BIKE-SHARING SYSTEMS:
PARKING RESERVATION POLICIES
AND MAINTENANCE OPERATIONS

By

Mor Kaspi

THESIS SUBMITTED TO THE SENATE OF TEL-AVIV UNIVERSITY
in partial fulfillment of the requirements for the degree of
"DOCTOR OF PHILOSOPHY"

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Under The Supervision of Prof. Michal Tzur and Dr. Tal Raviv
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Abstract

Vehicle sharing systems and in particular bike-sharing systems have become a common sight in many major cities around the world in the past decade. Currently there are more than 900 operating bike-sharing systems and at least another 200 are in planning. Such systems allow users to rent a bicycle in any of its stations scattered around the city, use it for a short period of time and return it back to any station. This novel type of transport has given rise to new planning challenges, regarding both efficient design and efficient operation of these systems.

In this dissertation we examine two aspects that can enhance the operation of bike-sharing systems: (1) implementation of parking reservation policies, and (2) planning of maintenance operations.

Under parking reservation policies, users may be required, upon renting a bicycle, to reserve a parking space (locker) at their destination and these spaces are kept for them. We propose several parking reservation policies, including a complete parking reservation (CPR) policy, under which all users are required to reserve parking spaces and the reserved places are kept for them until the bicycle is returned. Other policies require the users to make reservations in certain cases only, or not at all. Using a Markovian model of a simplified system and simulation of real world systems, we compare the proposed policies. In addition, we devise mathematical programming based bounds on the total excess travel time under any parking space reservation policy. Our results demonstrate the effectiveness of the simple CPR policy and suggest that parking space reservations should be used in practice.

Maintenance operations are crucial for the smooth operation of bike sharing systems. In these systems a small percentage of the bicycles become unusable every day. Currently, there is no reliable on-line information that indicates the usability of bicycles. We present a model that estimates the probability that a specific bicycle is unusable, based on available trip transactions data. Further on, we present some information based enhancements of the model and discuss an equivalent model for detecting locker failures.

Next, given reliable information regarding unusable bicycles, we study their impact on the quality of service. We model user dissatisfaction by a weighted sum of the expected shortages of bicycles and lockers as a function of the initial inventory of usable and unusable bicycles. We analyze and prove the convexity of the resulting bivariate function and accurately approximate it by a convex polyhedral function. The fitted polyhedral function can later be used in linear optimization models for operational and strategic decision making in bike sharing systems. Our numerical results demonstrate the high effect of the presence of unusable bicycles on user dissatisfaction. This in return emphasizes the need for having accurate real-time information regarding bicycle usability.
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1. Introduction

The research described in this dissertation deals with enhancing operations in bike sharing systems in order to improve the quality of service provided to the users of these systems. The dissertation is divided into two main parts: Part I - implementation of parking reservation policies and Part II – planning maintenance operations.

In this chapter, we present a background on bike-sharing systems and summarize the contributions of this study. In Section 1.1, we shortly describe what Vehicle Sharing Systems (VSS) and particularly bike-sharing systems are. We discuss the motivation for implementation of such systems and outline the development of vehicle-sharing systems in recent years. In Section 1.2, we introduce the concept of parking reservation policies, outline the methods we used in this research to examine the proposed parking reservation policies and discuss the main results and implications of implementing such policies. In Section 1.3, we discuss how maintenance operations in bike-sharing systems can be improved using available existing information. We present a framework for optimization of such operations that include detection of unusable bicycles, evaluation of the effect of unusable bicycles on the quality of service and planning of their collection.

1.1. Background

Public transportation in modern cities is typically composed of bus systems, trams, subway and taxi services. Such a mix can satisfy most of the citizens’ travel demands, but nevertheless many citizens still prefer to use private vehicles. This can be attributed to the fact that public transportation is usually limited as far as service areas (depending on the planned lines), operating hours, and service frequency. A private vehicle is available at any time and allows greater flexibility such as: out of city trips, traveling with cargo (groceries, baby carriages etc.). In addition, a private vehicle may be the choice of people who do not live within walking distance of public transportation stations (the ‘last mile’ problem) or who merely prefer not to travel with strangers. However, the cost of owning a private vehicle is typically higher compared to using public transportation. Moreover, the average car is used for less than an hour daily. The rest of the time it sits idle and takes up a parking space. This represents inefficient utilization of scarce resources (both the vehicle and the parking space), especially in city centers.

