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Existence and Uniqueness of Ordinal Nash Outcomes[1]

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In this paper we present necessary and sufficient conditions for existence and uniqueness of ordinal Nash outcomes. These outcomes are derived from the ordinal Nash solution—a reinterpretation and an extension of the Nash bargaining solution that allows bargainers to have preference relations that are more general than expected utility. Our task is undertaken by the construction of a new notion called "induced utilities". *Journal of Economic Literature* Classification Number: C78. © 2000 Academic Press

1. INTRODUCTION

Since its publication, the axiomatic Nash bargaining solution [13] has attracted considerable attention in the economic literature. In Nash's formulation, it is assumed that the players are expected utility (EU) maximizers. Recently, Rubinstein, Safra, and Thomson [15] reinterpreted Nash's bargaining problem. The ordinal Nash solution that they define is characterized by axioms that refer to the preference relations alone. Rather than specifying the Nash outcome as the one that maximizes a product of utilities, the ordinal Nash outcome is characterized as an outcome against which no player can successfully appeal. Rubinstein, Safra, and Thomson examine the family of preference relations for which a unique ordinal Nash solution exists and extend the domain of the solution beyond that of EU preference relations.

Building on the above, Grant and Kajii [7] extend the family of preference relations over which the ordinal Nash solution is well defined. The "disagreement linear" (DL) preference relations that they define behave like EU on the set of elementary lotteries (lotteries whose support consists of the disagreement outcome and at most one other outcome). In a related paper, Grant and Kajii [8] further extend the set of preference relations for which

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the ordinal Nash solution exists by considering a notion of marginal boldness (see Aumann and Kurz [1]).[2]

Several other papers also deal with the ordinal Nash solution. Houba, Tieman, and Brinksma [11] characterize ordinal Nash outcomes for preference relations with separable representations over elementary lotteries; Hanany and Safra [10] extend the ordinal Nash solution to other sets of preference relations; Valenciano and Zarzuelo [20] analyze an asymmetric ordinal Nash solution; finally, Denicolo [6] applies the ordinal Nash solution to a different domain of bargaining problems.[3]

Since these papers only present sufficient conditions for the existence of ordinal Nash outcomes, it is as yet unclear the extent to which the ordinal Nash solution is well defined. In this paper we suggest a solution to this problem by identifying *necessary* and *sufficient* conditions for the existence of ordinal Nash outcomes. Then, we present necessary and sufficient conditions for the existence of a *unique* ordinal Nash outcome. The conditions that we identify are advantageous since they refer to *individual* preference relations. This stands in contrast to existing alternative characterizations that are either too restrictive or that involve joint conditions, on both preference relations (in some works, conditions are even stated jointly on the preference relations and on the outcome set).

To begin, we develop an approach that treats every bargaining problem in a way that "locally" resembles the Nash EU treatment. This is achieved by defining new functions that are referred to as "induced utilities." The induced utility of a given player measures his willingness to switch from an existing bargaining position to an alternative, whilst considering the risk of disagreement. As shown in Section 2, a necessary and sufficient condition for an outcome to be immune to appeals is that it maximizes the Nash-product of the induced utilities, assuming the outcome itself is the existing bargaining position. In a sense, for any given bargaining position the induced utility approach creates two artificial EU players who agree with the original players in their desire to appeal against the existing outcome. If the original players are EU maximizers then the artificial EU players would be identical to them. Hence, when EU players are discussed, the induced utility approach is reduced to the classic Nash approach.

We then use the induced utility approach to characterize the set of preference relations over which the ordinal Nash outcome is well defined. Since an ordinal Nash outcome is both immune to appeals and Pareto efficient, we start by identifying conditions that refer to the former property.

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[2] Koskievic [12] presents similar results. Burgos, Grant, and Kajii [2] use the notion of marginal boldness for analysing a sequential bargaining problem.

[3] Two completely different extensions of the original Nash solution to non-EU preferences are suggested by Safra and Zilcha [19].

In Section 3 we identify conditions on individual preference relations that are both necessary and sufficient for the existence of outcomes that are immune to appeals. In other words, we define a set of preference relations with the following property: First, an outcome that is immune to appeals exists for each couple of preference relations in the set; second, if a preference relation does not belong to the set then there exists another preference relation such that the resulting bargaining problem has no appeals immune outcome. Other necessary and sufficient conditions (and another set of preference relations) are then identified, ensuring the existence of a unique outcome that is immune to appeals.

In Section 4 we add the requirement of Pareto efficiency and identify conditions on individual preference relations that are both necessary and sufficient for the existence of ordinal Nash outcomes. This is achieved by adding a weak form of risk-aversion to the conditions of Section 3. Finally, we identify necessary and sufficient conditions for the existence of a unique ordinal Nash outcome.

2. DEFINITIONS

We consider two-player bargaining games that are characterized by elements of the form $\langle \mathbf{X}, D, \succsim_1, \succsim_2 \rangle$ where $\mathbf{X}$ is a set of deterministic outcomes, D is the disagreement outcome and $\succsim_i$ $(i = 1, 2)$ are the players preference relations over $\mathscr{L}(\mathbf{X} \cup \{D\})$, the set of simple (finite) lotteries over $\mathbf{X} \cup \{D\}$. For a given D, a lottery in $\mathscr{L}(\mathbf{X} \cup \{D\})$ is denoted by $(\vec{\mathbf{x}}, \mathbf{p}) = (\mathbf{x}^1, ..., \mathbf{x}^n, p^1, ..., p^n)$, $\sum_{k=1}^{n} p^k \leqslant 1$, with the convention that p^k is the probability of $\mathbf{x}^k \in \mathbf{X}$ and $1 - \sum_{k=1}^{n} p^k$ is the probability of D. When $n = 1$ and $\mathbf{x} \succsim_i D$, $i = 1, 2$, the lottery is called an *elementary* lottery and is denoted $p\mathbf{x}$. We assume that $\mathbf{X} \subseteq \mathbb{R}_{+}^{2}$ is compact connected and fixed. Without loss of generality we consider $\mathbf{X} = \{\mathbf{x} = (x_1, x_2) \in \mathbb{R}_{+}^{2} \mid x_1 + x_2 \leqslant 1\}$.[4] We also assume that $D \in \dot{\mathbf{X}}$, where $\dot{\mathbf{X}} = \{\mathbf{x} \in \mathbf{X} \mid x_1 + x_2 < 1\}$. Denote the (relative to D) efficient frontier of $\mathbf{X}$ by

$$F(D) = \{\mathbf{x} \in \mathbf{X} \mid \mathbf{x} \geqslant D \text{ and } (\mathbf{y} \in \mathbf{X}) \wedge (\mathbf{y} \geqslant \mathbf{x}) \Rightarrow \mathbf{y} = \mathbf{x}\}.^5$$

Since $\mathbf{X}$ is fixed, we do not specify it in the notation below. Hence, for example, $\mathscr{L}(\mathbf{X} \cup \{D\})$ is denoted $\mathscr{L}(D)$. We also assume that, for every $\mathbf{x} \in \mathbf{X}$, the preference relation $\succsim_i$ depends only on the ith coordinate x_i. A lottery $l \in \mathscr{L}(D)$ can therefore be described by the pair (l_1, l_2) of its

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[4] The results of Sections 2 and 3 can be easily modified to handle the more general outcome sets. The situation is different with respect to the results of Section 4, where the definition of risk aversion explicitly utilizes the specific linear structure of the boundary of $\mathbf{X}$.

[5] We use the following notations: $\mathbf{x} \geqslant \mathbf{y} \Leftrightarrow x_i \geqslant y_i$, $i \in \{1, 2\}$ and $\mathbf{x} > \mathbf{y} \Leftrightarrow x_i > y_i$, $i \in \{1, 2\}$.

marginal distributions. Let $\mathscr{P}(D)$ denote the set of all preferences that satisfy the condition above and are complete, transitive, monotone with respect to the relation of first-order stochastic-dominance (with strict monotonicity when elementary lotteries are considered) and continuous (with respect to the topology of weak convergence) on the set of elementary lotteries. The preference set $\mathscr{P}(D)$ contains the familiar set of EU preferences as well as more general non-EU preferences. Without loss of generality, preferences in $\mathscr{P}(D)$ are considered to be defined over $\mathscr{L}([0, 1])$ —the set of lotteries over $[0, 1]$—and the preference set of player i denoted $\mathscr{P}(D_i)$. As usual, $\sim_i$ and $\succ_i$ denote the symmetric and asymmetric components of $\succsim_i$, respectively. Given two preference relations $\succsim_1 \in \mathscr{P}(D_1)$, and $\succsim_2 \in \mathscr{P}(D_2)$ let

$$\mathbf{F}(D, \succsim_1, \succsim_2) = \left\{ \ell \in \mathscr{L}(D) \,\middle|\, \begin{array}{c} \forall i \in \{1, 2\}\ \ell_i \succsim_i D_i \text{ and } \forall i \in \{1, 2\} \\ (\bar{\ell} \in \mathscr{L}(D)) \wedge (\bar{\ell}_i \succsim_i \ell_i) \Rightarrow \bar{\ell} \sim_i \ell \end{array} \right\}$$

be the set of Pareto efficient lotteries (relative to D). Finally, for any set $M \subseteq \mathbb{R}^2_+$, let M_i denote its projection on the ith coordinate of $\mathbb{R}^2_+$.

