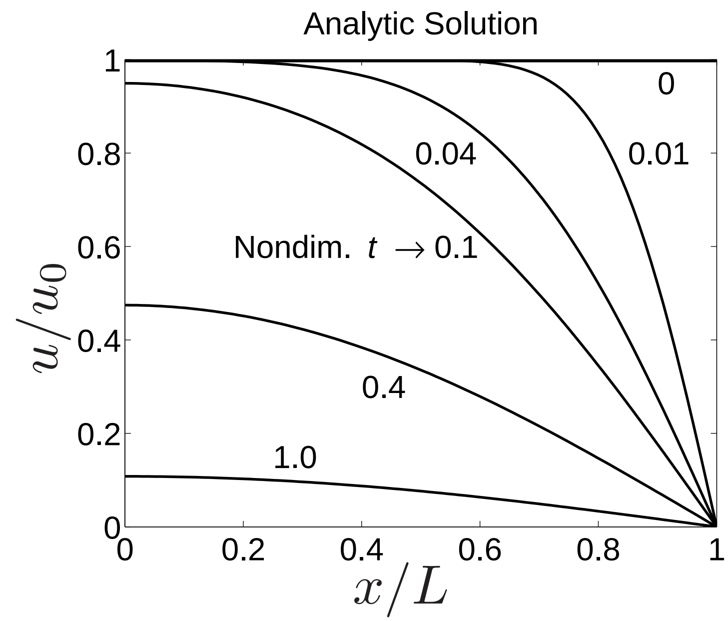
Computer Assignment No. 2

Problem

Heat flow in a slab $0 < x < L$.
$k =$ const., $f = 0$.
bc's: insulation at $x = 0$ and $u(L) = 0$.
ic's: $u|_{t=0} = u_0 =$ const.



Maximum Principle

Extreme values of the solution of the homogeneous heat equation occur at $t = 0$ or on the spatial boundary.

Computation

5 equal linear finite elements in space.
Backward Euler ($\alpha = 1$) in time.

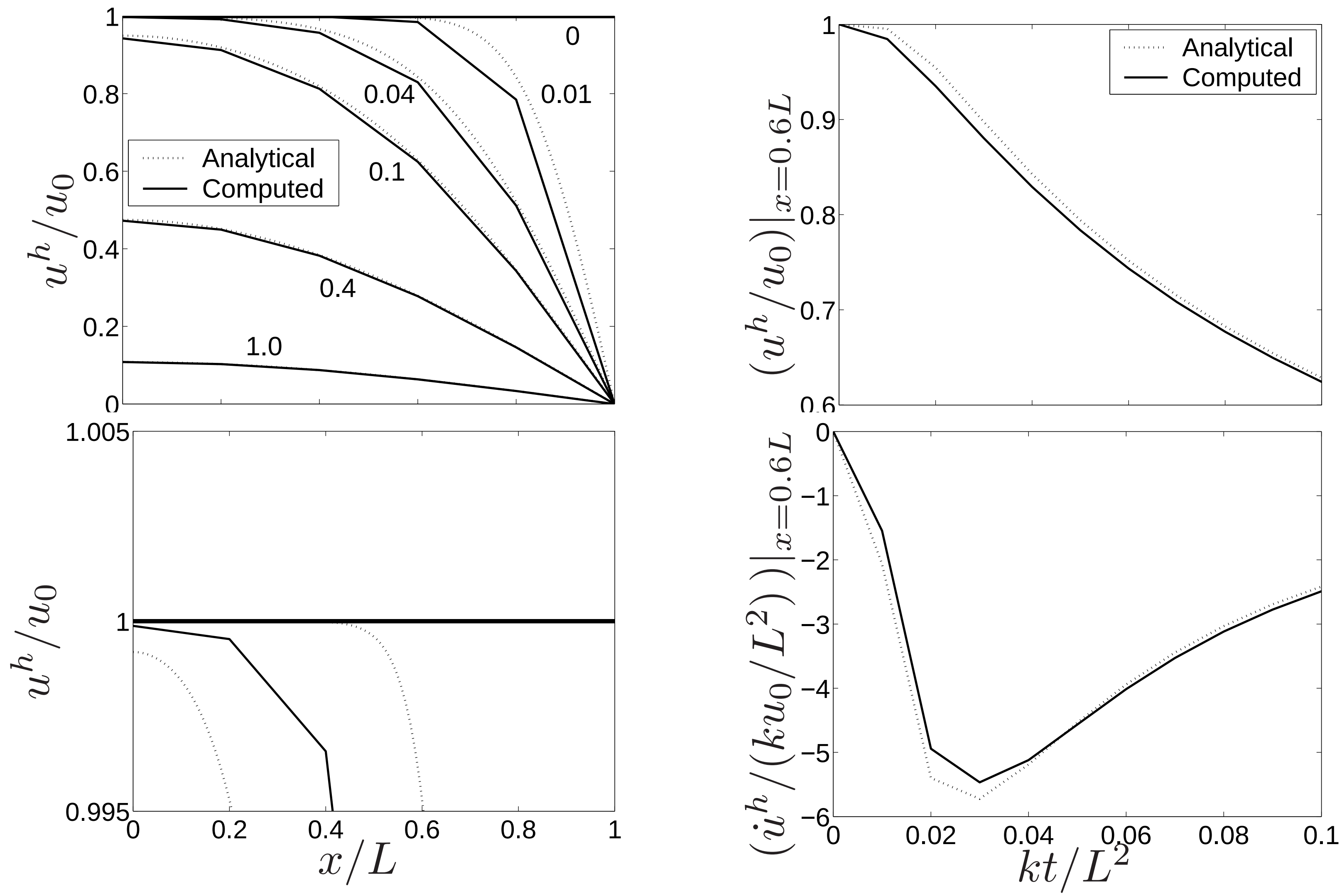
$$\Delta_t = \begin{cases} 0.25h^2/k = 0.01L^2/k \\ 0.025h^2/k \\ 0.0025h^2/k \end{cases}$$

Nondimensional time is

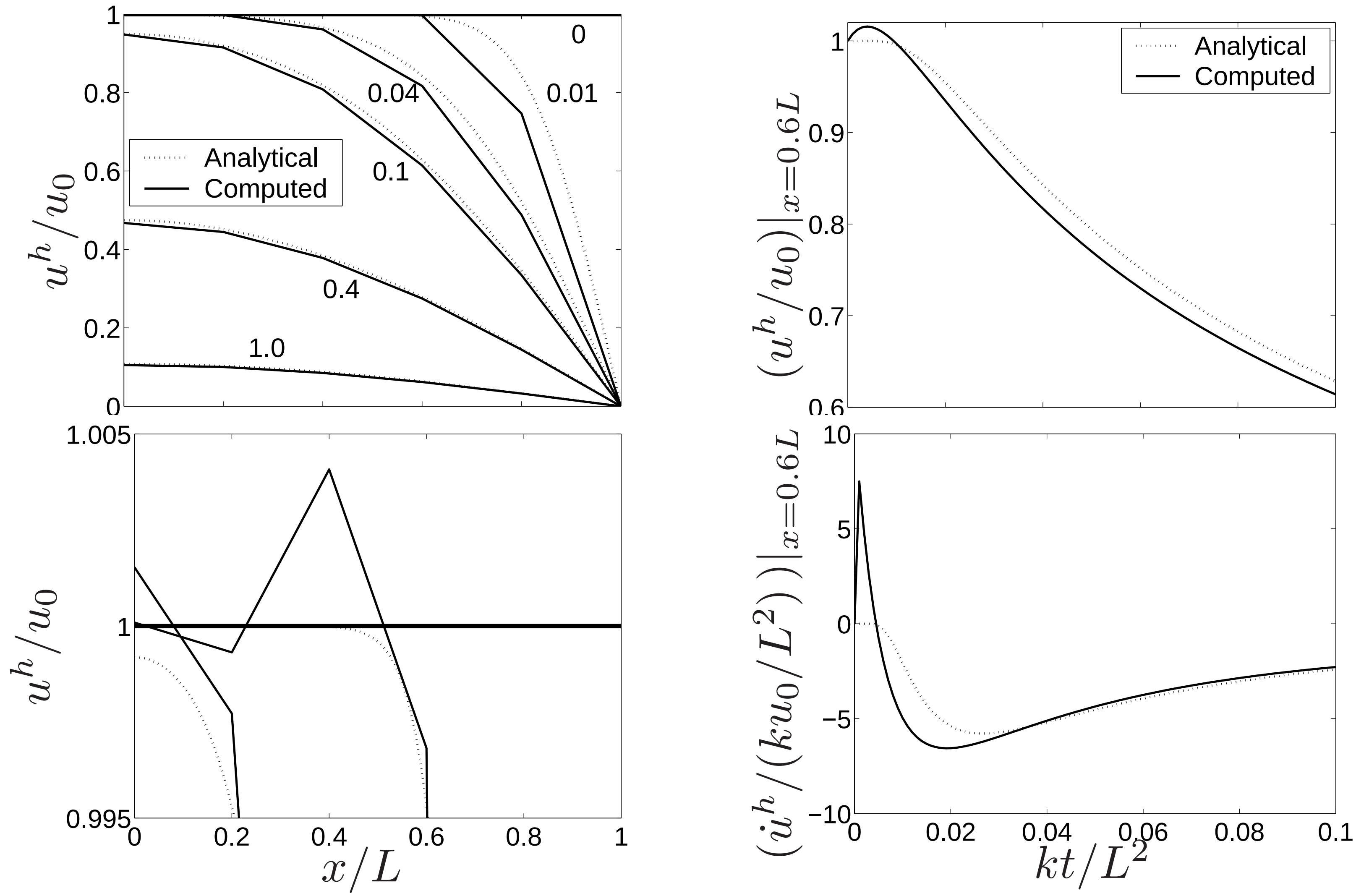
$$\frac{kt}{L^2}$$

[Carslaw-Jaeger, 1959, p. 97]

