HYBRID ABSORBING BOUNDARY CONDITIONS BASED ON FAST NONUNIFORM GRID INTEGRATION FOR NONCONVEX SCATTERERS

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1. INTRODUCTION

Differential-equation-based techniques, such as finite-element (FE), finite-difference time-domain (FDTD), and finite-difference frequency-domain (FDFD) methods are often preferred for analyzing electromagnetic problems involving inhomogeneous media. Truncation of the computational domain, as required in these methods, can be achieved using either global or local absorbing boundary conditions (ABCs). Global boundary conditions were the first to be formulated in the early days of finite-method implementation for electromagnetic problems, due to their similar nature to well-known integral-equation formulations. These ABCs have the advantages of giving accurate results and accommodating arbitrarily shaped boundaries [1]. In a global formulation, the field on the boundary is considered the outcome of sources within the computational domain via a Kirchoff-type integral. An example of such a formulation is found in [2], where the boundary is divided into basis functions in a fashion similar to the method of moments (MoM), but with a geometrical relation to the FE element edges. The resultant matrix is semi-sparse, with the interboundary interactions forming a fully populated block, the computational domain generating a sparse matrix, and the boundary-internal element interactions described by banded matrix blocks. In the global lookback scheme [1], the computational domain is truncated near the sources and the field components on its boundary are generated from those field values known at retarded times on an interior surface one cell away from it via an integral representation of the electromagnetic field. This approach is in essence the basis for the boundary-integration procedure employed in the formulation given below. The main disadvantage associated with global ABCs, however, is the creation of populated rather than sparse matrix blocks; hence, they are considered computationally expensive. For this reason, local ABCs are now more popular. Semiglobal techniques, such as the Green’s function method (GFM) [3, 4], provide a compromise between the efficiency and accuracy of the local and global ABCs, respectively. The GFM is the result of a purely discrete formulation of the Green’s function tailored to a given arbitrary surface, rather than using an outright discretization of the continuous Green’s function.

Early differential-based local ABCs used to truncate the FDTD computational domain have included those proposed by Engquist and Majda [5] and Mur [6]. Many other extensions of these differential-based ABCs have since been proposed. Numerically derived local ABCs have been also proposed [7]. Another type of essentially local ABCs is based on absorption provided by a fictitious layer of material surrounding the computational domain. The most notable advance in material-based ABCs was put forward by Berenger [8]. His ABC, termed the perfectly matched layer (PML) ABC, appears to yield a major improvement in the reduction of boundary reflections compared to any ABC proposed previously. Although PML approach is relatively new, it has enjoyed enormous attention. However, one major drawback common to all these local ABC formulations is the requirement for the boundary surface to be convex, since these boundary conditions are only designed to absorb the outgoing waves. The convexity requirement translates into a sizeable “white space” when treating an essentially concave geometry, and hence implies a significant additional computational cost.

Although global ABCs have traditionally been considered numerically inefficient, this work shows a way to exploit their advantages while gaining computational speed by using the fast method discussed below. It has already been shown in [9, 10] that the combination of global ABCs with suitable fast-integration algorithms provides a viable alternative to local formulations. The proposed method relies on surface integration accelerated by a two-level nonuniform Grid (NG) algorithm, introduced in [11]. This algorithm is shown to reduce the computational cost of evaluating the boundary integrals from O(N^2) to O(N^1.5), while a multilevel algorithm will ultimately attain an asymptotic complexity of approximately O(N log N). Here, N = kR, where k and R are the wavenumber and the radius of the smallest circle circumscribing the scatterer, respectively. In this work, the method presented in [10] is hybridized with a local ABC for enhanced accuracy and improved convergence.

2. HYBRID GLOBAL BOUNDARY CONDITION

Consider a two-dimensional (2D) problem of scattering by an arbitrarily shaped inhomogeneous scatterer illuminated by a transverse magnetic (TM) field. In other words, the geometry and the incident field are uniform along the z-axis and the field is z-polarized. Also, a harmonic time dependence is assumed. In the x–y plane, the scatterer can be circumscribed by a circle of radius R. A perfectly electrically conducting (PEC) surface S forming an open-ended cavity type scatterer (see Fig. 1) will serve as an example of an essentially concave scatterer.