Given a bargaining problem $\langle D, \succsim_1, \succsim_2 \rangle$, an outcome $\mathbf{x}$ is an *appeal against* $\mathbf{y}$ if there exist $i \in \{1, 2\}$ and $p \in [0, 1]$ such that $p\mathbf{x} \succ_i \mathbf{y}$ while $\mathbf{x} \succ_j p\mathbf{y} (j \neq i)$ (see Rubinstein, Safra, and Thomson). The intuition behind this definition is that, if both players perceive the probability of breakdown to be $1 - p$, player i is willing to take the risk of a possible breakdown when insisting on $\mathbf{x}$ while player j is unwilling to do so when insisting on $\mathbf{y}$. An outcome that is immune to all possible appeals can be defined as:

Definition 2.1. Let $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times \mathscr{P}(D_1) \times \mathscr{P}(D_2)$ be a bargaining problem. An outcome $\mathbf{y}^* \in \mathbf{X}$ is *immune to appeals* (*appeals-immune*) in $\langle D, \succsim_1, \succsim_2 \rangle$ if

$$\forall \mathbf{x} \in \mathbf{X}, \qquad \forall p \in [0, 1], \qquad \forall i, j \in \{1, 2\}, \qquad i \neq j, \qquad p\mathbf{x} \succ_i \mathbf{y}^* \Rightarrow p\mathbf{y}^* \succsim_j \mathbf{x}$$

Note that an appeals-immune outcome must belong to $F(D)$, the efficient frontier of $\mathbf{X}$. Sometimes, however, an appeals-immune outcome does not belong to the set of Pareto efficient lotteries $\mathbf{F}(D, \succsim_1, \succsim_2)$. This is the case, for example, when the preference relations are EU while the von Neumann–Morgenstern (vNM) utility functions are not concave. Note that if $\mathbf{y}^*$ is appeals-immune then $\mathbf{y}^* > D$. This is the case since $y_i^* \leqslant D_i$ implies that any $\mathbf{x}$ with $x_i > D_i$ is an appeal (made by i) against $\mathbf{y}^*$.

Definition 2.2. Let $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times \mathscr{P}(D_1) \times \mathscr{P}(D_2)$ be a bargaining problem. An appeals-immune outcome $\mathbf{y}^* \in \mathbf{X}$ is an *ordinal Nash* outcome of $\langle D, \succsim_1, \succsim_2 \rangle$ if it belongs to $\mathbf{F}(D, \succsim_1, \succsim_2)$.

As was shown by Rubinstein, Safra, and Thomson, the notion of an ordinal Nash outcome generalizes that of a Nash outcome. More explicitly, whenever both $\succsim_i$ are EU preference relations represented by the vNM utilities v_i and $[\ell \in \mathbf{F}(D, \succsim_1, \succsim_2) \Rightarrow \exists \mathbf{x} \in F(D)$ such that $x_i \sim_i \ell_i, i \in \{1, 2\}]$, the outcome that maximizes the product $\prod_i (v_i(x_i) - v_i(D_i))$ over $\{\mathbf{x} \in \mathbf{X} \mid \mathbf{x} \geqslant D\}$ is also an ordinal Nash outcome. Moreover, there exist bargaining problems in which the preference relations are not of the EU type for which the ordinal Nash outcome is well defined and unique.

Comment. Rubinstein, Safra, and Thomson introduced a *convexity* assumption on the bargaining problem (for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $\alpha \in [0, 1]$, there exist $\mathbf{z} \in \mathbf{X}$ such that $\mathbf{z} \sim_i \alpha\mathbf{x} + (1 - \alpha)\mathbf{y}$, for both i). For EU preference relations, this assumption guarantees $\mathbf{F}(D, \succsim_1, \succsim_2) = \mathbf{F}(D)$, hence every appeals-immune outcome is an ordinal Nash outcome as well. For non-EU preference relations, additional restrictions are required to ensure this property. Such is their *conditional certainty equivalent** assumption (for all $\mathbf{x} \in \mathbf{X}$, $\ell', \ell \in \mathscr{L}(D)$ and $\alpha \in [0, 1]$, $\mathbf{x} \succsim_i \ell' \Rightarrow \alpha\mathbf{x} + (1 - \alpha)\ell \succsim_i \alpha\ell' + (1 - \alpha)\ell$). Note that such conditions must involve lotteries whose total number of outcomes is unbounded. As is shown by Safra and Segal [16], non-EU preference relations exist that nevertheless behave like EU preferences on the set of all lotteries with n outcomes at most. Therefore, the cardinal convexity assumption introduced by Grant and Kajii [7] (for all $\mathbf{x} \neq \mathbf{y} \in \mathbf{X}$ there exists $\mathbf{z} \in \mathbf{X}$ such that for each i: $pB_i \sim_i \mathbf{x} \wedge qB_i \sim_i \mathbf{y} \Rightarrow \mathbf{z} \succsim_i \frac{1}{2}(p + q) B_i$, where B_i is ith best outcome in $\mathbf{X}$) may sometimes not be sufficiently strong to ensure the equality $\mathbf{F}(D, \succsim_1, \succsim_2) = F(D)$.

We now turn to the main new definition of this paper—that of induced utilities. Given a bargaining problem $\langle D, \succsim_1, \succsim_2 \rangle$, let $\mathscr{U}_i(D_i)$ be the set of all continuous functions u_i: $\{\mathbf{x} \in \mathbf{X} \mid \mathbf{x} \geqslant D\}_i \times \{\mathbf{x} \in \mathbf{X} \mid \mathbf{x} > D\}_i \to \mathbb{R}_+$ that increase in their first argument, decrease in the second, satisfy $u_i(t; t) = 1$, $u_i(D_i; t) = 0$ and $u_i(s; t)\, u_i(t; s) = 1$.

DEFINITION 2.3. Let $D \in \dot{\mathbf{X}}$ and let $i \in \{1, 2\}$. The *ith induced utility mapping* is the function IU_i: $\mathscr{P}(D_i) \to \mathscr{U}_i(D_i)$, that is defined by

$$IU_i(\succsim_i)(s; t) = \begin{cases} p & \text{if } \; s \sim_i pt \\ \dfrac{1}{p} & \text{if } \; t \sim_i ps \end{cases}$$

The function $u_i = IU_i(\succsim_i)$ is the induced utility of $\succsim_i$.

It is easy to see that the induced utilities u_i, whenever they are well defined, satisfy:

$$px_i \sim_i y_i \Leftrightarrow pu_i(x_i; x_i) = u_i(y_i; x_i) \Leftrightarrow pu_i(x_i; y_i) = u_i(y_i; y_i)$$

Therefore, if the second arguments are taken as reference points and only elementary lotteries are considered, the induced utilities locally behave like vNM utilities of some EU preference relations that are in agreement with the given preferences.

EXAMPLE 2.4. (1) *EU preference relations.* If $\succsim_i$ is an EU preference relation with a vNM utility function v_i then $u_i(x_i; y_i) = (v_i(x_i) - v_i(D_i))/(v_i(y_i) - v_i(D_i))$. It is straightforward to see that, qualitatively speaking, the induced utilities of an EU preference are independent of the reference point $\mathbf{y}$ and of the disagreement outcome D.

(2) *Multiplicatively separable preferences relations.* According to these preference relations, the utility value of an elementary lottery px_i to a player with a preference relation $\succsim_i$ is given by $g_i(p)\, v_i(x_i) + (1 - g_i(p))\, v_i(D_i)$, where g_i: $[0, 1] \to [0, 1]$ is increasing and onto and v_i: $[0, 1] \to \mathbb{R}$ is increasing. The probability transformation function g_i is uniquely determined while the utility function v_i is unique up to affine transformations. EU preference relations belong to this set (there, $g_i(p) = p$). The set of DL preferences, introduced by Grant and Kajii [7], is a subset of this set, too; it contains non-EU preferences for which $g_i(p) = p$ while the independence axiom does not hold for some non-elementary lotteries. Another non-EU family that is a subset of this set is the family of rank-dependent utility (RDU) preference relations (see Quiggin [14] and Weymark [21]). The value that an RDU preference relation $\succsim_i$ associates with a lottery ℓ is given by $V_i(\ell) = \int v_i(x_i)\, \mathrm{d}[1 - g_i(1 - F_\ell(x_i))]$, where F_ℓ is the cumulative distribution function of ℓ. Another non-EU family that is a subset of the set of multiplicatively separable preferences is Gul's [9] disappointment aversion (DA) family. The value that a DA preference relation $\succsim_i$ associates with a lottery ℓ is given by

$$V_i(\ell) = \frac{\gamma(\alpha)}{\alpha} \int_{x_i > c_i(\ell)} v_i(x_i)\, \mathrm{d}F_\ell(x_i) + \frac{1 - \gamma(\alpha)}{1 - \alpha} \int_{x_i < c_i(\ell)} v_i(x_i)\, \mathrm{d}F_\ell(x_i),$$

where $c_i(\ell) \sim_i \ell$ is the certainty equivalent of ℓ, α is the probability that ℓ yields an outcome above its certainty equivalent and $\gamma(\alpha) = \alpha/(1 + (1 - \alpha)\, \beta_i)$ for some number β_i. Clearly, $g_i(p) = p/(1 + (1 - p)\, \beta_i)$.

For multiplicatively separable preferences, if the utility v_i is chosen such that $v_i(D_i) = 0$ then

$$u_i(x_i; y_i) = \begin{cases} g_i^{-1}\left(\dfrac{v_i(x_i)}{v_i(y_i)}\right) & \text{if } y_i \geqslant x_i \\ \dfrac{1}{g_i^{-1}(v_i(y_i)/v_i(x_i))} & \text{if } x_i > y_i \end{cases}.$$

The following lemma demonstrates that every function in $\mathcal{U}_i(D_i)$ is the induced utility of at least one preference in $\mathcal{P}(D_i)$.

LEMMA 2.5. *Let* $D \in \dot{\mathbf{X}}$ *and* $i \in \{1, 2\}$. *For all* $u_i \in \mathcal{U}_i(D_i)$, $IU_i^{-1}(u_i) \neq \emptyset$.

Proof. See Appendix.

The properties of the induced utilities will now be exploited in order to characterize appeals-immune outcomes in a parallel manner to that of Nash outcomes. Consider a given bargaining problem $\langle D, \succsim_1, \succsim_2 \rangle$. Define a correspondence $\boldsymbol{\delta}$: $\{\mathbf{x} \in \mathbf{X} \mid \mathbf{x} > D\} \to 2^{\mathbf{X}}$ by

$$\boldsymbol{\delta}(\mathbf{y}) = \arg \max_{D \leqslant \mathbf{x} \in \mathbf{X}} \left\{ \prod_i u_i(x_i;\, y_i) \right\}.$$

That is, $\delta(\mathbf{y})$ is the Nash outcome of the Nash utility bargaining problem where the induced utilities substitute for the vNM utilities, and non-elementary lotteries are not considered. Note that the correspondence $\boldsymbol{\delta}$ is well defined. The following proposition makes use of this analogy.

