1 Introduction

Synthetic cellular materials manufactured from metals, ceramics, paper, and carbon are used in diverse engineering applications including crush worthiness, heat conductivity, and shock mitigation. Porous materials are also commonly found in nature, e.g., balsa wood, cork, bone, and plant stalks. In addition to their high strength to weight ratio, these structures possess a wealth of geometric and material parameters that offer numerous avenues for optimization. A comprehensive exposition of the subject can be found in a monograph by Gibson and Ashby [1] and a recent review article by Alghamdi [2].

The primary source for energy absorption in cellular structures, the subject of main interest in this work, is progressive buckling and folding of thin material elements of the structure. The specific behavior may be conveniently classified into two categories, depending on whether the compression load is applied normal to the cell axis (e.g., honeycombs and foams) or along it (e.g., thin-wall tubes). The latter case primarily differs from the former in the additional energy that is absorbed by stretching or compression along the periphery of the fold, and the fact that global buckling may interfere with the inner deformation process.

The response of cellular structures is generally characterized by the mean stress and the densification strain, the product that equals the specific crush energy. Systematic and detailed crush studies on hexagonal honeycombs [3], polymeric foams [4], and balsa wood [5], to name a few, show that the mean stress increases with the relative density or volume fraction of the cellular structure while the opposite trend occurs for the densification strain. In the case of tubes compressed along their axis, the pioneering treatments of the circular and rectangular section tubes by Alexander [6] and Abramowitz and Jones [7] are noted. Bardi et al. [8] carried out careful measurements of the geometric aspects of the fold pattern in circular tubes and identified the merit of various analytical models in terms of the ratio of tube radius and wall thickness. More particularized studies aimed at stabilizing the deformation sequence and improving the crush energy of tube structures include the effects of fillers [9–12] and circumferential grooves [13,14]. Another promising approach for enhancing the crush energy is offered by lateral constraints; compression studies on nested tubes [15,16] or polymeric foams [4], for example, show that lateral constraints may stabilize the deformation process and significantly increase the collapse load relative to the unconstrained structures.

A common characteristic in all the cell geometries detailed above is that the crush energy is mainly absorbed in filling porosities by means of plastic folding and possibly material stretching. We propose here the laterally confined column or bar configuration illustrated in Fig. 1 as yet another potential concept for energy absorption. In addition to the mechanisms noted above, lateral confinement may offer additional energy absorption through axial compression and friction between the confining walls and the bar. It is envisioned that this simple and instructive configuration may be used as a building block for multi-plate design possessing improved crush performance. Previous studies on the bi-laterally constrained column, conclusively limited to the elastic response [17–20], show that the deformation is characterized by a buckling mode transition process that results from secondary buckling of flattened segments of the bar. It was also pointed out that this configuration may lead to large energy dissipation due to friction between the plate and the confining walls [18]. In this work, the energy absorption characteristics of a polyvinyl chloride (PVC) bar that is confined on all of its four sides and subject to axial compression is studied in situ, with emphasis placed on the deep loading or the plastic deformation regime. Figure 1 shows a schematic of the test specimen. The thickness and length of the bar as well as the gap between two of the four confining walls are systematically varied. The experimental apparatus is detailed in Sec. 2 while the test results are discussed in Sec. 3. A simple analytic model is developed in Sec. 4 to account for energy dis-

Contributed by the Applied Mechanics Division of ASME for publication in the Journal of Applied Mechanics. Manuscript received January 18, 2005; final manuscript received March 15, 2005. Review conducted by S. Kyriakides. Discussion on the paper should be addressed to the Editor, Prof. Robert M. McMeeking, Journal of Applied Mechanics, Department of Mechanical and Environmental Engineering, University of California—Santa Barbara, Santa Barbara, CA 93106-5070, and will be accepted until four months after final publication of the paper itself in the ASME Journal of Applied Mechanics.
sipation as a function of the system parameters. Finally, Sec. 5 examines the crush performance of the confined bar relative to more traditional cellular structures.

2 Experimental

Tests are carried out to elucidate the energy absorption capability of a laterally confined rectangular bar under axial compression. Figure 1 details the test apparatus. Test samples of length \( L \), thickness \( t \), and width \( b \) are cut from flat PVC slabs. The latter is confined by four thick blocks, with two wall surfaces parallel to \( yz \) plane ("side walls") and the other two parallel to the \( xy \) plane ("transverse walls"). The side walls are spaced a distance \( h \) apart while the distance between the transverse walls matches the width of the bar, \( b \). The majority of the tests are carried out with the bar initially resting on one of the two side walls (see Fig. 1). Some tests in which the bar is positioned in the middle of the two side walls are also performed. No distinction between these two configurations is made in the following as the test results are found to be virtually indistinguishable. One of the transverse walls is made of transparent PMMA to allow for in situ observations. The lower and upper edges of the bar are bonded with small pieces of PVC to facilitate a clamped-type support there. The specimens are supported at their lower edge by a steel plate while the upper one is compressed by a rectangular steel rod whose cross section coincides with the confinement area, \( b \times h \). The upper part of the loading rod is firmly inserted into the loading head of the testing machine in order to increase its buckling resistance. To reduce geometric imperfections, a high degree of precision in the machining and alignment of all relevant components of the test apparatus is employed. As friction is a leading energy absorption mechanism in this study, the surfaces of the specimen and the confining walls are cleaned prior to testing with an alcohol solvent. Tests are carried out as a function of \( L/t \), i.e., from 4 to 30. The volume fraction \( t/h \) is varied by varying the wall to wall gap, \( h \), i.e., from 0.12 to 0.9.

The upper edge of the specimen is monotonically compressed using a screw driven testing machine (Instron). The deformation of the specimen is observed through the transparent transverse wall using a video camera. An additional video camera, whose image is superimposed on the main frame, is used to record the load. In all tests, the nominal strain (i.e., end displacement divided by the specimen length) is applied at a fixed rate of about 0.01/s. The axial load, \( P \), and the crosshead displacement are recorded during the tests using a National Instrument data acquisition package. To circumvent the effect of machine compliance and the deformation absorbed in the loading rod and other components of the system, effects that become more and more significant as the ratio \( t/h \) increases toward unity, the relative shortening of the specimen ends is directly evaluated from the video image rather than the crosshead displacement. Some tests using solid cylindrical bars are also performed in order to elucidate the post-yield response of the PVC material used, which is a key ingredient in the analysis. Initial tests using uniform section cylinders show a tendency for barreling following yielding. To reduce this undesirable effect, the central part of the cylinders is thinned down in a gradual manner using a circular head cutting tool to form a hyperbolid test section. The radius of the tool is varied from 5 mm to infinity (i.e., uniform section). Tests are performed in either compression or tension. The deformation of these specimens is also observed in real time in order to facilitate concurrent measures of specimen diameter and load. If \( d_0 \) and \( d \) are the initial and concurrent diameter of the central cross section of the specimen, and \( F \) is the applied axial force, then, assuming material incompressibility and a rigid plastic behavior, the true stress and true strain are related by

\[
\sigma_t = 4F/\pi d^2, \quad e_t = 2 \ln(d/d_0)
\]  

where positive quantities indicate compression.
3 Test Results

We first discuss the material response before moving on to consider the main problem of interest. The test results show that the stress-strain response depends somewhat on the pretest neck radius. However, the yield stress, defined at the time of a noticeable change in the slope of the stress vs. strain curve, is found to be little sensitive to that parameter. The mean values of the yield stress in tension and compression from a number of tests having different initial neck radii are found to be 53 and 83 MPa, respectively, with standard deviation on the order of 5%. (The larger value in compression as compared to tension is a common phenomenon in polymers, attributed to the pressure sensitivity of the yield behavior.) These results will be discussed later in this work.