To fill the gap between the advantages of using private vehicles and the services offered by public transportation, in recent years, many cities around the world have developed station-based one-way vehicle sharing systems such as car-sharing and bike-sharing systems. Such systems consist of a fleet of vehicles spread across the city, which users can rent for a short period of time. This type of service may be considered as an extension of the traditional public transportation, which offers more flexibility and enables more multi-modal journeys.

For example, a commuter who traveled in the morning using the subway and wishes to do some shopping at the end of the work day, can rent a car near her work and return it to a station located near her home. A user, who does not live within walking
distance from a public transportation station, may be able to rent a bicycle at a nearby station and return it near the public transportation station.

With this added flexibility, more citizens are able to shift from private vehicles to public transportation. In turn, this may result with a decrease in traffic congestion, a better utilization of land resources (since fewer parking spaces are needed) and a reduction in air pollution and greenhouse gasses emissions.

In this study, we focus on one-way, station-based, vehicle sharing systems such as most bike-sharing and some car-sharing systems. Such systems allow users to rent a vehicle in any of its stations scattered around the city (given that there is an available vehicle in that station), use it for a short period of time and return it back to any station in which a parking space is available. In the case of bike-sharing systems, the "parking space" is in fact a locker. However, here, we use the term parking space to refer to one unit of storage of a vehicle of any kind.

In their current form, vehicle-sharing systems rely on information and communication technology, such as 3G communication and GPS, to allow the operators and the users to check the availability, location and status of each vehicle in the system online. The widespread use of smartphones increased the usability of vehicle sharing systems by making this information accessible to the system’s users at any time and at any place.

In recent years, there has been a rapid increase in the deployment of car sharing and bike sharing systems around the world. As of October 2012, car sharing systems were operating in 27 countries, accounting for an estimated 1,788,000 shares over 43,500 cars (Shaheen and Cohen, 2012). While in June 2014, the estimated number of shared cars has increased to over 90,000 and the estimated total number of car sharing members worldwide was 4.94 million (Frost and Sullivan, 2014). An even steeper incline was observed in bike-sharing systems. In May 2011 there were an estimated 136 bike sharing programs, with 237,000 bicycles on the streets (Shaheen and Guzman, 2011). As of June 2015, these numbers were significantly higher, with about 910 active bike-sharing systems worldwide and more than 250 in planning, with over 1,062,000 bicycles in use in the active systems (DeMaio and Meddin, 2015).

A distinction should be made between round-trip vehicle-sharing systems and one-way vehicle-sharing systems. While in the former a user must return the vehicle to the same station in which it was rented, in the latter, a user may return the vehicle to any of the system’s stations. Clearly, the flexibility of one-way systems makes them more useful for their users. However, this flexibility creates an intricate challenge to the operators due to the need to redistribute vehicles in the system in order to meet the demand. For this reason, until recent years, most of the car sharing systems provided only round-trip service. The current trend is however towards one-way systems, see for example, studies by Jorge et al. (2015), Nourinejad and Roorda (2015) and Shaheen et al. (2015). Metropolitan bike sharing systems, however, are typically deployed as one-way systems since physical redistribution of bicycles can be carried out more easily.

Though the focus of this study is on bike-sharing systems, there are many similarities between bike sharing systems and one-way car sharing systems. In particular, the parking reservation policies that we propose in Part I of this dissertation
can be implemented in both types of systems in a similar manner. For additional information regarding vehicle sharing systems, see surveys by: Shaheen and Cohen (2007, 2010), Demaio (2009), Demaio and Meddin (2015), Shaheen et al. (2010) and Shaheen and Guzman (2011).

The operators of vehicle sharing systems face a difficult task of meeting the demand for vehicles and for available parking spaces. This difficulty arises mainly from the characteristics of the demand for journeys during the day. These demand processes are typically stochastic, a-symmetric and heterogeneous in time. The system is unable to satisfy the demand either when a user who wishes to rent a vehicle arrives at an empty station or when a user who wishes to return a vehicle arrives at a full station, i.e., a station with no vacant parking spaces. The latter scenario is typically perceived as more severe since a user who is unable to return a vehicle is “trapped” in the system because she cannot complete the renting transaction until she finds an available parking space. Contrary to that situation, a user who cannot rent a vehicle may decide to use an alternative mode of transportation. Typically, the system performance is measured as a function of these two types of undesired situations. We note that although users can check online the availability of vehicles and parking spaces, this does not guarantee that their requests would be satisfied, because by the time the user arrives at her destination, the space might be already occupied.

To reduce the occurrence of shortages in vehicles or parking spaces, the system operators may take either strategic or operational actions. Strategic actions include deploying more stations or enlarging existing stations. Operational actions may include actively or passively regulating the system.

By active regulation we refer to redistributing vehicles between the system’s stations, and possibly a central depot, using trucks (in the case of bikes-haring systems) or by designated drivers (in the case of car-sharing). This mode of operation is referred to in the literature as repositioning (or rebalancing), and is further classified into static repositioning and dynamic repositioning. Static repositioning operations are carried out during off-peak periods when demands are negligible (at night) in order to prepare for the demands of the following day. In most systems, repositioning operations are carried out also during peak periods, while vehicles are on the move. Such operations are referred to as dynamic repositioning.

By passive regulation, we refer to mechanisms used for redirecting demand in order to improve the performance of the vehicle sharing system. Such mechanisms do not affect the true demand for journeys but may instead shift users to rent (return) vehicles at stations different from their true origin (destination) station or to use alternative modes of transportation. Parking reservation policies, which are the focus Part I of this dissertation, are one example of a passive regulation. Additional examples are discussed in the literature review in Chapter 2.

1.2. Parking reservation policies
Reservations are often used to coordinate supply-and-demand mismatches in various systems. They provide a control over the flow of demands and, in addition, allow a better forecast of the system’s future state. Due to their ability to smooth demand peaks
and reduce uncertainty, this mechanism is widely used by many service providers such as hotels, restaurants, airline companies, healthcare facilities, etc.

In Part I, we focus on the reservations of parking spaces at the destinations. We note that since parking reservation policies can be implemented both in bike-sharing systems and in car-sharing systems, we use the terms vehicles and bicycles interchangeably throughout this part of the dissertation. In Chapter 3, we consider the following policy: when a user rents a vehicle, she declares her destination station and a vacant parking space in that station is reserved for her, if one is available. That is, the reserved vacant parking space at the destination becomes unavailable for other users from the moment the reservation is made, until the user reaches her destination. This assures that when she reaches her destination, she will be able to return her vehicle. If there is no available parking space at the destination, the transaction is denied, so that the user is unable to rent the vehicle. We refer to this policy as the Complete-Parking-Reservation (CPR) policy. It is complete in the sense that all users are obliged to reserve their parking space. By implementing this parking reservation policy, the system can secure an ideal ride for some users. On the other hand, it holds parking spaces without utilizing them for a certain amount of time, and by doing so, possibly rejecting other potential users. Due to this tradeoff, the effectiveness of using reservations is not obvious.

Technologically, the implementation of parking reservations requires mainly software updates, which need to be adjusted to include the above-mentioned renting process, and then present only empty and non-reserved parking spaces as available. As for the hardware, specific vehicle identification is already in place in the existing systems, therefore returning a vehicle to a reserved parking space may be allowed by the system only if the vehicle is identified as the one for which the reservation was made. In some systems (mainly bike-sharing), different light colors are used to signal whether a parking space (locker) is operative or out of order. An additional color can be used to signal that a certain parking space is reserved.

Managers of public transport system should aim to improve the quality of service provided to its users, subject to the available resources. In this part of the dissertation, we measure the quality of service by the total excess time users spend in the system as a result of shortages in vehicles or parking spaces. The excess time of a user is the difference between the actual time she spends in the system (her exiting minus entering time) and her ideal time, i.e., the riding time between her origin and destination stations. Indeed, we believe that time is a major consideration of commuters in an urban public transit system. This is especially true in cities where regular commuters can buy a monthly or annual subscription for the transit system, which excludes the cost consideration of each particular journey.

