Nonlinear Elasticity in the Interaction of Living Cells
with their Mechanical Environment

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The elastic cytoskeleton of biological cells contains molecular motors that produce mechanical forces by which cells attach to and pull on their surroundings. This mechanical interaction is responsible for many aspects of cellular function, from cell spreading and proliferation to stem-cell differentiation and tissue development. Both the cytoskeleton and the extracellular matrix comprise cross-linked, semi-flexible polymeric filaments, and as such they exhibit very nonlinear viscoelastic behavior that includes a power-law stiffening of the elastic moduli with increasing stress [1,2,8,9], see Figure 1. Additional nonlinearity comes from the dependence of the mechanical activity of cells on the rigidity of their environment [10].

Figure 1: Differential shear modulus vs shear stress is initially constant (as in linear elasticity) and then crosses over to power-law behavior.

Our theoretical work [7] is motivated by traction-force microscopy experiments of cells that adhere to soft gels, in which microbeads are used to measure the deformation field in the gel, and from which the traction forces that the cells exert are deduced. Recent experiments have shown that non-motile cells are dominated by force dipoles, comprised of equal and opposite contractile forces. However, the dependence of strain energy on the total dipole moment exhibits peculiar scaling laws which have not yet been explained [4].

We consider a spherical force dipole (see Figure 2) in an infinite isotropic, homogeneous, highly nonlinear elastic medium, with constitutive relations derived from a model inspired by [3]. For strong nonlinearity, the differential shear modulus diverges at a finite strain, and we may employ a small strain (but strongly nonlinear) expansion [5,6]. This leads to a nonlinear differential equation for
the condition of mechanical equilibrium, from which we extract scaling relations with non-trivial exponents.

![Figure 2: Spherical force dipole.](image)

References: