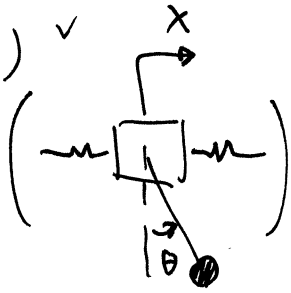
-נרשום מש' כוחות ומומנטים בנפרד עבור כל מסה עם מטוטלת:

M1

$$\sum F_x = -K_3X_1 + K_4(X_2-X_1) = m_1\ddot{X}_1 + m_4(\ddot{X}_1 + L\ddot{\theta}_1) \quad \checkmark$$



$$\sum M = -m_4 g L\theta_1 = m_4 L^2\ddot{\theta}_1 + m_4 L\ddot{X}_1 \quad \checkmark$$

M2

$$\sum F_x = -K_4(X_2-X_1) + K_5(X_3-X_2) - K_1X_2 \overbrace{\ }^{-K_2X_2} = m_2\ddot{X}_2 + m_5(\ddot{X}_2 + L\ddot{\theta}_2) \quad \checkmark$$

$$\sum M = -m_5 g L\theta_2 = m_5(\ddot{X}_2 + L\ddot{\theta}_2)L \quad \checkmark$$

M3

$$\sum F_x = -K_5(X_3-X_2) - K_6X_3 = m_3\ddot{X}_3 + m_6(\ddot{X}_3 + L\ddot{\theta}_3) \quad \checkmark$$

$$\sum M = -m_6 g L\theta_3 = m_6(\ddot{X}_3 + L\ddot{\theta}_3)L \quad \checkmark$$

בדיקה: נסכם את 3 משוואות הכוחות ונקבל:

$$-K_3X_1 - (K_1+K_2)X_2 - K_6X_3 = m_1\ddot{X}_1 + m_2\ddot{X}_2 + m_3\ddot{X}_3 + m_4(X_1 + L\ddot{\theta}_1) + m_5(X_2 + L\ddot{\theta}_2) + m_6(\ddot{X}_3 + L\ddot{\theta}_3)$$

קיבלנו:

$$\boxed{\checkmark} \quad \sum F_{ext} = \sum m_i a_i$$

- נסכם את המשוואות:

$$
\begin{cases}
(m_1+m_4)\ddot{X}_1 + m_4L\ddot{\theta}_1 + (k_3+k_4)X_1 - k_4X_2 = 0 \\
m_4L\ddot{X}_1 + m_4L^2\ddot{\theta}_1 + m_4gL\theta_1 = 0 \\
(m_2+m_5)\ddot{X}_2 + m_5L\ddot{\theta}_2 - k_4X_1 + (k_1+k_2+k_5+k_4)X_2 - k_5X_3 = 0 \\
m_5L\ddot{X}_2 + m_5L^2\ddot{\theta}_2 + m_5gL\theta_2 = 0 \\
(m_3+m_6)\ddot{X}_3 + m_6L\ddot{\theta}_3 - k_5X_2 + (k_5+k_6)X_3 = 0 \\
m_6L\ddot{X}_3 + m_6L^2\ddot{\theta}_3 + m_6gL\theta_3 = 0
\end{cases}
$$

- בצורה המטריצינית:

$$
\underbrace{\begin{bmatrix}
m_1+m_4 & 0 & 0 & m_4L & 0 & 0 \\
m_4L & 0 & 0 & m_4L^2 & 0 & 0 \\
0 & m_2+m_5 & 0 & 0 & m_5L & 0 \\
0 & m_5L & 0 & 0 & m_5L^2 & 0 \\
0 & 0 & m_3+m_6 & 0 & 0 & m_6L \\
0 & 0 & m_6L & 0 & 0 & m_6L^2
\end{bmatrix}}_{[M]}
\begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{Bmatrix}
+
\underbrace{\begin{bmatrix}
(k_3+k_4) & -k_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_4gL & 0 & 0 \\
-k_4 & (k_1+k_2+k_3+k_4) & -k_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & m_5gL & 0 \\
0 & -k_5 & (k_5+k_6) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & m_6gL
\end{bmatrix}}_{[K]}
\cdot
\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix}
= \underline{0}
$$

– עבור

$K_1, K_2 = 2K \quad , \quad K_3, K_4, K_5, K_6 = K$

$M_1, M_2, M_3 = 2M, \quad M_4, M_5, M_6 = M$

נקבל:

$$[M] = \begin{bmatrix} 3M & 0 & 0 & ML & 0 & 0 \\ ML & 0 & 0 & ML^2 & 0 & 0 \\ 0 & 3M & 0 & 0 & ML & 0 \\ 0 & ML & 0 & 0 & ML^2 & 0 \\ 0 & 0 & 3M & 0 & 0 & ML \\ 0 & 0 & ML & 0 & 0 & ML^2 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 2K & -K & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & MLg & 0 & 0 \\ -K & 6K & -K & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & MLg & 0 \\ 0 & -K & 2K & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & MgL \end{bmatrix}$$

נשים לב כי ע"מ לקבל מט' סימטרית יש לבצע reordering
הבא של מט' המשוואות:

$$[M] = \begin{array}{cc} \begin{bmatrix} 3M & 0 & 0 & ML & 0 & 0 \\ 0 & 3M & 0 & 0 & ML & 0 \\ 0 & 0 & 3M & 0 & 0 & ML \\ ML & 0 & 0 & ML^2 & 0 & 0 \\ 0 & ML & 0 & 0 & ML^2 & 0 \\ 0 & 0 & ML & 0 & 0 & ML^2 \end{bmatrix} & \begin{array}{l} \checkmark\ R_1 \leftarrow R_1 \\ \checkmark\ R_2 \leftarrow R_3 \\ \checkmark\ R_3 \leftarrow R_5 \\ \checkmark\ R_4 \leftarrow R_2 \\ \checkmark\ R_5 \leftarrow R_4 \\ \checkmark\ R_6 \leftarrow R_6 \end{array} \end{array}$$

$$[K] = \begin{bmatrix} 2K & -K & 0 & 0 & 0 & 0 \\ -K & 6K & -K & 0 & 0 & 0 \\ 0 & -K & 2K & 0 & 0 & 0 \\ 0 & 0 & 0 & MLg & 0 & 0 \\ 0 & 0 & 0 & 0 & MLg & 0 \\ 0 & 0 & 0 & 0 & 0 & MLg \end{bmatrix}$$

(בצורה דומה)

כעת: $[M]=[M]^T,\ [K]=[K]^T$

- המט' המטריציונית:

$$[M]\{\ddot{x}\}+[K]\{x\}=\{0\}$$

- ע"מ למצוא ע"ע ו"ע של המערכת נפתור בע"ת ע"ע מוכללת:

$$[K]\{u\}=\omega^2[M]\{u\}$$

- פתרון לא טריוויאלי:

$$\left|[K]-\omega^2[M]\right|=0$$

MATLAB code:

```
clear all
close all
clc

% define symbolic variables:
syms M K m l k g lambda

% define numeric values:
mnum=5;
knum=20;
lnum=7;
gnum=10;

% define mass & stiffness matrices for the system (symmetric):
M=[3*m 0 0 m*l 0 0; 0 3*m 0 0 m*l 0; 0 0 3*m 0 0 m*l; m*l 0 0 m*l^2 0 0; 0 m*l 0 0
m*l^2 0; 0 0 m*l 0 0 m*l^2];

K=[2*k -k 0 0 0 0; -k 6*k -k 0 0 0; 0 -k 2*k 0 0 0; 0 0 0 m*l*g 0 0; 0 0 0 0 m*l*g 0; 0 0 0 0 0
m*l*g ];

% substitute numerical values:
Mnum=subs(M,{m,l},{mnum,lnum});
Knum=subs(K,{k,l,m,g},{knum,lnum,mnum,gnum});

% solve generalize eigenvalue problem:
[V,lambda_n] = eigs(Knum,Mnum);

% obtain nat. frequencies:
wn=sqrt(diag(lambda_n));

% solve for initial excitation:

% normalize eigenvectors with mass matrix:
% (in general, not needed in MATLAB, since eig() does this by itself)
for i=1:6
   alpha(i)=V(:,i)'*Mnum*V(:,i);
   V(:,i)=V(:,i)/sqrt(alpha(i));
   % possible to check the normalization:
   % lambda(i)=eval(V(:,i)'*Knum*V(:,i));
end

% define initial excitation:

x0=[0.1 0.2 0.3 0.03 0 0.02]';
v0=[0.5 0 0 0 0 0]';

% define symbolic time and disp. array:
syms t x
```

```
% initialize x:
x=[0 0 0 0 0 0]';

digits(4)

% solution using the course book, p.148, 4.72:
for i=1:6
    x = x + ( sym(V(:,i)'*Mnum*x0,'d')*cos(sym(wn(i),'d')*t) + ...
                  sym(V(:,i)'*Mnum*v0,'d')*sin(sym(wn(i),'d')*t)/wn(i) ) * V(:,i);
end

% define numerical time:
% (taking 5 periods of the smallest nat. freq. mode,
% with time interval of 1/20 of the period for the largest nat. freq.
% mode)
tfin=5*2*pi/min(wn);
dt=2*pi/max(wn)/20;
tnum=[0:dt:tfin];

% substitute numerical values:
for i=1:length(tnum)
    xnum(:,i)=subs(x,t,tnum(i));
end

% plot time history:
figure(1)
color=['b' 'k' 'r' 'g' 'y' 'c'];

for i=1:6
    hold on
    plot(tnum,xnum(i,:),color(i))
end

hold off
```

Output:

- Matrices:

```
K =

[   2*k,    -k,     0,     0,     0,     0]
[    -k,   6*k,    -k,     0,     0,     0]
[     0,    -k,   2*k,     0,     0,     0]
[     0,     0,     0, m*l*g,     0,     0]
[     0,     0,     0,     0, m*l*g,     0]
[     0,     0,     0,     0,     0, m*l*g]


M =

[   3*m,     0,     0,   m*l,     0,     0]
[     0,   3*m,     0,     0,   m*l,     0]
[     0,     0,   3*m,     0,     0,   m*l]
[   m*l,     0,     0, m*l^2,     0,     0]
[     0,   m*l,     0,     0, m*l^2,     0]
[     0,     0,   m*l,     0,     0, m*l^2]
```

- Nat. frequencies and normalized modal matrix:

```
wn =

    3.7009
    2.2361
    2.0458
    1.1599
    1.0690
    1.0288


V =

   -0.0675    0.2152   -0.2001   -0.0056   -0.0609   -0.0733
    0.3004    0.0000   -0.0899    0.0247    0.0000   -0.0330
   -0.0675   -0.2152   -0.2001   -0.0056    0.0609   -0.0733
    0.0108   -0.0430    0.0434   -0.0128   -0.0348   -0.0300
   -0.0479   -0.0000    0.0195    0.0571   -0.0000   -0.0135
    0.0108    0.0430    0.0434   -0.0128    0.0348   -0.0300
```

- ***time history:***