Let $\sigma = P/bh$ and $\varepsilon = \Delta L/L$ be the nominal stress and strain, where $P$ and $\Delta$ are the axial load and the axial displacement, respectively. In this way, a plot of $\sigma$ vs. $\varepsilon$ produces the stress-strain behavior of an effective material having a cross-sectional area $bh$. We further denote $\varepsilon_t = t/h$ as the volume fraction of the material within the confinement space $bhL$. Figure 2 shows the deformation sequences for two different bars having a ratio $h/t$ of 4 (a) and 2 (b). As shown, the first stages in the deformation consist of column buckling, formation of a discrete contact point at the opposing wall, and the spread of the contact area.\footnote{The deformation of a bilaterally constrained column in the elastic range was studied in [18]. It was found that the deformation is characterized by a sequential mode transition process that results from secondary buckling. The latter initiates at contact zones formed between the column and the confining walls. This process was found to be inherently asymmetric, which leads to significant scatter in the buckling mode transition loads. An extension of this study to laterally constrained plates [21,22] shows that once a number of buckles are initiated, the buckling pattern in this case tends to approach the 1-D configuration.} This is followed by plate folding at the top end, leading to progressive buckling and folding toward the lower end. In the last stage of the deformation, the material is essentially completely compacted.

The deformation for relatively large gap ratio (a) is characterized by plate folding while that of the tighter confinement (b) exhibits pronounced shearing. As shown by the solid line curve in Fig. 3, the nominal stress for the case study of Fig. 2(a) fluctuates somewhat during the deformation, but generally remains small except near densification, the latter of which occurs at a nominal strain of about 0.75.

As shown in Fig. 2, the material response greatly depends on the volume fraction, $t/h$. Figure 4 shows fully compacted bars in the unloading state for a number of volume fractions, all pertaining to $L/t = 26$. The stress-strain behavior for some representative cases is given in Fig. 3. As shown, the deformation exhibits a cell-like pattern characterized by plate folding. The number of these cells decreases with increasing $h/t$, a consequence of the greater room available for deformation. When $h/t$ is decreased from 3, the deformation in a cell seems more associated with shearing than bending. These aspects of the deformation will be discussed further in Sec. 4 in connection with the analytic model.

As shown in Fig. 3, the effective stress fluctuates during the de-

---

Fig. 2 Video motion pictures showing the deformation of two different laterally confined PVC bars

Fig. 3 Nominal stress vs. nominal strain for three different volume fractions, $t=4$ mm, $L/t=20$

Fig. 4 Fully compacted specimens as they appear after unloading, $t=4$ mm, $L/t=26$
formation, the wavelength of which increases with increasing \( h/t \).

As the latter is decreased, the effective stress generally increases. One also observes that the densification strain decreases with \( h/t \).

The overall behavior described is very reminiscent of progressive collapse in other cellular structures (e.g., [3,8]).

The geometric aspects of the cell structure are an important ingredient in any analytic treatment. Figure 5 (symbols) details the variation of \( \delta/t \) with \( t/h \) for the three specimen thicknesses tested, where \( \delta \) denotes the cell wavelength. The data for this plot are generated from results as those shown in Fig. 4, e.g., by dividing the total number of cells in a given specimen by its original length, \( L \). As shown, \( \delta \) may be reasonably well approximated as

\[
\delta = A t, \quad A \approx 3.7
\]  

(2)

Similar results are also found for other values of \( L/t \) (i.e., 4 to 30). Another useful quantity that can be extracted from the tests is the number of folds in a single cell, \( m \). This quantity is established by dividing the total number of folds in the entire bar by the number of cells present. The results, shown as symbols in Fig. 6, seem to be well approximated by the dashed curve, given as

\[
m = h/t
\]  

(3)

Note that the results shown are limited to relatively small values of \( t/h \); as is apparent from Fig. 4, once the ratio \( h/t \) reduces from 3, the folds are not fully developed, with the deformation associated with shearing rather than bending.