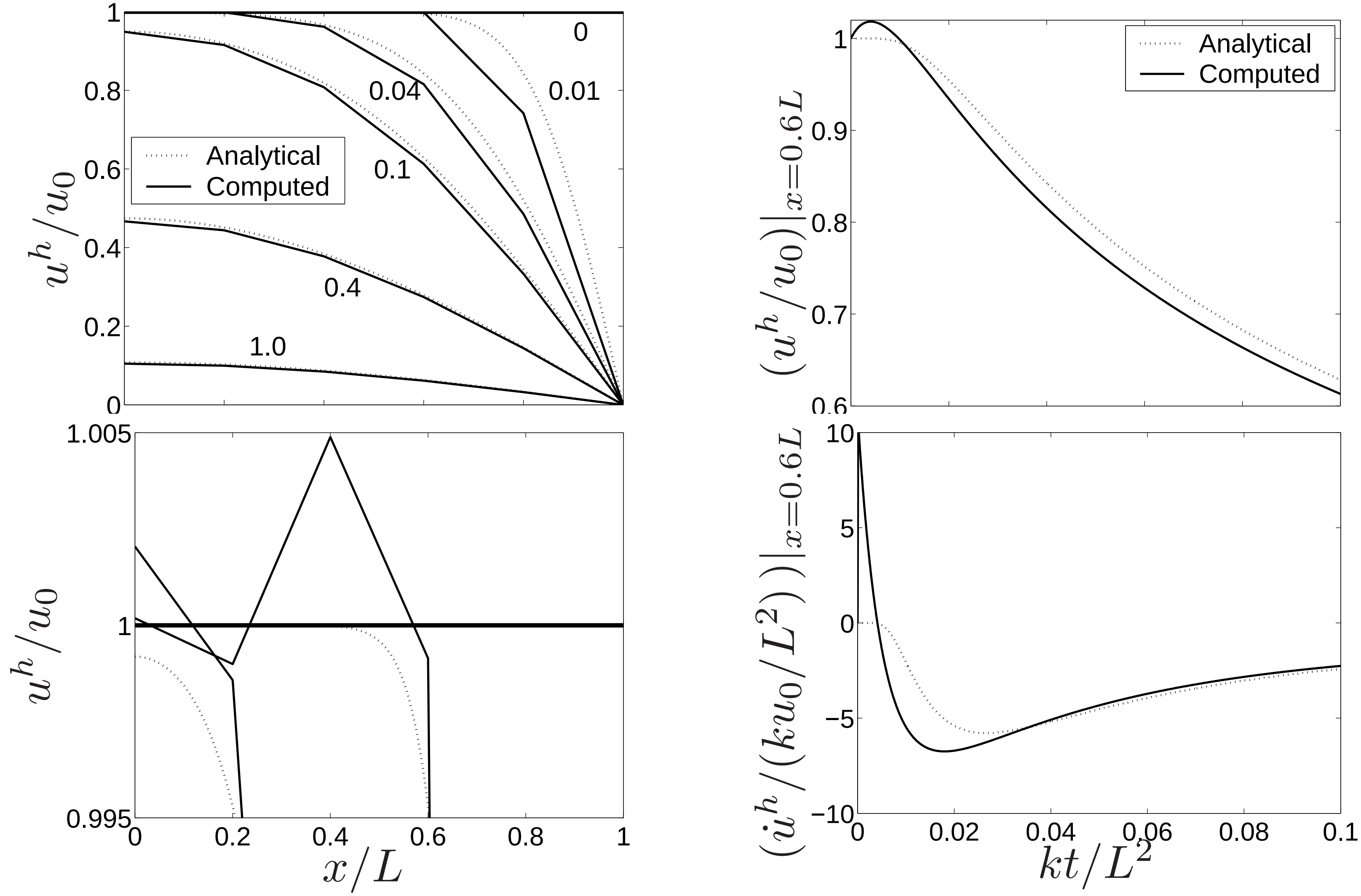
$$\Delta_t = 0.25h^2/k$$



$\Delta_t = 0.025h^2/k$



$\Delta_t = 0.0025h^2/k$



Exercise

1. Reproduce the numerical examples with the backward Euler method. Explain the results.
2. Repeat the numerical examples with the trapezoidal rule ($\alpha = 1/2$). Try also a larger time step $\Delta_t = 0.5h^2/k$ (is it over the oscillation limit?).
3. Repeat the numerical examples with the explicit forward Euler method ($\alpha = 0$, lumped "mass"). Try also a larger time step $\Delta_t = 0.625h^2/k$. Is it over the stability limit? Compare the stability limit based on the element-level estimate to the global value for this mesh.