PROPOSITION 2.6. *Let* $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times \mathcal{P}(D_1) \times \mathcal{P}(D_2)$ *be a bargaining problem. An outcome* $\mathbf{y}^* \in \mathbf{X}$ *is appeals-immune if, and only if,* $\mathbf{y}^* \in \boldsymbol{\delta}(\mathbf{y}^*)$.

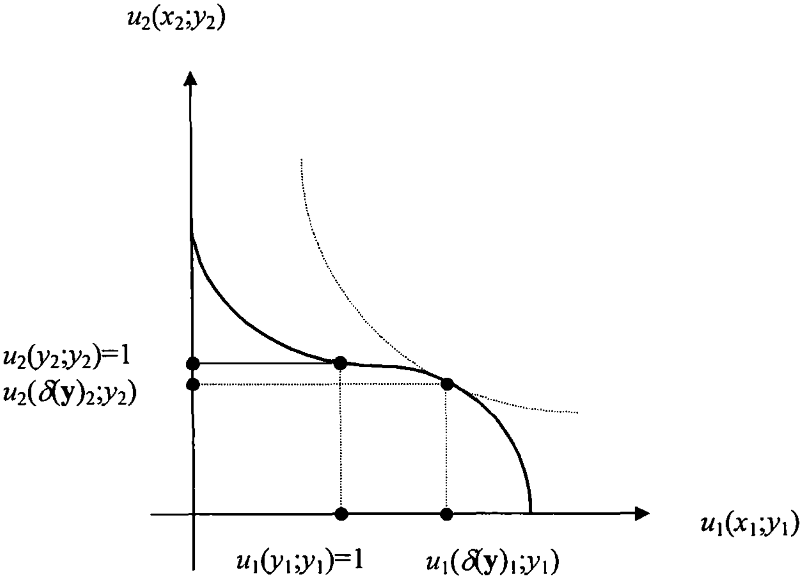

FIG. 1. The induced Nash utility problem w.r.t. $\mathbf{y} \in \mathbf{X}$, where $\delta(\mathbf{y}) \neq \mathbf{y}$.

Proof. Let $\mathbf{y}^* \in \mathbf{X}$. Then,

$$\begin{aligned} \mathbf{y}^* \in \arg\max_{D \leqslant \mathbf{x} \in \mathbf{X}} & \left\{ \prod_i u_i(x_i;\, y_i^*) \right\} \\ & \Leftrightarrow \forall \mathbf{x} \in \mathbf{X},\ \mathbf{x} \geqslant D \quad \prod_i u_i(x_i;\, y_i^*) \leqslant 1 \\ & \Leftrightarrow \forall \mathbf{x} \succ_i \mathbf{y}^* \text{ and } i \in \{1, 2\} \quad \frac{1}{u_i(x_i;\, y_i^*)} \geqslant u_j(x_j;\, y_j^*) \\ & \Leftrightarrow \forall \mathbf{x} \succ_i \mathbf{y}^*,\ \forall p \in [0, 1],\ i \in \{1, 2\} \quad p > \frac{1}{u_i(x_i;\, y_i^*)} \Rightarrow p \geqslant u_j(x_j;\, y_j^*) \\ & \Leftrightarrow \forall \mathbf{x} \succ_i \mathbf{y}^*,\ \forall p \in [0, 1],\ i \in \{1, 2\} \quad p\mathbf{x} \succ_i \mathbf{y}^* \Rightarrow p\mathbf{y}^* \succsim_j \mathbf{x}. \quad \blacksquare \end{aligned}$$

Figure 1 demonstrates a situation in which the induced utilities are drawn with respect to an outcome $\mathbf{y}$ that is not an appeals-immune outcome.

3. EXISTENCE AND UNIQUENESS OF APPEALS-IMMUNE OUTCOMES

Let $D \in \dot{\mathbf{X}}$ and restrict attention to elementary lotteries. We say that a preference relation $\succsim_i \in \mathscr{P}(D_i)$ is *smooth* if it has a utility representation (with the arguments p and x_i) that is twice differentiable and has positive partial derivatives on the subset $\{1\mathbf{x} \mid \mathbf{x} > D\}$ of degenerate elementary lotteries. The set of elementary lotteries for $D = (0, 0)$ is displayed in Fig. 2. Let $\mathscr{P}^{sm}(D_i)$ be the set of all smooth preferences in $\mathscr{P}(D_i)$. If $\succsim_i \in \mathscr{P}^{sm}(D_i)$ then the partial derivatives of the induced utility u_i at $(t;\, t)$ are well defined, as well as the second partial derivatives from either the left or the right of $(t;\, t)$. Hence, for all $t \in \mathbb{R}_{++}$, $(\partial/\partial s)\, u_i(s;\, t)|_t$, is well defined, finite, positive, bounded away from 0 and differentiable. Let $u_i'(t;\, t)$ denote this partial derivative. Assuming $v_i(D_i) = 0$, for EU preference relations $u_i'(t;\, t) = v_i'(t)/v_i(t)$ and for multiplicatively separable preference relations $u_i'(t;\, t) = v_i'(t)/(g_i'(1)\, v_i(t))$. Clearly $\mathscr{P}^{sm}(D_i)$ includes all EU preference relations with twice-differentiable vNM utilities that have positive derivative and hence is not empty. Note that the partial derivative $u'(t;\, t)$ is the absolute value of the slope of the player's indifference curve at the degenerate lottery $1\mathbf{t}$ (see Fig. 2).

The following lemma provides a necessary condition for an outcome $\mathbf{y}^*$ to be an appeals-immune outcome of the bargaining problem $\langle D, \succsim_1, \succsim_2 \rangle$.

LEMMA 3.1. *Consider a bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times \mathscr{P}^{sm}(D_1) \times \mathscr{P}^{sm}(D_2)$. *If* $\mathbf{y}^*$ *is an appeals-immune outcome then* $u_1'(y_1^*;\, y_1^*) = u_2'(y_2^*;\, y_2^*)$.

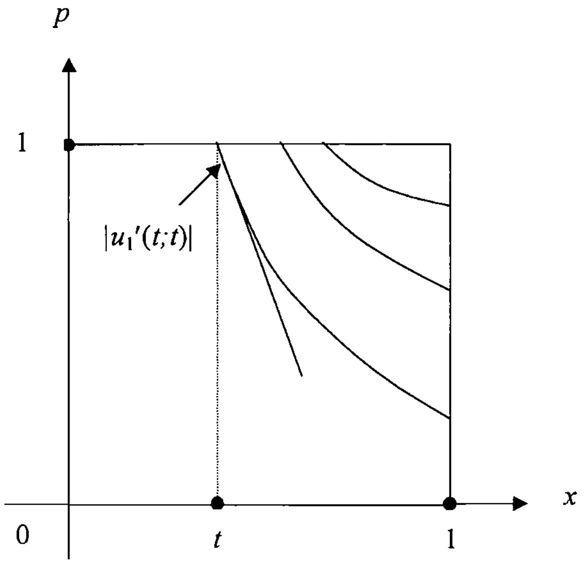

FIG. 2. Indifference curves over elementary lotteries, where $D = (0, 0)$.

Proof. According to Proposition 2.6, $\mathbf{y}^* \in \arg\max_{D \leqslant \mathbf{x} \in \mathbf{X}} \prod_i u_i(x_i; y_i^*)$. Therefore, $\mathbf{y}^* \in F(D)$ implies

$$0 = \frac{\mathrm{d}}{\mathrm{d}x_1} \prod_i u_i(x_i; y_i^*)\bigg|_{\mathbf{y}^*}$$

$$= \sum_i \left[\frac{\mathrm{d}}{\mathrm{d}x_1} u_i(x_i; y_i^*)\bigg|_{\mathbf{y}^*} u_j(y_j^*; y_j^*) \right] = u_1'(y_1^*; y_1^*) - u_2'(y_2^*; y_2^*). \quad \blacksquare$$

(i) *Existence of Appeals-Immune Outcomes*

The next theorem provides necessary and sufficient conditions for the existence of at least one appeals-immune outcome in $\mathbf{X}$. The theorem characterizes a set of preference relations with the property that an appeals-immune outcome exists for each couple of preference relations in the set. On the other hand, if a preference relation does not belong to the set then there exists another preference relation, that can even be chosen from the set, such that the resulting bargaining problem has no appeals-immune outcome.

THEOREM 3.2. *Let $\mathscr{P}^E(D_i) \subseteq \mathscr{P}^{sm}(D_i)$ be the set of all preference relations satisfying the following conditions:*

$$\lim_{y_i \to D_i} u_i'(y_i;\, y_i) = \infty \tag{E1}$$

$$\forall x_i \in F_i(D), \qquad \forall y_i \in F_i(D) \backslash \{D_i\}, \qquad u_i(x_i;\, y_i) \leqslant e^{u_i'(y_i;\, y_i)(x_i - y_i)} \tag{E2}$$

Then

(1) *Every bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times \mathscr{P}^E(D_1) \times \mathscr{P}^E(D_2)$ *has appeals-immune outcomes.*

(2) *If* $\succsim_1 \in \mathscr{P}^{sm}(D_1) \backslash \mathscr{P}^E(D_1)$ *then there exists* $\succsim_2 \in \mathscr{P}^E(D_2)$ *such that the bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle$ *has no appeals-immune outcomes.*

Proof. (1) Assume $\succsim_i \in \mathscr{P}^E(D_i)$. Condition (E2), together with the properties of the induced utilities, implies

$$\forall x_i,\, y_i \in F_i(D) \backslash \{D_i\}, \qquad e^{u_i'(x_i;\, x_i)(x_i - y_i)} \leqslant u_i(x_i;\, y_i) \leqslant e^{u_i'(y_i;\, y_i)(x_i - y_i)}$$

Thus, the functions $u_i'(y_i;\, y_i)$ are monotonically non-increasing with respect to y_i in the set $F_i(D) \backslash \{D_i\}$. Adding condition (E1), continuity implies the existence of $\mathbf{y}^* \in F(D)$ with $u_1'(y_1^*;\, y_1^*) = u_2'(y_2^*;\, y_2^*)$. Therefore,

$$\forall D \leqslant \mathbf{x} \in \mathbf{X}, \qquad \prod_i u_i(x_i;\, y_i^*) \leqslant \prod_i e^{u_i'(y_i^*;\, y_i^*)(x_i - y_i^*)} \leqslant 1$$

which implies that $\mathbf{y}^*$ is an appeals-immune outcome.