The contribution of Chapter 3 is as follows: for the first time, we study a parking reservation policy in one-way vehicle sharing systems and compare its performance to a base policy, entitled No-Reservation (NR) policy. We propose measuring the performance of the system by the total excess time users spend in the system due to unfulfilled renting requests or delays in returning the vehicles. We show that implementing parking reservations, as suggested, will improve the performance of one-
way vehicle sharing systems in most realistic settings. In particular, using Markovian models, we prove our main theoretic result, stating that if the demand faced by the system is not extremely high, the CPR policy outperforms the NR policy. In other words, under the CPR policy users will spend less excess time in the system. We further demonstrate through a small example that in some cases the CPR is superior for any demand rate. Finally, the main theoretic result is reinforced by a numerical study based on a discrete event simulation of a real-world vehicle sharing system with a realistic user behavior model. The bike sharing system in Tel-Aviv, Tel-O-Fun, was used as our case study. The behavior model assumes users are strategic, i.e., in case of an unfulfilled renting request or unavailability of a parking space, the user determines her alternative route so as to minimize the expected time she spends in the system. Through this numerical study, we also demonstrate that the CPR policy outperforms the NR policy under several additional performance measures.

In Chapter 4 we extend our analysis of parking reservation policies beyond the complete policies (CPR and NR). Under the CPR policy, a reserved parking space remains empty until the user returns her vehicle. In the meantime, other users cannot use this resource, i.e., it is blocked. The tradeoff in implementing such a policy is that while some users are guaranteed an ideal service (since they will certainly be able to return their vehicle at their desired destination) other users may receive poorer service due to the blocking of parking spaces. That is, it might be beneficial not to require that all users make parking reservations. In Chapter 4 we examine whether and to what extent a further reduction in the total excess time may be achieved by any other passive regulation, and especially by any other parking reservation policy. Towards that, we use mathematical programming models to devise lower bounds on the total excess time that users spend in the system under any passive regulation and under any parking reservation policy. We consider the benefit of limiting the requirement to make a reservation to only some of the journeys. We refer to these policies as partial reservation policies, which combine the two extreme (complete) policies in different ways. We evaluate the performance of all policies and compare them to the lower bounds.

The contribution of Chapter 4 is as follows. First, using mathematical programming models, we provide for the first time lower bounds on the performance of a vehicle sharing system, measured by the total excess time, under any passive regulation and under any parking reservation policy. Second, we introduce the concept of partial reservation policies. We examine three different partial policies, each is based on a simple sound principle, which is easy to control by the system’s managers and communicate to the users. We define the user behavior under these policies and examine their performance using discrete event simulation of real world systems. Third, we examine the potential benefit of parking space overbooking.

Our results indicate that, surprisingly, the CPR policy achieves a significant part of the improvement that can theoretically be attained by any passive regulation, compared to applying no regulation (the NR policy). In addition, we show that under three reasonable partial reservation policies the performance of the system is inferior to its performance under CPR. In fact, we were unable to come up with a partial reservation
policy that outperforms the CPR. This reinforces the advantage of implementing the CPR policy. Nevertheless, our numerical study shows that the examined partial policies can significantly improve the performance of the system compared to the NR policy. Therefore, if, due to any considerations or constraints, the system managers are reluctant to implement CPR, then most of its advantages can be obtained by implementing a partial reservation policy. Finally, parking space overbooking is ruled-out as a direction for improving the results obtained by the CPR policy. We show that on one hand it cannot yield a significant improvement in terms of the expected performance and on the other hand it introduces undesirable uncertainty to the system.

1.3. Maintenance operations

Maintenance operations of bike-sharing systems have not been so far at the focus of Operation Research or Operations Management studies. To the best of our knowledge, this topic is studied for the first time in Part II of this dissertation. An underlying assumption in the existing literature on bike sharing systems is that all bicycles that are in the system are in a usable state at all times. In practice, however, a certain amount of bicycles becomes unusable and require repair every day. These bicycles block additional system resources, i.e., the lockers. Therefore, it is highly important to monitor and report the usability level of bicycles. The amount of bicycles that become unusable every day is certainly not negligible and therefore this phenomenon should be taken into account in the planning of daily operational activities. See, for example, the monthly operating reports published by NYC Bikeshare, the operator of the bike-sharing system in New-York, Citibike: https://www.citibikenyc.com/system-data/operating-reports.

