Formation of spatial patterns with increasing complexity within a complex nonlinear medium was modeled. The predictions were verified experimentally with patterns emitted from ring shaped VCSELs.
The formation of a variety of stable complex transverse light patterns in nonlinear optical cavities was studied both in one and two dimensions [1-3]. A laser, which is based on a nonlinear resonator, can emit spontaneously some of these patterns [3,4]. A ring shaped laser is a special case of a two dimensional structure that imposes periodic boundary conditions in the azimuthal coordinate and therefore may emit interesting and complex patterns. Vertical cavity surface emitting lasers (VCSELs) are unique within the family of semiconductor lasers by having a 2-dimensional resonator and may serve as an ideal tool for the exploration and exploitation of complex electrical fields. Previous research on pattern formation in nonlinear active cavities [5-7] explained the on-switching of higher order lasing modes and of complex pattern by the gain-loss balance modified by the gain saturation (spatial hole burning). Here we show that the switching of patterns is caused mainly by a dynamic effect - the Modulation Instability (MI) of the self consistent field patterns and present the threshold conditions for the formation of the patterns.

We modeled the evolution of the electrical field in the semiconductor laser medium using the 1D wave equation which is applicable for large enough ring shaped lasers. The paraxial wave equation, which is valid for the VCSEL geometry, is written for the electrical field:

\[
\frac{\partial E}{\partial z} + i \frac{\partial^2 E}{\partial x^2} - \frac{1}{2} [g_0 - \alpha_{\text{tot}}] \cdot E + \frac{k_0 n_2}{R} |E|^2 E = 0
\]  

(1)

where \( k_0 \) is the wave number, \( n_0 \) – the linear index of refraction, \( R \) – the anti-guiding factor and \( \alpha_{\text{tot}} \) – the loses. \( g_0 \) is the unsaturated gain defined as \( g_0(N_p) = \Gamma \alpha \cdot (N_p - N_0) \) where \( \Gamma \) is the confinement factor, \( \alpha \) - the differential gain, \( N_p \) – the carrier density due to the pump and \( N_0 \) – the carrier density for transparency. \( n_2 \) the effective nonlinear refraction index is given by \( n_2 = \frac{R \cdot g_0}{2k_0 I_{\text{sat}}} \), where \( I_{\text{sat}} \) is the saturation energy - \( I_{\text{sat}} = \hbar v/(\Gamma \alpha_{\text{sp}}) \).

For simplicity we introduce the following variables - \( x' = k_0 x, \ z' = k_0 z / 2n_0 \) and \( E(x,z) = \sqrt{1/n_0 n_2} \cdot A(x',z') \). Substituting them into (1) and dropping the primes, yields the normalized field equation:

\[
\frac{\partial A}{\partial z} - i \frac{\partial^2 A}{\partial x^2} - 2i |A|^2 A = \gamma \cdot A - \frac{2}{R} |A|^2 A \quad \text{where} \quad \gamma = n_0 \cdot (g_0 - \alpha_{\text{sat}}) / k_0
\]  

(2)

Equation (2) has soliton like solutions [9] but also uniform Tilted Wave (TW) solutions, \( A = A_0 e^{ikx - \gamma z} \):

\[
|A_0| = \frac{\sqrt{R}}{2}; \quad k = \frac{2\pi}{L} \cdot \eta; \quad \eta = k^2 - R; \quad \eta = 0,1,2...
\]
L is the ring’s perimeter. The quantization of k is caused by the cyclic boundary conditions imposed by the ring geometry. In order to explore the existence range of each solution we analyzed its stability to small spatial harmonic perturbation. When the examining the nth order solution is being, a field component (noise) with a spatial frequency \( k + \Omega \) will experience exponential growth if it is larger than k and lower than a certain frequency \( k_{\text{max}} \):

\[
k < k + \Omega < \sqrt{k^2 + 2\gamma R}
\]  

(3)

The MI range depends on the pumping, the laser parameters and the specific steady state solution. For a given net gain (\( \gamma \)), higher order solutions have smaller MI range and are therefore more stable. Pattern switching will occur when the positive MI gain curve of the actually lasing pattern could excite the next order solution, which is (for the same pump rate) stable. From the perturbation analysis we can derive a threshold gain for the lasing of the (n+1)th solution:

\[
\gamma = \frac{2\pi^2}{R \cdot L^2} (2n + 1)
\]  

(4)

Thus, a necessary condition for achieving thresholds for the higher multiple soliton solutions is R>0, satisfied by all semiconductor lasers, however may be not satisfied for other types of lasers. Further more, the threshold gains are smaller for larger rings and thus the appearance of complex pattern will require lower injection currents in larger lasers. The ring shaped VCSEL has, therefore, many threshold currents. The first one is the conventional uniform field lasing threshold (corresponds to \( \gamma=0 \)). The other threshold levels, derived from the MI consideration and not from gain-loss, correspond to the thresholds of the emergence of multiple solitons arrays.

Another two important conclusions that rise from the perturbation analysis are: 1) The MI mechanism is unidirectional in the sense that it enables energy transfer only from lower solutions to higher solutions. 2) A TW solution is unstable to perturbation of its complex conjugate TW (solution n is unstable to solution –n). The lasing of the nth TW solution would excite its complex conjugate and a standing wave azimuthal pattern would emerge.

Figure 1 shows BPM simulation of Eq. (1) when the current is below the second \( (n=1) \) threshold. The perturbed electric field reached a uniform steady state distribution and in Fig. 2 - the BPM simulation above the third \( (n=2) \) threshold evolved into four-lobed solution.

All the above predictions were tested and verified experimentally. The near-field intensity pattern emerging from a proton implanted ring shaped VCSELs (20-40\( \mu \)m in diameter) with three 8nm In\(_{0.2}\)Ga\(_{0.8}\)As QW, emitting at \(~0.95 \mu\)m [8] were monitored. The near-field patterns were examined at room temperature under pulsed operation. A uniform near-field was registered until the injected current was increased to \(~1.5 \) times the threshold current. Then the pattern switched to an azimuthal standing pattern. As the current was increased, the number of light lobes increased, but not monotonically – probably due to defects and the inhomogeneous nature of the laser. Fig. 3 shows the near-field intensity pattern of 40\( \mu \)m and 28\( \mu \)m diameter lasers at various currents.

The semi-analytical solutions of eq. (1) based on the perturbation analysis will be presented and compared with the experimentally measured near-fields and the BPM simulation results. It should be emphasized that the theory presented here is in principle valid for every semiconductor laser structure with cylindrical symmetry (disks, coupled rings etc.) [3]. The pattern selection mechanism shown here is, therefore, a general rule that applies for every cylindrical symmetric problem.
REFERENCES


Figure 1 – Stability for current below the second threshold, $I_{sat}=20$, $I=185$, $g_0=0.01004$, $\alpha_{sat}=0.01$

Figure 2 – Stability at current above second threshold

Figure 3 – Ring VCSEL near-field patterns - experimental results