(2) See Appendix. ∎

Condition (E1) is natural since it is equivalent to satisfying a form of first-order stochastic-dominance near the degenerate lottery that yields D_i with certainty. It can be examined easily in a graphical representation as in Fig. 2, where it is satisfied if indifference curves near (0, 1) are almost vertical. For EU preference relations, (E1) is satisfied if $v_i'(D_i) > 0$. The intuition behind condition (E2) is that an upper bound on the induced utility functions ensures that appeals are more difficult to conduct. As in part (1) of the proof above, condition (E2) guarantees that no appeals can be made against the point $\mathbf{y}^*$, where marginal utilities are equal, since the product of the induced utilities does not exceed one.

Remark. Consider an EU preference relation $\succsim_i$ with a twice-differentiable vNM utility function v_i that has a positive derivative. It follows that log-concavity of v_i is necessary and sufficient to ensure that $\succsim_i$ belongs to $\mathscr{P}^E(D_i)$. Clearly, concavity of the vNM utility function is a sufficient condition for log-concavity. For convex functions, log-concavity is satisfied as long as the (Arrow–Pratt) measure of risk loving, $v_i''(y_i)/v_i'(y_i)$, is bounded from above by $v_i'(y_i)/v_i(y_i)$. Next consider an RDU preference relation $\succsim_i$

with $v_i(x_i) = x_i$, as in Yaari's dual theory [22]. Condition (E2) is stated for such a preference by

$$\forall y_i \geqslant x_i \geqslant D_i,$$

$$g_i[e^{(1/g'_i(1))[1-(y_i-D_i)/(x_i-D_i)]}] \leqslant \frac{x_i-D_i}{y_i-D_i} \leqslant g_i\,[e^{(1/g'_i(1))[(x_i-D_i)/(y_i-D_i)-1]}].$$

Therefore, the condition

$$\forall p \in [0, 1], \qquad 1 + g'_i(1)\ln(p) \leqslant g_i(p) \leqslant \frac{1}{1 - g'_i(1)\ln(p)}$$

is a necessary and sufficient condition on $g_i(p)$ that ensures $\succsim_i \in \mathscr{P}^E(D_i)$ (see Fig. 3).

We now present an example of a bargaining problem with two well behaved players (one with a RDU preference relation and the other with an EU preference relation) for which there is no appeals-immune outcome. The example demonstrates the role of condition (E2) in the characterization of the set $\mathscr{P}^E(D_i)$.

EXAMPLE 3.3. Let $D = (0, 0)$ and consider a RDU preference relation for Player 1 that is given by $v_1(x_1) = x_1^{\alpha_1}$ and $g_1(p) = p/[1 + (1-p)\beta_1]$,

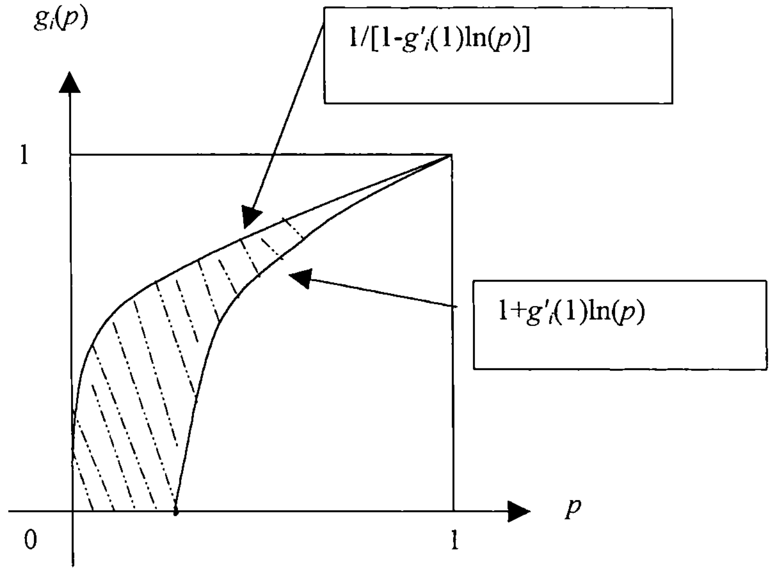

FIG. 3. The range of the function g_i that is necessary and sufficient for a RDU preference relation with linear v_i to belong in the set P^E.

where $\alpha_1 = \beta_1 = 3$. For this preference, the induced utility function is given by

$$u_1(x_1; y_1) = \begin{cases} (1+\beta_1) \Big/ \left[\left(\dfrac{y_1}{x_1} \right)^{\alpha_1} + \beta_1 \right] & \text{if } x_1 \leqslant y_1 \\ \left[\left(\dfrac{x_1}{y_1} \right)^{\alpha_1} + \beta_1 \right] \Big/ (1+\beta_1) & \text{if } x_1 > y_1 \end{cases}$$

and it violates condition (E2). To see this, take $\mathbf{w}^1 = (0.55, 0.45)$, $\mathbf{w}^2 = (0.5, 0.5)$ and note that

$$u_1(w_1^1; w_1^2) = \frac{1}{1+\beta_1} \left[\left(\frac{w_1^1}{w_1^2} \right)^{\alpha_1} + \beta_1 \right] = \frac{1}{4} [1.1^3 + 3] = 1.082$$

while

$$e^{u_1'(w_1^2;\, w_1^2)(w_1^1 - w_1^2)} = e^{(\alpha_1/((1+\beta_1)\, w_1^2))(w_1^1 - w_1^2)} = e^{(3/2)\, 0.05} = 1.077.$$

Thus $\succsim_1 \notin \mathscr{P}^E(D_1)$. Following Case 1 in the proof of Theorem 3.2, $\succsim_2 \in \mathscr{P}^E(D_2)$ is chosen for every $\varepsilon \in (0, 1.5)$ with an EU preference relation, where the vNM utility function is $v_2^\varepsilon(x_2) = x_2^\varepsilon e^{(1.5 - 2\varepsilon) x_2}$. The unique candidate for an appeals-immune outcome is $\mathbf{w}^2$, satisfying $u_1'(w_1^2; w_1^2) = u_2'(w_2^2; w_2^2) = 1.5\ \forall \varepsilon \in (0, 1.5)$. However, for every $\varepsilon \in (0, 0.84)$, $u_1(w_1^1; w_1^2) > u_2(w_2^2; w_2^1)$, thus player 1 can successfully appeal against $\mathbf{w}^2$ by suggesting $\mathbf{w}^1$. Consequently, no appeals-immune outcome exists.

(ii) *Uniqueness of Appeals-Immune Outcomes*

We now present necessary and sufficient conditions for the existence of a unique appeals-immune outcome. Similarly to Theorem 3.2, the next theorem characterizes a set of preference relations with the property that a unique appeals-immune outcome exists for each couple of preference relations in the set. Moreover, if a preference relation does not belong to the set then there exists another preference relation, that can even be chosen from the closure of the set, such that the bargaining problem either has multiple appeals-immune outcomes or none at all.

THEOREM 3.4. *Let $\mathscr{P}^U(D_i) \subseteq \mathscr{P}^{sm}(D_i)$ be the set of all preference relations satisfying the following conditions:*

$$u_i'(y_i; y_i) \textit{ decreases in } y_i, \qquad \forall y_i \in F_i(D) \backslash \{D_i\} \textit{ and} \tag{U1}$$

$$\lim_{y_i \to D_i} u_i'(y_i; y_i) = \infty$$

$$\forall x_i \in F_i(D), \qquad \forall y_i \in F_i(D) \backslash \{D_i\}, \qquad u_i(x_i; y_i) \leqslant e^{u_i'(y_i;\, y_i)(x_i - y_i)} \tag{U2}$$

Then

(1) *Every bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times \mathscr{P}^U(D_1) \times \mathscr{P}^U(D_2)$ *has a unique appeals-immune outcome.*

(2) *If* $\succsim_1 \in \mathscr{P}^{sm}(D_1) \backslash \mathscr{P}^U(D_1)$ *then there exists* $\succsim_2 \in \mathscr{P}^E(D_2)$ *such that the bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle$ *either has multiple appeals-immune outcomes or none at all.*

Proof. (1) Assume $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times \mathscr{P}^U(D_1) \times \mathscr{P}^U(D_2)$. Condition (U1) implies that there exists a unique $\mathbf{y}^* \in F(D)$ with $u_1'(y_1^*; y_1^*) = u_2'(y_2^*; y_2^*)$. It now follows from part (1) of the proof of Theorem 3.2 that $\mathbf{y}^*$ is the unique appeals-immune outcome of the bargaining problem.

(2) See Appendix. ▌

Note that the set $\mathscr{P}^E(D_i)$ is the closure of $\mathscr{P}^U(D_i)$ (recall from part (1) of Theorem 3.2's proof that, by (E2), $u_i'(y_i; y_i)$ weakly decreases with y_i). In fact, relative to the set of preference relations that satisfy condition (E2) (=(U2)), $\mathscr{P}^U(D_i)$ is the interior of $\mathscr{P}^E(D_i)$. In this sense, "almost all" bargaining problems have a unique appeals-immune outcome. Also note that the set of preference relations that is given by the set of conditions {(E1), strict (E2)}, where "strict (E2)" is derived from (E2) requiring a strict inequality, cannot replace the set $\mathscr{P}^U(D_i)$ (since this set is a proper subset of $\mathscr{P}^U(D_i)$).

Remark. It is easy to see that the former two theorems can be immediately extended to cases in which the outcome set is given by

$$\mathbf{X} = \{\mathbf{x} = (x_1, x_2) \in \mathbb{R}_+^2 \mid a_1 x_1 + a_2 x_2 \leqslant b,\ a_1, a_2, b > 0\}.$$

The necessary condition for $\mathbf{y}^*$ to be an appeals-immune outcome would then become $(1/a_1)\, u_1'(y_1^*; y_1^*) = (1/a_2)\, u_2'(y_2^*; y_2^*)$.

(iii) *Other Possible Conditions*

A natural question that may arise is whether there exists other sets of preference relations for which results like those presented in Theorems 3.2 and 3.4 hold. The set of EU preference relations, for example, have the property that an appeals-immune outcome exists for any two players in the set. Hence, a similar result holds for the set of DL preferences. As is shown in the following proposition, a result similar to Theorem 3.2 can indeed be achieved for the set $\mathscr{P}^{DL}(D_i) \subseteq \mathscr{P}^{sm}(D_i)$ of smooth DL preference relations (relative to D). Since the sets $\mathscr{P}^{DL}(D_i) \backslash \mathscr{P}^E(D_i)$ and $\mathscr{P}^E(D_i) \backslash \mathscr{P}^{DL}(D_i)$ are both non empty, it may seem that there is no reason to justify the choice of $\mathscr{P}^E(D_i)$ over that of $\mathscr{P}^{DL}(D_i)$. Nevertheless, as is shown in Proposition 3.6, if uniqueness of appeals-immune outcomes is required then the most natural set to consider is $\mathscr{P}^U(D_i)$, the interior of $\mathscr{P}^E(D_i)$. We think this is a convincing justification for the choice of conditions that we previously made.

Proposition 3.5. (1) *Every bargaining problem* $\langle D, \succsim_1, \succsim_2\rangle \in \dot{\mathbf{X}} \times \mathscr{P}^{DL}(D_1) \times \mathscr{P}^{DL}(D_2)$ *has appeals-immune outcomes.*

(2) *If* $\succsim_1 \in \mathscr{P}^{sm}(D_1) \backslash \mathscr{P}^{DL}(D_1)$ *then there exists* $\succsim_2 \in \mathscr{P}^{DL}(D_2)$ *such that the bargaining problem* $\langle D, \succsim_1, \succsim_2\rangle$ *has no appeals-immune outcomes.*

Proof. (1) Let v_i be the disagreement linear utility function of $\succsim_i \in \mathscr{P}^{DL}(D_i)$. Since $u_i(x_i; y_i) = (v_i(x_i) - v_i(D_i))/(v_i(y_i) - v_i(D_i))$ then any

$$\mathbf{y}^* \in \arg\max_{D \leqslant \mathbf{x} \in \mathbf{X}} \left\{ \prod_i \left[v_i(x_i) - v_i(D_i) \right] \right\}$$

satisfies

$$\mathbf{y}^* \in \arg\max_{D \leqslant \mathbf{x} \in \mathbf{X}} \left\{ \prod_i u_i(x_i; y_i^*) \right\}$$

and is therefore an appeals-immune outcome.

(2) See Appendix. ▌

Proposition 3.6. (1) *Every bargaining problem*

$$\langle D, \succsim_1, \succsim_2\rangle \in \dot{\mathbf{X}} \times (\mathscr{P}^{DL}(D_1) \cap \mathscr{P}^{U}(D_1)) \times (\mathscr{P}^{DL}(D_2) \cap \mathscr{P}^{U}(D_2))$$

has a unique appeals-immune outcome.

(2) *If* $\succsim_1 \in \mathscr{P}^{DL}(D_1) \backslash \mathscr{P}^{U}(D_1)$ *then there exists* $\succsim_2 \in \mathscr{P}^{DL}(D_2) \cap \mathscr{P}^{E}(D_2)$ *such that the bargaining problem* $\langle D, \succsim_1, \succsim_2\rangle$ *has multiple appeals-immune outcomes.*

Proof. See Appendix. ▌

(iv) *Disagreement-free Conditions*

We conclude this section with a theorem that immediately follows from Theorems 3.2 and 3.4. The theorem deals with sets of preference relations that are independent of the disagreement outcome D. Let $\mathscr{P}^{sm} = \bigcap_{D \in \dot{\mathbf{X}}} \mathscr{P}^{sm}(D_i)$, $\mathscr{P}^{E} = \bigcap_{D \in \dot{\mathbf{X}}} \mathscr{P}^{E}(D_i)$ and $\mathscr{P}^{U} = \bigcap_{D \in \dot{\mathbf{X}}} \mathscr{P}^{U}(D_i)$. These sets are non-empty since they all contain EU preference relations with concave, strictly increasing and twice-differentiable vNM utility functions.

Theorem 3.7. (Existence). *Every bargaining problem* $\langle D, \succsim_1, \succsim_2\rangle \in \dot{\mathbf{X}} \times (\mathscr{P}^{E})^2$ *has appeals-immune outcomes.*

(Non-existence). *If* $\succsim_1 \in \mathscr{P}^{sm} \backslash \mathscr{P}^E$ *then there exists* $\langle D, \succsim_2 \rangle \in \dot{\mathbf{X}} \times \mathscr{P}^E(D_2)$ *such that the bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle$ *has no appeals-immune outcome.*

(Uniqueness). *Every bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times (\mathscr{P}^U)^2$ *has a unique appeals-immune outcome.*

(Non-uniqueness). *If* $\succsim_1 \in \mathscr{P}^{sm} \backslash \mathscr{P}^U$ *then there exists* $\langle D, \succsim_2 \rangle \in \dot{\mathbf{X}} \times \mathscr{P}^E(D_2)$ *such that the bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle$ *either has multiple appeals-immune outcomes or none at all.*

4. EXISTENCE AND UNIQUENESS OF ORDINAL NASH OUTCOMES

As in the last part of Section 3, the conditions derived below are independent of the disagreement outcome. Let $\mathscr{P}^{ra}$ be defined by $\mathscr{P}^{ra} = \{ \succsim \in \mathscr{P}^{sm} \mid \forall \ell \in \mathscr{L}([0, 1]), \mu(\ell) \succsim \ell \}$ where $\mu(\ell)$ represents the expected value of the lottery ℓ. Every preference relation in $\mathscr{P}^{ra}$ displays a weak form of risk aversion. This notion is weaker than the one that is implied by aversion to mean preserving spreads; see Safra and Zilcha [18] for a possible taxonomy of risk aversion notions (together with their efficiency implications). The set $\mathscr{P}^{ra}$ contains risk-averse EU preferences with twice-differentiable and strictly increasing vNM utility functions, hence it is not empty. The next two theorems provide necessary and sufficient conditions for existence (Theorem 4.1) and uniqueness (Theorem 4.2) of ordinal Nash outcomes.

THEOREM 4.1. (1) *Every bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times (\mathscr{P}^E \cap \mathscr{P}^{ra})^2$ *has ordinal Nash outcomes.*

(2) *If* $\succsim_1 \in \mathscr{P}^E \backslash \mathscr{P}^{ra}$ *then there exist* $\langle D, \succsim_2 \rangle \in \dot{\mathbf{X}} \times (\mathscr{P}^E(D_2) \cap \mathscr{P}^{ra})$ *such that the bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle$ *has no ordinal Nash outcome.*

Proof. (1) Let $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times (\mathscr{P}^E \cap \mathscr{P}^{ra})^2$ be a bargaining problem and let $\mathbf{y}^*$ be any appeals-immune outcome (its existence follows from Theorem 3.7). If $\mathbf{y}^* \notin \mathbf{F}(D, \succsim_1, \succsim_2)$ then there exists $\ell \in \mathbf{F}(D, \succsim_1, \succsim_2)$ such that $\ell \succ_i \mathbf{y}^*$, $\ell \succsim_j \mathbf{y}^* (i, j \in \{1, 2\})$. By risk aversion $\mu(\ell_i) > y_i^*$ and $\mu(\ell_j) \geqslant y_j^*$, which implies $\mathbf{y}^* \notin F(D)$; a contradiction. Hence, any appeals-immune outcome is an ordinal Nash outcome too.

(2) Let $\succsim_1 \in \mathscr{P}^E \backslash \mathscr{P}^{ra}$. There exists $\ell_1 \in \mathscr{L}([0, 1])$ with $\ell_1 \succ_1 \mu(\ell_1)$. Let $c_1(\ell_1)$ satisfy $\ell_1 \sim_1 c_1(\ell_1)$ and choose $D \in \dot{\mathbf{X}}$ that satisfies $\mu(\ell_1) < D_1 < c_1(\ell_1)$ and $D_2 > 1 - c_1(\ell_1)$. Consider the bargaining problem $\langle D, \succsim_1, \succsim_2 \rangle$ where player 2 is an expected value maximizer. Clearly $\succsim_2 \in (\mathscr{P}^E \cap \mathscr{P}^{ra})$ and any appeals-immune outcome $\mathbf{y}^*$, if it exists, satisfies $\mu(\ell_1) < y_1^* < c_1(\ell_1)$ and $1 - c_1(\ell_1) < y_2^* < 1 - \mu(\ell_1)$. Define ℓ_2 by $\ell_2(x_2) = \ell_1(1 - x_2)$. Then

$\ell = (\ell_1, \ell_2) \in \mathscr{L}(D)$ and $\mu(\ell_2) = 1 - \mu(\ell_1)$. It follows that $\ell_i \succ_i y_i^*$, for both i, which implies that $\mathbf{y}^*$ cannot be an ordinal Nash outcome. Hence, no ordinal Nash outcome exists. ▌

Note that a result analogous to Proposition 3.5 is of no interest here since, for $\mathscr{P}^{DL} = \bigcap_{D \in \dot{\mathbf{X}}} \mathscr{P}^{DL}(D_i)$, $(\mathscr{P}^{DL} \cap \mathscr{P}^{ra}) \subset (\mathscr{P}^{E} \cap \mathscr{P}^{ra})$. This is yet another justification for the choice of the sets $\mathscr{P}^{E}(D_i)$ and $\mathscr{P}^{U}(D_i)$ that we previously made.

The proof of the next theorem follows directly from Theorems 3.7 and 4.1.