Towards solving the problem, we consider a scattered field discretized via an FDFD or FE formulation. Enclosing the scatterer within a closed convex boundary ΓWnovel [see Fig. 1(a)], one may apply a number of discretized local ABCs on this boundary, all of which take the following general form

\[ \sum_{i=1}^{M} c_i E(r_i) = 0. \]  

For the sake of simplicity, we consider a local point numbering, where M is the number of points within the ABC stencil. As an example, for the Mur ABC on a Cartesian finite-difference mesh, we have M = 6 where the sampling points reside on a double-layered boundary Γo as shown in Figure 2.

The electrical size of the problem is characterized by N = kR, which serves as a large parameter in evaluating the asymptotic...
computational complexity. For the discretized domain within the convex boundary, the number of unknowns is of $O(N^2)$. Note that majority of the unknowns are needed to mesh the "white space" formed inside the cavity as a result of the convexity of the boundary, dictated by the application of a local ABC. When solving the problem iteratively, such formulation would require $O(N^2)$ operations per iteration. In contrast, we note that for the thin shell-type of scatterer depicted in Figure 1, the number of unknowns on its surface is of $O(N)$. Thus, it appears tempting to deform the convex boundary $\Gamma_{\text{convex}}$ to be conformal with the scatterer. This new boundary, designated $\Gamma_o$, [see Fig. 1(b)], is assumed to follow the scatterer surface at a fixed distance (independent of $N$). The total number of unknowns is then reduced from $O(N^2)$ to $O(N)$ at the expense of making the outer boundary nonconvex, thus prohibiting the usage of local ABCs. Extending Eq. (1) to concave boundaries requires the introduction of a boundary-integral-type correction term. This leads to a hybrid boundary-integral formulation, as detailed below.

Let $E(r_i)$ denote the scattered field at point $r_i$ on $\Gamma_o$, that can be calculated using a Kirchoff-type integral as follows:

$$E(r_i) = \int_{C} \left[ \frac{\partial G(r, r')}{\partial n'} - G(r, r') \frac{\partial E(r')}{\partial n'} \right] ds', \quad (2)$$

where $C$ is an arbitrarily shaped closed contour between the scatterer and $\Gamma_o$, and $G(r, r') = 1/4\pi H_0^{(2)}(kr - kr')$ is the 2D Green's function, where $H_0^{(2)}(\cdot)$ is the Hankel function of the second kind and order 0. By analogy with [9], we propose a general hybrid boundary condition, given by

$$\sum_{i=1}^{M} c_i [E(r_i) - \tilde{E}(r_i)] = 0. \quad (3)$$

Eq. (3) is readily satisfied for convex $\Gamma_o$, since both $E(r_i)$ and $\tilde{E}(r_i)$ should individually satisfy the local ABC (1), that is, $\sum_{i=1}^{M} c_i E(r_i) = 0$ and $\sum_{i=1}^{M} c_i \tilde{E}(r_i) = 0$. Condition (3) is also satisfied over concave (as well as convex) portions of $\Gamma_o$ because the application of Eq. (2) leads to $E(r_i) = \tilde{E}(r_i)$, thus annulling each term in the sum individually. Alternatively, this generalized hybrid boundary condition may be viewed as a preconditioning operation performed on the formulation in [10] by means of combining the generalized local ABC in Eq. (1) and the radiation integral formulation $E(r_i) = \tilde{E}(r_i)$. As previously stated, the formulation employing the conformal concave boundary reduces the number of unknowns to $O(N)$, however, the global ABC (3), with integrals of Eq. (2) evaluated directly, still requires $O(N^2)$ operations per iteration. Thus, in order to produce computational savings as compared to purely local ABCs, we need to reduce the computational cost of evaluating the integrals of Eq. (2) for $O(N)$ observation points on $\Gamma_o$ using $O(N)$ quadrature points on $C$ below $O(N^2)$.

Figure 1 Scattering from a rectangular open-ended cavity with (a) convex and (b) concave ABCs

Figure 2 Example of discretized 6-point Mur ABC
3. THE NONUNIFORM GRID ACCELERATION SCHEME

