constant and $\pi^*_{\text{competitive pooling}} - \pi^*_{\text{capacity}}$ approaches $\frac{s^2}{2m\theta(\theta-2)}[(\theta-1)(f^*_{\text{competitive pooling}})^2 - \theta(f^*_{\text{capacity}})^2]$. Since $(\theta-1)(\frac{2\theta}{2\theta-1})^2 - \theta = -\frac{\theta}{(2\theta-1)^2} < 0$, $\pi^*_{\text{competitive pooling}} < \pi^*_{\text{capacity}}$. This proves the existence of a threshold $\tilde{h}(m,\beta)$ that determines where the profit ranking reverses.
Finally, (5) is verified because $\overline{\alpha} > \underline{\alpha}$ implies that $\frac{\theta^2(\theta+y-1)m(\overline{\alpha}-\underline{\alpha})^2}{2(2\theta+y-1)^2}$ increases as y changes from 0 to 1. ■

Proof of Proposition 7. (1) and (2) hold because the consumer surplus and total social welfare are increasing in β and f^*, and because f^* is increasing in β by Proposition 2.3. To prove (3), consider the first order condition for maximizing the total social welfare with respect to the level of variety offered, given by

$$H_W \equiv -\frac{s^2\theta f}{2m\theta(\theta-2)} + s\beta[\ln(1+lf) + \frac{lf}{1+lf}] + s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c = 0.$$

Since H_W is strictly concave in f and equals $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c > 0$ at $f = 0$, there exists a unique f^W satisfying this first order condition, thus maximizing the total social welfare. Fixing the value of l, since f^* for $r = 0$ is less than f^* for $r = 1$ by Proposition 2.3, and the difference

$$H_W - H = s\beta(l-1)r\frac{f}{1+fl} + \frac{s^2 f}{2m\theta(\theta-2)}(\theta - r) > 0,$$

increasing r from 0 to 1 while fixing l improves the total social welfare. There is an improvement also when l increases alone or together with r because the total social welfare is increasing in l. This proves the comparison for the first four agreement types, except when comparing no agreement and capacity pooling. In this case note that a higher level of variety offered under the capacity pooling implies higher total social welfare because both l and f^* increase. However there is a range of values where l and f^* move in opposite directions such that no agreement results in a higher level of variety offered but lower total social welfare. For sufficiently high values of $s\frac{\overline{\alpha}+\underline{\alpha}}{2} - c$, as the levels of variety offered move further apart, no agreement results also in higher total social welfare, thus proving the higher threshold result in (3).

Finally, (4) is verified because $\overline{\alpha} > \underline{\alpha}$ implies that $\frac{\theta(\theta+y-1)(3\theta+y-1)m(\overline{\alpha}-\underline{\alpha})^2}{2(2\theta+y-1)^2}$ increases as y changes from 0 to 1. ∎