The specific crush energy (i.e., the energy dissipated during the entire deformation history per unit confinement volume, \( bhL \)), \( U \), is an important mechanical property. This quantity, given by the area under the stress-strain curves in plots such as those shown in Fig. 3, is shown in Fig. 7 (symbols) as a function of \( t/h \) for all three plate thicknesses studied. The variation of the densification strain with \( t/h \) for these tests is depicted in Fig. 8 as open symbols. (These data are obtained by dividing the end shortening at densification with the initial length of the bar, with the end shortening determined separately for each test from the video records.) Both plots, corresponding to the case \( L/t = 26 \), show that the so-normalized results are quite insensitive to the specimen thickness. As shown in Fig. 7, the specific energy increases from zero, reaches a peak value at \( t/h = 0.5–0.7 \), and steadily declines thereafter. The densification strain steadily declines from unity toward zero as \( t/h \) is increased. Another set of tests is carried out to determine the crush energy as a function of the bar length for a fixed value of \( h/t = 1.5 \). The results, corresponding to two choices of \( t \) (i.e., 2 and 4 mm), are shown as symbols in Fig. 9. As shown, while the specific energy is little insensitive to the bar thickness, it greatly increases with the length of the bar. Clearly, such behavior must be effected by frictional forces.
4 Analytical Model

The external work done by the applied compression force is made up of the strain energy absorbed in the material and the energy dissipated by frictional forces transmitted by the constraining walls. Analytical expressions for these contributions are now derived by neglecting the elastic part of the strain energy and assuming that the material is rate independent, incompressible, and rigid perfectly plastic with yield stress \( \sigma_y \). Accordingly, the densification strain is given by

\[
\epsilon_d = 1 - \frac{t}{h}
\]

(4)

It should be noted that this relation provides an upper limit to the densification strain since, as can be seen from the last prints in Figs. 2(a) and 2(b), there always exist some gaps or cavities in the compressed material. Figure 8 illustrates the deformation sequence used in the analytic model. Initially, the densification strain was determined from the cross-sections of the compacted material. Figure 8 shows that as the material is compressed, the cross-sections become elliptical, with the major axis of the ellipse being parallel to the direction of loading.

The densification strain is given by

\[
\epsilon_d = 1 - \frac{t}{h}
\]

where \( t \) is the thickness of the bar and \( h \) is the height of the bar. This relationship is valid for small deformations and can be used to determine the densification strain in the analytical model.

4.1 Compression Model. For sufficiently small clearance \( h \), a uniform state of compression may be assumed [see Fig. 11(a)]. Under the previous simplifying assumptions, the specific energy in this case equals \( \sigma_y \epsilon_d \), where \( \epsilon_d = \ln (A/\delta) \) is the true strain in compression. The strain energy for a fully compacted specimen is then given by \( \Delta U = \frac{1}{2} \sigma_y (A h) \ln (h/t) \). Making use of Eqs. (2) and (5), one has

\[
U = \frac{1}{2} \sigma_y (A h) \ln (h/t),
\]

(6)

where \( W \) is the work done by the external load \( P \) and \( U \) denotes the specific work. The latter provides a useful baseline for optimizing the energy absorption based on the space available for deformation.

4.2 Shear Model. Consider an initial segment of length \( L \) which undergoes buckling such that the buckle just touches the opposing side wall [see print I of Fig. 11(b)]. Let the associated end shortening be \( \eta \). Assuming no friction, the work done in fully compacting the cell, i.e., from illustration (I) to (II) in Fig. 11(b), is given by \( \frac{1}{2} \sigma_y \eta A \), where \( \eta \) is the axial load acting on the section of the bar, taken as \( t b \tau_0 \), where \( \tau_0 \) is the yield stress in shear. Following [18], \( \eta = \frac{1}{2} \int_0^b (v')^2 dy \), where \( v \) denotes the normal displacement of the bar and the prime sign indicates derivation with respect to the vertical coordinate, \( y \), that extends from the base of the buckle. Substituting \( v = (h - c)(1 - \cos 2\pi y/\lambda) \) and integrating, one has \( \eta = \left( \frac{\pi^2}{4A} \right) h (1 - t/h)^2 \) [18]. Making use of the preceding relations, the specific energy due to shearing in a bar of length \( L \) containing \( n \) cells is thus

\[
U_s = \left( \sigma_y \sqrt{\frac{3}{5}} \right) \frac{1}{2} \sigma_y (A h) \ln (h/t) \left[ 1 - (1 - t/h)(\pi/2A)^2 \right]
\]