Reduction of the computational complexity of Eq. (2) from \( \mathcal{O}(N^3) \) to \( \mathcal{O}(N^{1.5}) \) is achieved by using the two-level NG domain-decomposition algorithm [11], reviewed below. The integral in (2) is decomposed into separate near-field and far-field contributions. This is done by subdividing \( \Gamma_p \) into \( P \) disjoint subdomains \( \Gamma_p \) of roughly equal size, such that \( \Gamma_p = \bigcup_{p=1}^{P} \Gamma_p \). The field \( \tilde{E}_p(\mathbf{r}) \) is then computed as a superposition of partial fields \( E_p(\mathbf{r}) \). Each partial field, being the outcome of the sources on \( \Gamma_p \), is evaluated either as a far-field or near-field contribution, depending upon the location of the observation point \( \mathbf{r} \) relative to the center of \( \Gamma_p \), denoted as \( \tilde{\mathbf{r}}_p \). In the near field of \( \Gamma_p \), \( E_n(\mathbf{r}) \) is obtained by direct integration of (2). If, however, \( \mathbf{r} \) is located in the far-field region of \( \Gamma_p \) (defined by \( \rho_p = |\mathbf{r} - \tilde{\mathbf{r}}_p| > \Omega_p R_p \), with \( R_p \) the radius of the smallest circle circumscribing \( \Gamma_p \) and \( \Omega_p > 1 \)), we first compute the phase and amplitude-compensated partial electric field, defined as

\[
E_p(\mathbf{r}) = \tilde{\rho}_p^{1/2} e^{-jk\phi} \int_{\Gamma_p} \left[ E(\mathbf{r}, \mathbf{r}') \frac{\partial G(\mathbf{r}, \mathbf{r}')}{\partial n'} - G(\mathbf{r}, \mathbf{r}') \frac{\partial E(\mathbf{r}')}{\partial n} \right] ds',
\]

(4)

where \( \tilde{\rho}_p(\mathbf{r}) = (\rho_p^2(\mathbf{r}) + \tilde{R}_p^2/2)^{1/2} \). The factor \( \tilde{\rho}_p^{1/2} e^{-jk\phi} \) is designed to cancel out the rapid oscillations and amplitude drop-off typical of the asymptotic behavior of the Hankel functions (in the Green’s function and its derivative) for large arguments. The smoothly varying compensated partial field (4) can now be sampled at a coarse rate that is compatible with the size of the selected subdomain. The design of the optimal sampling grid for \( E_p(\mathbf{r}) \) was derived in [11], where it was shown that this compensated field is essentially bandlimited as a function of the polar angle \( \phi_p \) and reciprocal radius \( \alpha_p = 1/\rho_p \) in the local polar-coordinate system centered at \( \tilde{\mathbf{r}}_p \). The required sampling rates in \( \alpha_p \) and \( \phi_p \) were found to be \( \alpha_p = \Omega_p k R_p^2/4 \pi \) and \( \phi_p = \Omega_p k R_p^2/2 \pi \), respectively. The optimal sampling grid is thus uniform in \( \phi_p \) and \( \alpha_p \), as shown in Figure 3, hence constituting an NG with exceedingly sparse sampling as \( \rho_p \) increases. The usage of \( \tilde{\rho}_p \) rather than \( \rho_p \) in the phase-correction factor in Eq. (4) was shown to provide an additional reduction in the radial-sampling rate by a factor of two.

The aforementioned removal of the fast variations allows for the evaluation of the compensated field (4) on the NG. This computation benefits from the coarseness of the NG, that is, the field is evaluated at a substantially smaller number of points. Subsequently, the compensated field is interpolated from the NG to the relevant set of points \( \{ \mathbf{r}_i \} \subset \Gamma_p \). Following the interpolation, the actual far-field contribution of the partial electric field is restored by offsetting the compensation factor of (4):

\[
\tilde{E}_p(\mathbf{r}) = \tilde{\rho}_p^{1/2} e^{-jk\phi} \tilde{E}_p(\mathbf{r}); \quad \rho_p = \Omega_p R_p.
\]

(5)

Finally, the total field is attained by aggregating all the partial ones as follows:

\[
E(\mathbf{r}) = \sum_{p=1}^{P} E_p(\mathbf{r}).
\]

(6)

Figure 3

Schematic nonuniform polar grid for a selected subdomain

Figure 4

Electric field along the x-axis for \( \phi = 0 \): (a) amplitude; (b) phase
The computational burden of computing and aggregating $P$ partial fields is given by [11]:

$$C \approx \frac{N}{N} (t_1 \tilde{N}^2 + t_2 N),$$

(7)

where $\tilde{N}$ is the number of samples in a typical subdomain of $\Gamma_s$, and $t_1$ and $t_2$ are implementation-specific constants. It is readily verified that the lowest asymptotic complexity of $\mathcal{O}(N^{1.5})$ is obtained for $\tilde{N} = O(N^{1/2})$. This result provides a distinct advantage over the cost of the straightforward evaluation of (2). Note also that this complexity estimate is similar to the two-level FMM algorithm.