We envision a framework for the planning of these maintenance operations that includes three processes: (1) detection of unusable bicycles; (2) analysis of the effect of the presence of unusable bicycles on the quality of service provided to the users; (3) collection of unusable bicycles to maintenance shops or repairing them on-site. The first process is at the focus of Chapter 5, the second is at the focus of Chapter 6 and an example for an application of the third process is given in Appendix D.

The information systems installed in bike-sharing systems present to the public online aggregated information about each station. In particular, using smartphones or stations’ kiosks, users may query the state of each station in terms of the number of available bicycles and the number of available lockers. Internally, the information system stores a log of the transactions that were carried out. Each transaction is featured by its type (renting, repositioning, maintenance), start time, end time, start station ID, start locker ID, end station ID, end locker ID, bike ID, User ID (either a regular user of the system or a maintenance personnel), etc. Some operators share a subset of the fields in this log with the public, see for example the CitiBike trip history: http://www.citibikenyc.com/system-data.

In existing bike-sharing systems, information about unusable bicycles is received either from users or from repositioning workers when they service the stations. The probability that a user will report on an unusable bicycle is rather low if other bicycles that are parked in the station can be rented. That is, a user will typically complain about an unusable bicycle only when there is no alternative in the station. In addition, not all
stations are serviced by repositioning workers on a daily basis. Therefore an unusable bicycle may be parked at a station for a long period of time before being detected and collected.

In some systems, such as CitiBike, each locker is equipped with a maintenance button that the users may push in order to signal to the operator that the bicycle should be serviced. While through this mechanism, more information about bicycles that should be repaired is obtained, this also generates a fair amount of false alarms. In the CitiBike system, about 36% percent of the reported bicycles are actually usable (Lewis, 2015) and, more importantly, many unusable bicycles are not reported by the users.

Undetected unusable bicycles appear in the information systems as available ones. This inaccuracy may adversely affect users' route choices and result in an inferior service level. For example, a user may go to a station with such undetected unusable bicycles only to find out that there are actually no available usable bicycles in the station. If the system could provide her with accurate information in advance she could save time by planning her trip differently, e.g., start her trip at a neighboring station or select a different mode of transportation.

The contribution of Chapter 5 is as follows: we propose using data that is already collected by existing bike sharing systems to estimate the probability that each bicycle is usable. We formulate a Bayesian model that makes use of on-line transactions data to constantly update these probabilities and propose a method to approximate these probabilities in real-time. Subsequently, we present some possible extensions of the model and explain how additional information such as user complaints can be incorporated in the model. In addition, we discuss how an equivalent model can be used for detection of locker failures.

In Chapter 6, we examine the effect of unusable bicycles on the service level provided in a single station. Obviously, the effect of the presence of unusable bicycles may differ between stations, depending on their capacities and their demand processes for bicycles and lockers. Better understanding of this effect will assist in better planning of the collection and repair operation. Clearly, in order to optimize the quality of service all the unusable bicycles should be removed from the stations or should be repaired on site as soon as possible. However, since the transportation and maintenance resources of the operator are limited, a method to estimate the expected effect of the unusable bicycles at each station can help in prioritizing these operations.

The contribution of Chapter 6 is as follows: we introduce an Extended User Dissatisfaction Function (EUDF) that represents the weighted number of users that are unable to rent or return a bicycle at a station during a given period as a bivariate function of the initial number of usable and unusable bicycles at the station. This is an extension of the User Dissatisfaction Function (UDF) that was presented in Raviv and Kolka (2013), which builds on the assumption that bicycles are always usable. We prove some discrete convexity properties of the EUDF. In addition, we propose a method for calculating a convex polyhedral function that has nearly identical values to the EUDF in its range. This polyhedral approximation can be used to optimize the initial bicycle inventories in the system subject to various constraints using linear programming, as demonstrated by an example in Appendix D.
2. Literature review

In this chapter, we provide a literature review of Operations Management and Operations Research studies on vehicle sharing systems. We note that parking reservation policies and maintenance operations were not studied before. The research presented in Chapters 3-6 represent novel additions to the existing literature. The focus of this review is mainly on bike-sharing related studies. However, car-sharing systems (especially one-way systems) are similar in many aspects to bike-sharing systems. Hence, we present also some car-sharing studies.