(8)

where \( \tau_0 \) is taken as \( \sigma_y / \sqrt{3} \).
4.3 Bending Model. Referring to Fig. 11(c), it is assumed that the strain energy contained in a fully compacted cell is due to the creation of plastic hinges, the latter of which is indicated as small, solid circles in this print. Invoking plastic limit analysis, the strain energy needed to create a single fold is given by \( M_0 \pi \), where \( M_0 = \sigma_b b^2/4 \) is the bending moment acting at the plastic hinge [1]. As is apparent from Fig. 6, the number of plastic hinges in a single cell, \( m \), may be approximated as \( h/t \). As indicated earlier, this expression is nominally valid for \( t/h < 1/3 \). (Kinematical admissibility suggests that the area occupied by a single plastic hinge is \( \lambda l m = \pi r^2 \).) For a bar having \( n \) cells, the total number of complete folds is \( nm \). Using previous expressions, the specific energy density due to bending is thus

\[
U_b = (\pi/4A)\sigma_b f/h
\]

(9)

4.4 Friction Model. Friction forces are assumed to develop on all four confining walls. The frictional energy dissipated during the complete deformation process may be broken into two parts. The first is associated with the individual compaction of all cells as shown in Fig. 10(b), and the second is due to the downward motion of all cells toward the compact configuration shown in Fig. 10(c). Consider first the first mechanism from Fig. 11(d), where \( f_s \) and \( f_t \) denote the total frictional forces exerted on a single cell as it moves downward by the two side walls and the two transverse walls, respectively. The frictional work expended on compacting \( n \) individual cells of initial length \( \lambda \) is thus

\[
W_f = n(f_s + f_t)(\lambda - \delta)/2.
\]

The second contribution, arising from the translation of all individual cells into the compact configuration shown in Fig. 10(c), is given by

\[
W_f = n(f_s + f_t)(\lambda - \delta)(0 + 1 + 2 + \cdots + (n-1)) = (f_s + f_t)(\lambda - \delta)n(n-1)/2.
\]

The combined frictional work is thus

\[
W_f = (f_s + f_t)(\lambda - \delta)n^2/2
\]

(10), and making use of preceding relations, one find

\[
U_t = (2/A)\mu \sigma_b (L/h) (t/h)^2(1 - t/h)(1 + 0.25h/b)
\]

(11)

Equation (11) shows that the frictional energy is proportional to the friction coefficient, the yield stress, and the length of the bar. The contribution from the transverse walls is represented by the term \( 0.25h/b \). The latter vanishes for very wide bars ("plane strain" conditions).

Figure 12 shows the variation of the specific energy, normalized by the yield stress, with the volume fraction \( t/h \) for all four energy absorption models, where \( A \) is taken as 3.7 (Fig. 5), and, in the case of the frictional energy, \( \mu_l L/t, t/b \) are taken as (0.3, 26, 0.25). Note that the dotted lines in this figure correspond to regions outside the nominally applicable range of the corresponding models. It is apparent that for relatively large volume fractions, the contributions from compression and friction are dominant. One observes that the frictional energy is maximized at \( t/h = 0.65 \).