THEOREM 4.2. (1) *Every bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle \in \dot{\mathbf{X}} \times (\mathscr{P}^{U} \cap \mathscr{P}^{ra})^2$ *has a unique ordinal Nash outcome.*

(2) *If* $\succsim_1 \in \mathscr{P}^{U} \setminus \mathscr{P}^{ra}$ *then there exists* $(D, \succsim_2) \in \dot{\mathbf{X}} \times (\mathscr{P}^{E}(D_2) \cap \mathscr{P}^{ra})$ *such that the bargaining problem* $\langle D, \succsim_1, \succsim_2 \rangle$ *has no ordinal Nash outcome.*

Theorem 4.2 characterizes the maximal domain of individual preference relations for which the ordinal Nash solution is always well defined. That is, whenever a pair of preference relations belongs to the set $(\mathscr{P}^{U} \cap \mathscr{P}^{ra})^2$, a unique ordinal Nash outcome exists. On the other hand, if one of the preference relations does not belong to this set then two different scenarios, both lead to non-existence of ordinal Nash outcomes, may hold. More explicitly, assume that $\succsim_1$ does not belong to $\mathscr{P}^{U} \cap \mathscr{P}^{ra}$. If $\succsim_1 \in \mathscr{P}^{E} \setminus \mathscr{P}^{ra}$ then, by Theorem 4.1, there exists another player with whom every appeals-immune outcome is not Pareto efficient. If, however, $\succsim_1 \notin \mathscr{P}^{E}$, then Theorem 3.7 implies the existence of another player with whom the bargaining problem has no appeals-immune outcomes at all.

We conclude this section with a detailed example that characterizes the intersections of familiar sets of preference relations with the set $\mathscr{P}^{U} \cap \mathscr{P}^{ra}$.

EXAMPLE 4.3. (1) *EU preferences relations.* As noted above, the intersection of $\mathscr{P}^{U} \cap \mathscr{P}^{ra}$ with this set consists of all EU preference relations with concave, strictly increasing and twice-differentiable vNM utility functions.

(2) *RDU preference relations.* We restrict attention to preference relations with twice differentiable utility function v that has a positive derivative and with strictly increasing g satisfying $g'(1) \neq 0$. These preference relations belong to $\mathscr{P}^{U}$ if v is strictly log-concave and if

$$\forall y \geqslant x \geqslant D_i,$$

$$g\left[e^{(1/g'(1))(v'(x)/(v(x)-v(D_i))[x-y]}\right] \leqslant \frac{v(x)-v(D_i)}{v(y)-v(D_i)}$$

$$\leqslant g\left[e^{(1/g'(1))(v'(y)/(v(y)-v(D_i))[x-y]}\right].$$

For RDU preference relations to display risk-aversion with respect to mean-preserving spreads, it is necessary and sufficient that both v and $-g$ are concave (see Chew, Karni, and Safra [5]). Our weaker notion of risk-aversion does not require the concavity of v (see Chateauneuf and Cohen [3]). A characterization of a set of preference relations that displays an intermediate notion of risk-aversion (monotone risk-aversion) appears in Chateauneuf, Cohen, and Meilijson [4]. Hence, adding the conditions of this paper to those required for $\mathscr{P}^U$ gives a set of RDU preference relations that is included in $\mathscr{P}^U \cap \mathscr{P}^{ra}$.

Detecting the inclusion in the set $\mathscr{P}^U$ becomes easier for RDU preference relations with a function g of the form $g(p) = p^\alpha$, $\alpha > 0$. This is the case since these preference relations belong to the class DL and hence they belong to $\mathscr{P}^U$ if, and only if, v is strictly log-concave.

A complete characterization of the set $\mathscr{P}^U \cap \mathscr{P}^{ra}$ for the set of RDU preference relations with linear utility (Yaari's model) is presented now. The remark following Theorem 3.2 immediately implies

$$\succsim\, \in \mathscr{P}^U \Leftrightarrow \forall p \in [0, 1], \qquad 1 + g'(1)\ln(p) \leqslant g(p) \leqslant \frac{1}{1 - g'(1)\ln(p)}.$$

By Yaari, our notion of risk-aversion is equivalent to $g(p) \leqslant p$. Hence,

$$\succsim\, \in \mathscr{P}^U \cap \mathscr{P}^{ra} \Leftrightarrow \forall p \in [0, 1], \qquad 1 + g'(1)\ln(p) \leqslant g(p) \leqslant p.$$

Interestingly, the same characterization holds for RDU preference relations with utility functions of the form $v(x) = x^\alpha$, $0 < \alpha \leqslant 1$. Deriving the conditions on $\mathscr{P}^U$ is rather complicated and hence omitted. Risk-aversion, on the other hand, is easy to detect. Since all utility functions are concave, our notion of risk-aversion is still characterized by the equation $g(p) \leqslant p$.

(3) *DA preference relations.* As before, we restrict attention to preference relations with twice differentiable utility function v that has positive derivative. By monotonicity with respect to the relation of first-order stochastic-dominance, $\beta > -1$. These preference relations belong to $\mathscr{P}^U$ if v is strictly log-concave and if

$$\forall y \geqslant x \geqslant D_i,$$

$$g\left[e^{(1/g'(1))(v'(x)/(v(x) - v(D_i))[x - y]}\right] \leqslant \frac{v(x) - v(D_i)}{v(y) - v(D_i)}$$

$$\leqslant g\left[e^{(1/g'(1))(v'(y)/(v(y) - v(D_i))[x - y]}\right].$$

where $g(p) = p/(1 + (1 - p)\beta)$. Following Gul, risk-aversion with respect to mean-preserving spreads is satisfied if, and only if, v is concave and $\beta > 0$. Hence, adding the conditions gives a set of DA preference relations that is included in $\mathscr{P}^U \cap \mathscr{P}^{ra}$.

We now present a complete characterization of the set $\mathscr{P}^U \cap \mathscr{P}^{ra}$ for the set of DA preference relations with linear utility. This class of preference relations is characterized by Safra and Segal [17]. It can be shown that our notion of risk-aversion is satisfied if, and only if, $\beta > 0$. Applying the condition presented for RDU preference relations with linear utility and using the special form of the g function gives

$$\succsim \in \mathscr{P}^U \cap \mathscr{P}^{ra} \Leftrightarrow \beta > 0 \qquad \text{and} \qquad \forall p \in [0, 1],$$

$$1 + (1 + \beta)\ln(p) \leqslant \frac{p}{1 + (1 - p)\beta}$$

which is equivalent to

$$\succsim \in \mathscr{P}^U \cap \mathscr{P}^{ra} \Leftrightarrow \beta > 0.$$

APPENDIX

Proof of Lemma 2.5. First, we construct the preference relation over elementary lotteries on $\{\mathbf{x} \in \mathbf{X} \mid \mathbf{x} \geqslant D\}_i$. Consider the elementary lottery $p\mathbf{x}$. By continuity of u_i and by its strict monotonicity with respect to its first argument, there exists a unique $t_i(p\mathbf{x}) \in [D_i, x_i]$ such that $u_i(t_i(p\mathbf{x}); x_i) = p$. The preference $\succsim_i$ is defined to satisfy $px_i \sim_i t_i(p\mathbf{x})$. Consequently, $px_i \succsim_i \bar{p}\bar{x}_i \Leftrightarrow t_i(p\mathbf{x}) \geqslant t_i(\bar{p}\bar{\mathbf{x}})$.

Finally, for any lottery $(\vec{\mathbf{x}}, \mathbf{p}) \in \mathscr{L}(D)$ $(x_i^k \neq D_i)$, $\succsim_i$ is defined by

$$(\vec{\mathbf{x}}, \mathbf{p})_i \sim_i \left(\sum_{x_i^k > D_i} p^k \right) (\max\{x_i^1, \ldots, x_i^n, D_i\})$$

Proof of Theorem 3.2. (2) Assume that $\succsim_1 \in \mathscr{P}^{sm}(D_1) \backslash \mathscr{P}^E(D_1)$. If (E1) is not satisfied then, clearly, $\lim_{y_1 \to D_1} u_1'(y_1; y_1) < \infty$. Below we construct a second player with an EU preference relation such that $\succsim_2 \in \mathscr{P}^E(D_2)$ and $\forall \mathbf{y} > D$ in $F(D)$, $u_1'(y_1; y_1) < u_2'(y_2; y_2)$. According to Lemma 3.1, no appeals-immune outcome would exist.

Choose a differentiable function $h\colon (D_2, 1-D_1] \rightarrow \mathbb{R}_{++}$ that is decreasing with respect to y_2 such that $\int_{D_2}^{y_2} h(z_2)\, dz_2$ is well defined, $\lim_{y_2 \rightarrow D_2} h(y_2) = \infty$ and $\forall \mathbf{y} > D$ in $F(D)$, $u_1'(y_1; y_1) < h(y_2)$. Assume further that, for some $k > 0$ and $\delta > 0$ sufficiently small, $h(y_2) = k/(y_2 - D_2)$ for all $y_2 < D_2 + \delta$. Let

$$v_2(y_2) = \begin{cases} \delta^k e^{\int_{D_2+\delta}^{y_2} h(z_2)\, dz_2} & \text{if } \ y_2 \geqslant D_2 + \delta \\ (y_2 - D_2)^k & \text{if } \ y_2 < D_2 + \delta \end{cases}$$

be a vNM utility function. Then $v_2(D_2) = 0$, $u_2(x_2; y_2) = v_2(x_2)/v_2(y_2)$ and $u_2'(y_2; y_2) = v_2'(y_2)/v_2(y_2) = h(y_2)$. By construction, (E1) is satisfied. Clearly, $u_2(x_2; y_2)$ is log-concave and therefore, (E2) is satisfied.[6] Thus, player 2 satisfies all the required properties, $\succsim_2 \in \mathscr{P}^E(D_2)$ and the bargaining problem $\langle D, \succsim_1, \succsim_2 \rangle$ has no appeals-immune outcomes.

Now assume that (E1) is satisfied while (E2) is not. Consider two possible cases: (Case 1) the function $u_1'(y_1; y_1)$ is monotonically non-increasing in y_1 on $F_1(D) \backslash \{D_1\}$ and (Case 2) the function $u_1'(y_1; y_1)$ is somewhere increasing in y_1.