The structure of this review is as follows: in Section 2.1, we discuss studies that characterize existing bike-sharing systems through data analysis. In Section 2.2, we present studies that deal with strategic planning aspects. In Section 2.3, we review studies that devise methods for determining the desired daily initial inventories. In Section 2.4, we describe papers that focus on active regulation, i.e., the rebalancing of the system using repositioning trucks. In Section 2.5, we discuss papers that propose passive regulations as means of rebalancing the inventory levels.

2.1. Data analysis

The existing bike sharing systems are equipped with information systems that accumulate trip transactions data and provide on-line information to the users regarding the inventory levels at the stations. Analysis of such data can be used to better understand the demand patterns in the different stations of a system and can be used to predict the near-future state of the system. The outcome of this analysis may serve as an input both for strategic decisions regarding the expansion of the systems and for operational purposes. With the increasing implementation of bike-sharing systems in the past decade, several studies that analyze the demand characteristics in a single system were published. In this section, we present a review of some studies that focus on data analysis of bike-sharing systems. In what follows, we discuss several features that affect demand, which are examined in these studies. Though some of these features are quite intuitive, the results of the following studies highlight the need for considering them in the demand prediction and planning processes.

Kaltenbrunner et al. (2010) use data regarding the bicycle inventory levels at the stations of Bicing, the bike-sharing system in Barcelona, to analyze the demand patterns at different parts of the city along the day. They propose several prediction models for the number of bicycles in a station based on time series of the number of available bicycles and lockers in the station and in surrounding stations with similar demand patterns.

Vogel et al. (2011) derive bike activity patterns of the stations of Citybike Wien, the bike-sharing system in Vienna. They use data mining techniques to cluster stations with similar spatio-temporal activity patterns in order to better understand spatial and temporal causes of imbalances. They conclude that the activity patterns greatly depend on the stations’ locations.
Nair et al. (2013) present an empirical analysis of the Vélib’ bike sharing system in Paris. One of their key finding is that close coupling of public transit stations and bike-sharing stations can lead to higher utilization of the bike-sharing system.

Rudloff and Lackner (2013) model the demand for bicycles and lockers using data from Citybike Wien. They study the influence of weather and full/empty surrounding stations on the demand in a single station using several count models. The results of the study demonstrate that weather has an impact on the usage of the bike sharing system and in particular, temperature and precipitation have an influence on the demand. In addition, demand spillovers from full/empty surrounding stations are shown to have an effect on the demand in a single station.

Côme et al. (2014) use Latent Dirichlet Allocation, a text categorization algorithm, to analyze Origin-Destination data from the Vélib’ bike-sharing system. They identify a reduced set of demand profiles and demonstrate that these profiles can be summarized by few OD-templates, which are typical and temporally localized.

Corcoran et al. (2014) examine the effect of weather conditions and calendar events on the demand patterns in CitiCycle, the bike sharing system in Brisbane. They conclude that rain and strong winds reduce the amount of trips taken, and especially the longer trips. Calendar events are shown to not have an effect on the total demand in the system, but they do affect the distribution of trips, as fewer trips are taken between housing and working areas during holidays.

O’Mahony and Shmoys (2015) present a framework for improving the operation of a bike sharing system, motivated by their collaboration with CitiBike, the New York city bike sharing system. They analyze the demand patterns at each station during the morning and evening rush-hour periods and use a k-means algorithm to distinct between three types of stations: stations that accumulate bikes, stations that lose bikes and stations that are “self-balanced”. This information is later used to set target fill levels in each station and to plan both static and dynamic repositioning activities.

De Chardon and Caruso (2015) discuss two types of available data in bike sharing systems: inventory levels and trip transactions (OD-data). Since the latter is only made publicly available in few systems, they propose a method to estimate the total daily number of trips based on the inventory level data. They argue that the quality of service provided by bike sharing operators is not easily verifiable and that their method can assist in revealing spatial and temporal frequencies of rebalancing completed by the operators. This in turn, should allow municipal oversight of service agreements. In their paper De Chardon and Caruso (2015) provide a comprehensive list of studies on the topic of demand data analysis in bike sharing systems.