4.5 Comparison With the Tests. As indicated in the derivation of the analytic models, the frictional model operates over the entire range of \( t/h \) while the bending and shearing models nominally apply in the range \( t/h < 1/3 \) and \( 1/3 < t/h < 0.5 \), respectively. The compression model is nominally limited to \( t/h \geq 0.5 \) as for smaller ratios the shearing mode becomes operational. Clearly, some interaction among the various deformation modes is expected at the interfaces between these regimes. However, explicit relations in these cases appear to be difficult if not impossible to elucidate. Based on these observations and limitations, the following hybrid relationship is proposed:

\[
U_c = \frac{1}{3} \frac{U_t}{A} + \frac{2}{3} \frac{U_b}{A}
\]

(12)

where \( U_t \) and \( U_b \) are the specific energy contributions due to friction and bending, respectively. This relationship satisfies both the limiting conditions and the general expression of Eq. (11).
\[ U = U_1 + U_2, \quad t/h \gg 0.5 \quad (12a) \]
\[ U = U_b + U_c, \quad t/h \ll 1/3 \quad (12b) \]
\[ U = (U_1 + U_2) + (U_3 - U_1)(h/t - 3), \quad 1/3 < t/h < 0.5 \quad (12c) \]

Equation (12c) implies a linear variation of the specific energy between its two extreme end points, i.e., friction plus compression and friction plus shear. It also leads to a small discontinuity in \( U \) at \( t/h = 1/3 \), reflecting the difference between \( U_b \) and \( U_c \). In order to compare this prediction with the test results, the yield stress and the friction coefficient need be specified. (The cell parameter \( A \) is taken as 3.7.) As discussed in Sec. 3, the yield stress in compression and tension differ, being equal to 53 and 83 MPa, respectively. Because buckling and folding involve both types of deformations, we shall take the average of these two values, namely \( \sigma_y = 68 \) MPa. Published values for the friction coefficient of PVC on steel substrates vary from 0.21 to 0.5. In view of this large scatter, the friction coefficient was chosen such as to fit best the experimental data, i.e., 0.3. The analytical prediction from Eq. (12) based on these choices is shown as solid line curves in Figs. 7 and 9. As shown, the analysis seems to agree reasonably well with the test results over the range of parameters studied. The results in Fig. 7 show that the aforementioned discontinuity at \( t/h = 1/3 \) could be removed by replacing \( U_c \) with \( U_b \) in Eq. (12c) without significant loss of accuracy. It also appears that the neglect of the possible interaction among the various deformation modes is justified from a practical viewpoint.

5 Discussions

The crush performance of cellular structures is generally characterized by the mean stress, \( \sigma_m \), and the relative density or volume fraction, \( \nu_l = (t/h) \) (in the present application). The specific crush energy is then given by \( U = \sigma_m \nu_l^2 = \sigma_m (1 - \nu_l) \). Then, from Eq. (12b), the mean stress for the laterally confined bar under the conditions of relatively small volume fraction and \( t/h \ll 1 \) is
\[ \sigma_m^* \sigma_0 = 0.54 \mu (L/t) U^2 + 0.21 \nu_f (1 - U^2). \quad (13) \]

It is interesting to compare this result with those from more common cellular structures such as honeycombs, foams, and hollow tubes. In the case where the cellular material is compressed in a direction normal to the cell axis, the mean stress may be expressed as
\[ \sigma_m^* \sigma_0 = B \nu_l^C \quad (14) \]
where \( B \) and \( C \) are constants, given as (0.3, 1.5) and (0.28, 2) for open cell foams and hexagonal cell honeycombs, respectively [1]. [Note that the constants for the hexagonal cell honeycomb are based on the relation \( \nu_l = (8/3) t/c \), where \( c \) is the distance between adjacent cell walls, with four of the wall cells being of thickness \( t \) while the other two of thickness \( 2t \).] Now consider the case where the loading is applied along the tube axis. For a circular section tube, the mean load, \( P_m \), and the wavelength of the fold, \( \gamma \), may be approximated as \( P_m = 11.2 \sigma_0^2 (R/t)^{1/2} \) and \( \gamma / R = 1.85 \nu_l^2 (R/t)^{1/2} \) [23], where \( R \) and \( t \) are the mean tube radius and the wall thickness, respectively. An effective volume fraction value for this configuration may be obtained by considering the tube radius in the fully crushed state, i.e., \( R + \gamma / 2 \), rather than the initial state. This gives \( \nu_l = 2 \pi R t / [(\pi (R + \gamma / 2))^2] \). Making use of this relation in the preceding expressions, and taking \( \sigma_m = P_m / [ (\pi (R + \gamma / 2))^2] \), one has
\[ \sigma_m^* \sigma_0 = 1.26 \nu_l^{1.5} / [(1 + 0.66 \nu_l^{0.5})^2] \quad (15) \]