4. NUMERICAL RESULTS

By applying the NG algorithm given in sections 3 to the example below, we choose linear interpolation for both $\phi_p$ and $\alpha_p$ and set the default oversampling factors at $\Omega_p = \Omega_\phi = 8$. The near-field region is defined by $\Omega_R = 3$.

We study the rectangular open-ended cavity-type scatterer shown in Figure 1 with dimensions $L = W = 10\lambda$, with $\Gamma_s$ positioned at a distance of $\lambda/2$ from the scatterer and $\lambda$ from $\Gamma_n$. $\Gamma_n$ is in fact composed of two layers, as shown in Figure 2, spaced one mesh size $h$ apart. Finite-difference discretization with $h = \lambda/20$ results in $N_o = 1440$ points along the external boundary of the computational domain. Plots of the amplitude and phase of the normalized electric field computed using the NG fast GBC algorithm and via a straightforward implementation of (2) are shown in Figures 4 and 5 along the $x$- and $y$-axes, respectively. Clearly, the results of the direct computation and of the proposed algorithm visibly coincide for both amplitude and phase. Figure 6 presents the normalized RMS and maximum errors on $\Gamma_n$ versus the oversampling parameter $\Omega = \Omega_p = \Omega_\phi = \Omega_R = 3$. The results demonstrate the quadratic convergence of the method (as expected) for linear interpolation. Figure 7 presents the number of floating point operations (FLOPS) spent on the computation of a single iteration versus the number of subdomains $P$. This result demonstrates the existence of an optimal subdomain size (that is, $P = 36$ subdomains).

Computational complexity is assessed by considering scatterers of various sizes, obtained by scaling the dimensions of the cavity of Figure 1. The computational cost of a single iteration (in FLOPS) versus the number of samples on $\Gamma_n$ for $h = \lambda/20$, is
depicted in Figure 8 for three implementations as follows: (i) the Mur ABC with \( L^{\text{cconv}} \), (ii) a straightforward implementation of Eq. (2), and (iii) the NG Fast GBC algorithm. The number of subdomains is duly scaled with the scatterer size, that is, \( P \propto N^{1/2} \). The slopes of both the direct computation and the six-point Mur boundary condition are in line with the expected \( O(N^2) \) behavior. The trend of the NG Fast GBC algorithm, as observed in this curve, validates the estimated complexity of \( O(N^{1.5}) \) for the NG algorithm. Extrapolation of these three curves leads to the conclusion that the NG Fast GBC algorithm with the two-level domain decomposition scheme becomes more attractive for scatterer dimensions of \( L, W > 50a \).

5. CONCLUSION

A novel fast iterative algorithm based on a hybrid boundary integral formulation, coupled with a nonuniform polar-grid (NG) scheme has the potential of overcoming some of the limitations of local ABC schemes. The algorithm applies to scattering problems with arbitrarily shaped exterior boundaries and allows for a significant reduction in the size of the “white space” in cases characterized by essentially concave geometries. In our numerical example, we considered a PEC scatterer, although arbitrary inhomogeneousscatterer can be treated the same way. An efficient and accurate multidimensional interpolation scheme is key to the implementation of the method, especially if the technique is employed in a multilevel computational scheme.

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REFERENCES


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FABRY–PEROT-BASED APPROACH FOR THE MEASUREMENT OF COMPLEX PERMITTIVITY OF SAMPLES INSERTED IN RESONANT CAVITIES

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ABSTRACT: A new approach to measure the complex dielectric permittivity of materials placed inside a microwave resonant cavity is presented. We exploit the similarity between the microwave cavity and an optical cavity to develop a new measurement technique, which is used to obtain the complex permittivity of a polytetrafluoroethylene (PTFE).


Key words: resonant microwave cavity; material characterization; complex permittivity; optical cavity

1. INTRODUCTION

The increasing demand for materials in microwave applications has triggered the development of techniques for the measurement of complex dielectric permittivity. Several techniques have been proposed for the measurement of permittivity, such as the cavity-resonant method [1, 2].

In this method, the resonance peak frequency and the quality factor of the cavity, with and without sample, can be used to obtain the complex dielectric permittivity of the material. We measure the shift in the resonant frequency of the cavity, \( \Delta f \), caused by the insertion of the sample, which can be related to the real part of the complex permittivity, \( \varepsilon' \). The change in the inverse of the quality factor of the cavity, \( \Delta(1/Q) \), gives the imaginary part, \( \varepsilon'' \). The relations are simple when we consider only the 1st-order perturbation in the electric field caused by the sample [3], using the small