Case 1. Since (E2) is violated, there must exist $\mathbf{w}^1 = (w_1^1, w_2^1)$, $\mathbf{w}^2 = (w_1^2, w_2^2) \in F(D)$ such that $\mathbf{w}^i > D$, $\mathbf{w}^1 \neq \mathbf{w}^2$ and $u_1(w_1^1; w_1^2) > e^{u_1'(w_1^2; w_1^2)(w_1^1 - w_1^2)}$. Note that $\mathbf{w}^1$ need not be close to $\mathbf{w}^2$ and that their order need not be given. For every $\varepsilon > 0$ choose a function $h^\varepsilon\colon (D_2, 1-D_1] \rightarrow \mathbb{R}_{++}$ that is monotonically decreasing with respect to y_2 such that $\int_{D_2}^{y_2} h^\varepsilon(z_2)\, dz_2$ is well defined, $\lim_{y_2 \rightarrow D_2} h^\varepsilon(y_2) = \infty$, $h^\varepsilon(w_2^2) = u_1'(w_1^2; w_1^2)$, $h^\varepsilon(y_2) \neq u_1'(y_1; y_1)\ \forall \mathbf{y} \neq \mathbf{w}^2$ and $|h^\varepsilon(y_2) - h^\varepsilon(w_2^2)| < \varepsilon$ for all y_2 that satisfies $\min(w_2^1; w_2^2) < y_2 < \max(w_2^1; w_2^2)$. As above, construct v_2 and u_2 such that the second player has EU preferences in $\mathscr{P}^E(D_2)$ (where $\delta < \min(w_2^1; w_2^2)$ should hold). Then,

$$\begin{aligned} \ln u_1(w_1^1; w_1^2) &> u_1'(w_1^2; w_1^2)(w_1^1 - w_1^2) = h^\varepsilon(w_2^2)(w_2^2 - w_2^1) \\ &= \int_{w_2^1}^{w_2^2} h^\varepsilon(z_2)\, dz_2 + \phi(\varepsilon) = \ln u_2(w_2^2; w_2^1) + \phi(\varepsilon) \end{aligned}$$

where $\lim_{\varepsilon \rightarrow 0} \phi(\varepsilon) = 0$. Hence, for a small enough $\varepsilon > 0$, $u_1(w_1^1; w_1^2) > u_2(w_2^2; w_2^1)$. Thus $\prod_i u_i(w_i^1; w_i^2) > 1$, $\mathbf{w}^1$ may be suggested as an appeal against $\mathbf{w}^2$ and according to Lemma 3.1, no appeals-immune outcome exists.

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[6] (E2) is implied by the property $(u_i'(x_i; y_i)/u_i(x_i; y_i))\, \mathrm{sign}(x_i - y_i) \leqslant u_i'(y_i; y_i)\, \mathrm{sign}(x_i - y_i)$, $\forall \mathbf{x} \in F(D)$ and $\forall \mathbf{y} > D$ in $F(D)$. This can be obtained by integrating from y_i to x_i each side of the inequality. Furthermore, the property is implied by the log-concavity of u_i on $F(D)$, $\forall \mathbf{y} > D$ in $F(D)$.

Case 2. In this case there must exist $\mathbf{w}^2 \in F(D)$ with $0 < (\mathrm{d}/\mathrm{d}y_1)\, u_1'(y_1; y_1)|_{w_1^2} < \infty$. Thus $u_1(\cdot; w_1^2)$ is log-convex at one side of w_1^2.[7] Note that if w_1^1 belongs to this side and is sufficiently close to w_1^2, then $u_1(w_1^1; w_1^2) > e^{u_1'(w_1^2; w_1^2)(w_1^1 - w_1^2)}$. Choose a monotonically decreasing and continuously differentiable function $h\colon (D_2, 1 - D_1] \to \mathbb{R}_{++}$ that satisfies $\lim_{y_2 \to D_2} h(y_2) = \infty$, $h(w_2^2) = u_1'(w_1^2; w_1^2)$, and $h(y_2) \neq u_1'(y_1; y_1)\ \forall \mathbf{y} \neq \mathbf{w}^2$.

First assume that $u_1(\cdot; w_1^2)$ is log-convex to the left of w_1^2. Define a preference relation $\succsim_2$ over elementary lotteries $p\mathbf{x}$ as follows: $p\mathbf{x} \sim_2 D$ if $p(x_2 - D_2) = 0$; otherwise, $p\mathbf{x} \sim_2 \mathbf{y}$ for the unique $y_2 \leqslant x_2$ that satisfies $p = e^{h(y_2)(y_2 - x_2)}$.[8] Note that $\succsim_2$ is monotone with respect to first-order stochastic-dominance since, for any given $\mathbf{y}$ with $y_2 > D_2$, p decreases with x_2. By definition, the induced utility u_2 satisfies

$$u_2(x_2; y_2) = \begin{cases} e^{h(x_2)(x_2 - y_2)} & \text{if} \quad x_2 \leqslant y_2 \\ e^{h(y_2)(x_2 - y_2)} & \text{if} \quad x_2 > y_2 \end{cases}$$

and $u_2'(y_2; y_2) = h(y_2)$. By Lemma 2.5, $\succsim_2$ can be extended to $\mathscr{L}(D)$. Now, by construction,

$$\ln u_1(w_1^1; w_1^2) > u_1'(w_1^2; w_1^2)(w_1^1 - w_1^2) = h(w_2^2)(w_2^2 - w_2^1) = \ln u_2(w_2^2; w_2^1)$$

hence $u_1(w_1^1; w_1^2) > u_2(w_2^2; w_2^1)$ and $\mathbf{w}^2$, the unique candidate for being an appeals-immune outcome, fails to satisfy its requirements.

Next assume that $u_1(\cdot; w_1^2)$ is log-convex to the right of w_1^2 and choose $\mathbf{z} \in F(D)$ such that $u_1(\cdot; w_1^2)$ is log-convex in $[w_1^2, z_1]$. Define a continuous function $\phi\colon (D_2, 1 - D_1) \to (D_2, 1 - D_1)$ such that $\phi(y_2) > y_2$ and $(\mathrm{d}/\mathrm{d}x_2)\, h(x_2)(y_2 - x_2) < 0$ for any $x_2 \in [y_2, \phi(y_2)]$.[9] Choose $\bar{z}_2$ that satisfies $\phi(y_2) > w_2^2$ for all $y_2 \in [\bar{z}_2, w_2^2]$. Assume, without loss of generality, that $z_2 \geqslant \bar{z}_2$. Choose $\mathbf{w}_1 \in F(D)$ for which $w_2^1 \in (z_2, w_2^2)$ and let $\mu = e^{h(w_2^2)(w_2^1 - w_2^2)} < 1$. Now define a preference relation $\succsim_2$ over elementary lotteries $p\mathbf{x}$ as follows: (1) for all $x_2 \in [w_2^1, w_2^2]$ and $p \in [\mu, 1]$, $p\mathbf{x} \sim_2 \mathbf{w}^1 \Leftrightarrow p = e^{h(x_2)(w_2^1 - x_2)}$; (2) if

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[7] Since $u_1(y_1; y_1) = u_1(x_1; y_1)\, u_1(y_1; x_2) = 1$, it then follows that $0 = [(\partial^2/(\partial x_1)^2) + (\partial^2/(\partial z_1)^2) + 2(\partial^2/\partial x_1\, \partial z_1)]\, u_1(x_1; z_1)|_{(y_1; y_1)} + [(\partial^2/(\partial x_1)^2) + (\partial^2/(\partial z_1)^2) + 2(\partial^2/\partial x_1\, \partial z_1)]\, u_1(x_1; z_1)|_{(y_1^+; y_1^+)} = 4[u_1'(y_1; y_1)]^2 + 2(\partial^2/\partial x_1\, \partial z_1)\, u_1(x_1; z_1)|_{(y_1; y_1)} + 2(\partial^2/\partial x_1\, \partial z_1)\, u_1(x_1; z_1)|_{(y_1^+; y_1^+)}$. Consequently, $(\mathrm{d}^2/(\mathrm{d}x_1)^2)[\ln u_1(x_1; y_1^-)]|_{y_1} + (\mathrm{d}^2/(\mathrm{d}x_1)^2)[\ln u_1(x_1; y_1^+)]|_{y_1^+} = (\partial^2/(\partial x_1)^2)\, u_1(x_1; z_1)|_{(y_1; y_1)} + (\partial^2/(\partial x_1)^2)\, u_1(x_1; z_1)|_{(y_1^+; y_1^+)} - 2[u_1'(y_1; y_1)]^2 = [(\partial^2/(\partial x_1)^2) + (\partial^2/\partial x_1\, \partial z_1)]\, u_1(x_1; z_1)|_{(y_1; y_1)} + [(\partial^2/(\partial x_1)^2) + (\partial^2/\partial x_1\, \partial z_1)]\, u_1(x_1; z_1)]|_{(y_1^+; y_1^+)} = 2(\mathrm{d}/\mathrm{d}x_1)\, u_1'(x_1; x_1)|_{y_1}$.

[8] Uniqueness and existence follow from: (i) $(\mathrm{d}/\mathrm{d}y_2)\, h(y_2)(y_2 - x_2) = h'(y_2)(y_2 - x_2) + h(y_2) > 0$ for $y_2 \leqslant x_2$, (ii) $h(x_2)(x_2 - x_2) = 0$ and (iii) the boundary condition on h.

[9] Existence is guaranteed by: (i) $(\mathrm{d}/\mathrm{d}y_2)\, h(y_2)(y_2 - x_2) = h'(y_2)(y_2 - x_2) + h(y_2)$, (ii) $(\mathrm{d}/\mathrm{d}y_2)\, h(y_2)(y_2 - x_2)|_{x_2} = h(x_2) > 0$ and (iii) continuity of $h'(y_2)$.

$p(x_2 - D_2) = 0$ then $p\mathbf{x} \sim_2 D$ and (3) for all other lotteries $\succsim_2$ is defined as an extension satisfying all the properties required from preference relations in $\mathscr{P}(D)$ and such that the slopes of its indifference curves at lotteries of the form $1\mathbf{y}$ are equal to $-h(y_2)$. Monotonicity with respect to first-order stochastic-dominance is assured by the way the function ϕ was constructed. By construction, the induced utility u_2 satisfies $u_2(w_2^2; w_2^1) = e^{h(w_2^2)(w_2^2 - w_2^1)}$ and $u_2'(w_2^2; w_2^2) = h(w_2^2)$. The proof concludes similarly to the proof of the former case.

Proof of Theorem 3.4. (2) Assume $\succsim_1 \in \mathscr{P}^{sm}(D_1) \setminus \mathscr{P}^U(D_1)$. In order for appeals-immune outcomes to exist, conditions (E1) and (E2) must hold for both players. Suppose, therefore, that condition (U1) is violated in such a way that $u_1'(y_1; y_1)$ is monotonically non-increasing with respect to y_1, but is not strictly decreasing at some point of $F_1(D) \setminus \{D_1\}$. Thus, there exist $\mathbf{w}^1, \mathbf{w}^2 \in F(D)$, $\mathbf{w}^2 \succ_1 \mathbf{w}^1$ and $\mathbf{w}' > D$ such that $\forall \mathbf{w}^2 \succsim_1 \mathbf{x}$, $\mathbf{y} \succsim_1 \mathbf{w}^1$, $u_1'(x_1; x_1) = u_1'(y_1; y_1)$. As in the proof of Theorem 3.2, construct player 2 with EU preferences for whom $h(y_2) = u_1'(y_1; y_1)$, $\forall \mathbf{w}^2 \succsim_1 \mathbf{y} \succsim_1 \mathbf{w}^1$. Since both preferences satisfy conditions (E1) and (E2), all $\mathbf{y} \in F(D)$ between $\mathbf{w}^1$ and $\mathbf{w}^2$ are appeals-immune outcomes of the problem, violating uniqueness.

Proof of Proposition 3.5. (2) Assume that $\succsim_1 \in \mathscr{P}^{sm}(D_1) \setminus \mathscr{P}^{DL}(D_1)$. If $\forall x_1, y_1 \in F_1(D) \setminus \{D_1\}$, $u_1(x_1; y_1) = e^{\int_{y_1}^{x_1} u_1'(t;\, t)\, dt}$, then $v_1(x_1) = e^{\int_{1-D_2}^{x_1} u_1'(t;\, t)\, dt}$ $= u_1(x_1; 1 - D_2)$ can be taken to be a disagreement linear utility function and hence $\succsim_1 \in \mathscr{P}^{DL}(D_1)$; a contradiction. Thus, there must exist $\mathbf{w}^1 \neq \mathbf{w}^2$ $\in F(D)$ such that $\mathbf{w}' > D u_1(w_1^1; w_1^2) > e^{\int_{w_1^2}^{w_1^1} u_1'(t;\, t)\, dt}$. Similarly to the proof of Theorem 3.2, construct player 2 with an EU preference, such that for every $\varepsilon > 0$, $h^\varepsilon \colon (D_2, 1 - D_1) \to \mathbb{R}_{++}$ satisfies $h^\varepsilon(w_2^2) = u_1'(w_1^2; w_1^2)$, $h^\varepsilon(y_2) \neq u_1'(y_1; y_1)$ $\forall \mathbf{y} \neq \mathbf{w}^2$ and $|h^\varepsilon(y_2) - u'(y_1; y_1)| < \varepsilon$ for all $\mathbf{y} \in F(D)$ that satisfies $\min\{w_2^1, w_2^2\} < y_2 < \max\{w_2^1, w_2^2\}$. Then

$$
\begin{aligned}
\ln u_1(w_1^1; w_1^2) &> \int_{w_1^2}^{w_1^1} u_1'(t;\, t)\, dt = \int_{1 - w_2^2}^{1 - w_2^1} h^\varepsilon(1 - t)\, dt + \phi(\varepsilon) \\
&= \int_{w_2^1}^{w_2^2} h^\varepsilon(s)\, ds + \phi(\varepsilon) = \ln u_2(w_2^2; w_2^1) + \phi(\varepsilon)
\end{aligned}
$$

where $\lim_{\varepsilon \to 0} \phi(\varepsilon) = 0$. Hence, for small $\varepsilon > 0$, $u_1(w_1^1; w_1^2) > u_2(w_2^2; w_2^1)$ and $\mathbf{w}^1$ is as an appeal against $\mathbf{w}^2$. According to Lemma 3.1, no appeals-immune outcome exists.

Proof of Proposition 3.6. (1) This follows immediately from Theorem 3.4.

(2) Assume that $\succsim_1 \in \mathscr{P}^{DL}(D_1) \setminus \mathscr{P}^U(D_1)$. Thus there exist $\mathbf{w}^1, \mathbf{w}^2 \in F(D)$, $\mathbf{w}^2 \succ_1 \mathbf{w}^1$ and $\mathbf{w}' > D$ such that $u_1'(y_1; y_1)$ is non-decreasing

$\forall \mathbf{w}^2 \succsim_1 \mathbf{y} \succsim_1 \mathbf{w}^1$. Similarly to the proof of Theorem 3.2, construct $\succsim_2 \in \mathscr{P}^{DL}(D) \cap \mathscr{P}^{E}(D)$ to be an EU preference for which $h(y_2) = u_1'(y_1; y_1)$ $\forall \mathbf{w}^2 \succsim_1 \mathbf{y} \succsim_1 \mathbf{w}^1$ and $h(y_2) \neq u_1'(y_1; y_1)$ elsewhere. Under Proposition 3.5 the problem $\langle D, \succsim_1, \succsim_2 \rangle$ has an appeals-immune outcome $\mathbf{y}^* \in F(D)$ and under Lemma 3.1 $\mathbf{w}^2 \succsim_1 \mathbf{y}^* \succsim_1 \mathbf{w}^1$. Then, $\forall \mathbf{w}^2 \succsim_1 \mathbf{x} \succsim_1 \mathbf{w}^1$,

$$\begin{aligned} u_1(x_1; y_1^*) &= e^{\int_{y_1^*}^{x_1} u_1'(t;\, t)\, dt} = e^{\int_{1-y_2^*}^{1-x_2} h(1-t)\, dt} \\ &= e^{\int_{x_2}^{y_2^*} h(s)\, ds} = u_2(y_2^*; x_2), \end{aligned}$$

thus $\prod_i u_i(x_i; y_i^*) = 1$. Therefore, every $\mathbf{w}^2 \succsim_1 \mathbf{y} \succsim_1 \mathbf{w}^1$ satisfies $\mathbf{y} \in \arg\max_{D \leqslant \mathbf{x} \in \mathbf{X}} \{\prod_i v_i(x_i)\}$, where v_i are the disagreement linear utility functions of $\succsim_i$. Hence, the problem $\langle D, \succsim_1, \succsim_2 \rangle$ has multiple appeals-immune outcomes.

REFERENCES

1. R. Aumann and M. Kurz, Power and taxes, *Econometrica* **45** (1977), 1137–1161.
2. A. Burgos, S. Grant, and A. Kajii, "Bargaining and Boldness," mimeo, Australian National University, 1997.
3. A. Chateauneauf and M. Cohen, Risk-seeking with diminishing marginal utility in a non-expected utility model, *J. Risk. Uncertainty* **9** (1994), 77–91.
4. A. Chateauneauf, M. Cohen, and I. Meilijson, "More Pessimism than Greediness: A Characterization of Monotone Risk Aversion in the Rank-Dependent Expected Utility Model," mimeo, Université de Paris I, 1997.
5. S. H. Chew, E. Karni, and Z. Safra, Risk aversion in the theory of expected utility with rank dependent probabilities, *J. Econ. Theory* **42** (1987), 370–381.
6. V. Denicolo, "The Nash Bargaining in the Space of Physical Alternatives," mimeo, Universita di Bologna, 1996.
7. S. Grant and A. Kajii, A Cardinal characterization of the Rubinstein–Safra–Thomson axiomatic Bargaining theory, *Econometrica* **63** (1995), 1241–1249.
8. S. Grant and A. Kajii, "Bargaining, Boldness, and Nash Outcomes with Non-Expected Utility," ANU Working Paper #264, Australian National University, 1994.
9. F. Gul, A theory of disappointment aversion, *Econometrica* **59** (1991), 667–686.
10. E. Hanany and Z. Safra, "Reformulation of the Bargaining Problem with Induced Utilities," mimeo, Tel Aviv University, 1996.
11. H. Houba, X. Tieman, and R. Brinksma, "The Nash- and Kalai–Smorodinsky Bargaining Solution for Decision Weight Utility Functions," Fu Working Paper, 1996.
12. J.-M. Koskievic, "Négociations et espérance d'utilité dépendante du rang," mimeo, Université de Paris I, 1997.
13. J. Nash, The bargaining problem, *Econometrica* **28** (1950), 155–162.
14. J. Quiggin, A theory of anticipated utility, *J. Econom. Behav. Organ.* **3** (1982), 323–343.
15. A. Rubinstein, Z. Safra, and W. Thomson, On the interpretation of the Nash bargaining solution and its extension to non-expected utility preferences, *Econometrica* **60** (1992), 1171–1186.
16. Z. Safra and U. Segal, How complicated are betweenness preferences?, *J. Math. Econom.* **24** (1995), 371–381.
17. Z. Safra and U. Segal, Constant risk aversion, *J. Econ. Theory* **83** (1998), 19–42.

18. Z. Safra and I. Zilcha, Efficient sets with and without the expected utility hypothesis, *J. Math. Econom.* **17** (1988), 369–384.
19. Z. Safra and I. Zilcha, Bargaining solutions without the expected utility hypothesis, *Games Econom. Behav.* **60** (1993), 1171–1186.
20. F. Valenciano and J. M. Zarzuelo, On the interpretation of non-symmetric bargaining solutions and their extension to nonexpected utility preferences, *Games Econom. Behav.* **7** (1994), 409–430.
21. J. A. Weymark, Generalized Gini inequality indices, *Math. Social. Sci.* **1** (1981), 95–115.
22. M. E. Yaari, The dual theory of choice under risk, *Econometrica* **55** (1987), 95–115.