In the case of a square section tube, the mean load may be approximated as \( P_m = 12.2 \sigma_0^2 (c/t)^{1/2} \) [7], where \( t \) and \( c \) are the wall thickness and the wall to wall distance, respectively. Again, the effective volume fraction value for this configuration is constructed by considering the dimension of the section in the fully crushed state, i.e., \( c + 2H \), where \( H = (\pi c / 2)^{0.5} \) [10]. The volume fraction then becomes \( \nu_l = 4 \pi c / [(c + 2H)] \), from which
\[ \sigma_m^* \sigma_0 = 1.3 \nu_l^{1.5} / [(1 + 1.25 \nu_l^{0.5})^2] \quad (16) \]
The variations of the normalized mean stress with volume fraction for the various cellular structures detailed in Eqs. (13)–(16) are shown in Fig. 13. In the case of the laterally constrained bar, results are plotted for two choices of the friction coefficients, i.e., \( \mu = 0 \) and 0.3, with \( L/t = 26 \); note that the case \( \mu = 0 \) corresponds to the bending model alone, and it is independent of \( L/t \). For all cellular structures, the mean stress monotonically increases with the volume fraction. The performance of the foam, the square and the circular section tubes, and the frictionless bar is quite similar. The result for honeycomb is a bit lower, but this may be due to the fact that the loading in this case is normal to the tube axis. With friction included, the constrained bar exhibits a significant increase in the crush energy over the more common structures.

6 Summary and Conclusions

A combined experimental/analytical effort is carried out to elucidate the energy absorption characteristics of laterally confined bars under monotonically increasing edge displacement. Real-time observations show that the deformation of the bar is characterized by progressive buckling and folding. In the context of crushworthiness the main contribution of the confinement is in the solicitation of further energy dissipation by means of axial deformation between adjacent folds as well as by friction between the bar and the walls. The overall deformation consists of repeating cells whose wavelength approximately equals four times the bar thickness. The specific energy delivered to the bar during the complete deformation process seems to be little sensitive to the thickness of the bar while strongly varying with the volume fraction, \( t/h \).

Simple analytic expressions, based on limit plasticity and incompressible material behavior in the post-yield regime, are developed. The main energy absorption mechanisms are simple compression, bending arising from plate folding, and friction. The compression model is limited to a relatively large volume fraction while the bending model nominally applies for \( \nu_l < 1/3 \). The combination of friction and compression provides good predictions of the actual behavior over the range 0.5 < \( \nu_l < 1 \). For a less dense structure, a combination of the friction and bending models takes
over. For frictionless surfaces and a small volume fraction, the energy dissipation is mostly due to plate folding, similarly to the behavior observed for common cellular structures such as honeycombs, foams, and tubes. Frictional effects greatly improve the crush worthiness of the confined bar, with the friction contribution linearly increasing with the friction coefficient or the length of the bar. With friction included, the specific energy is maximized at a volume fraction in the neighborhood of 0.5–0.7. The lower end will probably be more useful because it entails a greater densification strain.

The constrained bar seem to provide a simple and instructive vehicle for understanding the crush performance of more complex configurations. This configuration may be used as a building block in a multi-plate structure to achieve improved crush worthiness over a large working space. The implementation of this in practice does not appear straightforward, however.

References