# Static Broadcasting

Nadav Shulman and Meir Feder
Department of Electrical Engineering - Systems,
Tel-Aviv University, Tel-Aviv, 69978, ISRAEL
{shulman,meir}@eng.tau.ac.il

*Abstract* — **In this paper we present a different view on the broadcast channel that fits better an asynchronous setting, where each receiver can "listen" to the broadcasted data at different time intervals. In this scenario, there is a "static" fixed amount of data, that needs to be transmitted to all receivers. Each receiver wants to minimize the receiving time, and this way get the static data at the highest possible rate.**

## I. Concept Definition

In this work we define and analyze *static broadcasting.* In this broadcasting scenario, the sender has only a fixed common information to transmit to all receivers. We suggest the following definition of the rate - the number of reliably received bits divided by the number of symbols the receiver has *used* to retrieve these bits (or, divided by the information gathering time). Under this definition, in principle, a receiver that listen through a better channel, may gather less channel symbols in order to estimate the transmitted message, and by this to increase its rate. In the saved time it can fetch more information from other transmitters. The term *static broadcasting* comes from the notion that the information the transmitter sends is fixed, static, and the same for all receivers.

In this work, a broadcast channel is composed of single transmitter and $d$ memoryless channels $W_i$, $1 \leq i \leq d$, with common input alphabet through which the transmitter broadcasts to $d$ receivers. The capacity region is defined as the closure of the set of all possible achievable rates. A rate $(R_1, R_2, \ldots, R_d)$ is said to be achievable if for any $\epsilon > 0$ there exists a code with $M$ words such that for all $i$, the $i$th receiver can decode, with error probability smaller than $\epsilon$, the codeword using the first $\lfloor \log M / R_i \rfloor$ channel symbols. The achievable rate region is given by the following theorem.

**Theorem 1** *$(R_1, R_2, \ldots, R_d)$ is in the capacity region iff, for any $\delta \geq 0$ there exist input priors $P_1, P_2, \ldots$ and a number $K$ such that $\frac{1}{n_i}\sum_{t=1}^{n_i} I(P_t; W_i) \geq R_i - \delta$ for all $1 \leq i \leq d$, where $n_i = \lfloor \frac{K}{R_i} \rfloor$.*

In defining the capacity region for static broadcasting we utilized the possibility of transmitting the information at a higher rate if the receivers are not forced to be synchronously and simultaneously connected to the transmitter. The fact that there are various possible definitions of the capacity for the broadcast channel, depending on the subset of time the data is received, has been pointed out in, e.g., [1]. However, the setting we propose is novel.

The proposed setting was further extended in [2]. For example, in [2] there is a setting where the receivers start receiving at different arbitrary times, which may fit an IP Multicast scenario. Another extension corresponds to data transmission over an unknown channel, using infinitely long codes (to allow a channel with unbounded small capacity). Finally, universal and sequential decoding schemes were investigated.

## II. Examples of the Capacity Region

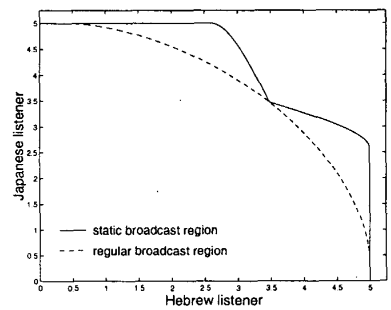
A general method to find the capacity region for static-broadcasting to 2 channels, is as follows. Assume the channels conditional probabilities are $W_1(y|x), W_2(y|x)$ and the corresponding capacities are $C_1$, $C_2$. Using the convexity of the mutual information, we may assume that the input prior to the channel is changed only at time points of the form $t = n_i + 1$. Hence, in the case of 2 receivers, we start with prior $P$ and after one of the receivers got all the information, it will quit, and in order to maximize the rate to the second receiver, we shall change the input prior to the one that achieves its capacity. Assuming $I(P; W_1) \geq I(P; W_2)$. Then,

$$R_1 = I(P; W_1), \qquad R_2 = \frac{I(P; W_1)C_2}{C_2 + I(P; W_1) - I(P; W_2)}.$$

Of course, any point $r_1 \leq R_1, r_2 \leq R_2$ is in the capacity region. To get the complete capacity region we should take the union of the region above over all possible values of the initial input prior, $P$.

In case where the capacity achieving prior is the same for all channels we can achieve simultaneously their point-to-point capacity. For example, two binary symmetric channels noisy and noiseless. In that case a simple code can be shown. Take any good systematic code for the noisy channel. The systematic part (the information bits are the prefix of each codeword) is sufficient, of course, for the noiseless channel and impose an effective rate of 1 for that channel. The noisy channel receives the information at a rate determined by the code.

In static broadcasting, unlike regular broadcasting, time sharing between two strategies is not a valid strategy. Hence, the capacity region is not necessarily convex. For example, suppose one communicates using 31 Japanese words and 31 Hebrew words. A Japanese listener can differentiate 32 different symbols (since all Hebrew words sound the same to him) and the same goes to a Hebrew listener. This broadcast channel leads to the capacity region in the figure below.




## References

[1] Thomas M. Cover. Comment on broadcast channels. *IEEE Trans. on Inform. Theory*, IT-44:pp.2524–2530, October 1998.

[2] Nadav Shulman. *Communication in Unknown Channels.* PhD thesis, Tel-Aviv University, in preparation.