Computer Assignment No. 2

Problem
Heat flow in a slab $0 < x < L$.
$k = \text{const.}, f = 0$.
bc’s: insulation at $x = 0$ and $u(L) = 0$.
ic’s: $u|_{t=0} = u_0 = \text{const.}$.

Maximum Principle
Extreme values of the solution of the homogeneous heat equation occur at $t = 0$ or on the spatial boundary.

Computation
5 equal linear finite elements in space.
Backward Euler ($\alpha = 1$) in time.

$$\Delta t = \begin{cases} 
0.25h^2/k = 0.01L^2/k \\
0.025h^2/k \\
0.0025h^2/k 
\end{cases}$$

[Carslaw-Jaeger, 1959, p. 97]
\[ \Delta_t = \frac{0.25h^2}{k} \]
\( \Delta t = 0.025 \frac{h^2}{k} \)
\[ \Delta_t = 0.0025h^2/k \]
**Exercise**

1. Reproduce the numerical examples with the backward Euler method. Explain the results.
2. Repeat the numerical examples with the trapezoidal rule \((\alpha = 1/2)\). Try also a larger time step \(\Delta t = 0.5h^2/k\) (is it over the oscillation limit?).
3. Repeat the numerical examples with the explicit forward Euler method \((\alpha = 0\), lumped “mass”). Try also a larger time step \(\Delta t = 0.625h^2/k\). Is it over the stability limit? Compare the stability limit based on the element-level estimate to the global value for this mesh.