We note that the trip data available for the operators does not necessarily represent the true demand patterns but merely the trips that users realize. Specifically, there is no available information regarding events in which users do not rent/return a bicycle due to bicycle/locker shortages. In future research, external data should be considered in order to fill this gap. Such data includes utilization of nearby public transportation stations, in-city commuting surveys, data retrieved from cellular networks or GPS based systems, and data that can be obtained from bike-sharing users reports through a designated smartphone application. The parking reservation policies studied in Part I of this
dissertation offer a partial remedy to this issue, as under such policies, users are required to state their destination station at the renting time. This reveals a characteristic of their true demand.

2.2. Strategic planning
This section reviews studies that have dealt with strategic planning aspects of vehicle sharing systems such as the location of stations, the location of bike lanes and the fleet size. In order to be able to perform a high-level analysis, all of the following studies make some simplifying assumptions regarding the capacities of the stations, the initial inventory levels and the manner in which operational activities take place.

The following studies take a deterministic centralized approach, as often done in studies of traditional logistics networks: Lin and Yang (2011) formulate a non-linear binary model to determine the locations of stations and bicycle lanes to be built, so as to minimize a weighted sum of the users total travel costs and the operators’ infrastructure setup costs. Lin et al. (2013) present a similar linear model that explicitly takes into account the inventory levels at each station. Both models assume that the true origin and destination of the users are given in advance and that the capacities of the stations are not binding.

Correia and Antunes (2012) present a MILP model to determine the location of car-sharing stations with the objective of maximizing revenues minus operational costs. The model assumes that trip requests are known in advance and a central planner is allowed to select which trips to serve.

The following studies formulate stochastic models and assume users are strategic, i.e., a decentralized approach. George and Xia (2011) formulate a closed queueing network model of a car-sharing system and use it to optimize the fleet size, assuming time homogenous demand and no parking capacity constraint.

Shu et al. (2013) develop a stochastic network flow model to estimate the flow of bicycles within the network and the utilization rate of bicycles given the initial inventory at each station. An upper bound on the number of trips taken is obtained using a LP model. Through numerical experiments, this bound is shown to be a good approximation of the stochastic model.

Nair and Miller Hooks (2015) formulate an equilibrium network design model to determine the locations of bike sharing stations in Capital BikeShare, the bike-sharing system in Washington D.C. A bi-level mixed integer program is devised, where the upper level objective is to maximize flow potential along bicycle sharing links and the lower level objective is to minimize users’ travel time. The model ignores both the initial inventory levels and stations’ capacities.

2.3. Initial bicycle allocation
An important tactical task that bike-sharing system operators need to address is to determine the inventory levels at the beginning of the day (or before rush hours). The goal is to set the inventory levels such that between visits of repositioning trucks, as much of the requested demand for bicycles and lockers would be satisfied. At the implementation stage, the common practice in some bike sharing systems was to use
target fill levels of 50%, namely, to fill each station with half of its capacity at the beginning of the day. However, this approach was soon discarded, due to better understanding of the demand patterns. Indeed, some stations exhibit high demand for bicycles in the mornings and therefore should be almost full at the beginning of the day, while other stations exhibit high demand for lockers in the mornings and therefore should be almost empty. We next discuss several studies that devise analytical methods for determining the desired daily initial inventories in the stations.

Raviv and Kolka (2013) formulate a stochastic model of a single station. In their study, the expected number of shortages during the planning horizon is represented as a function of the initial inventory, referred to as the User Dissatisfaction Function (UDF). The UDF takes as an input the time-dependent demand distributions for bicycles and for lockers in each station during the planning horizon (e.g., during the next day). The convexity of the UDF is proved and an efficient method to approximate it is presented.

In Chapter 6, we extend the UDF to calculate the expected number of shortages as a function of the initial inventory of unusable and unusable bicycles. Some convexity properties of the resulting bivariate function are proven and an accurate approximation by a convex-polyhedral function is provided.

Schuijbroek et al. (2013) define service level requirements as the proportions of satisfied requests for bicycles and lockers. They formulate a queuing model to represent the state of a single station, assuming stationary demand patterns between consecutive visits of repositioning staff. The model is used to calculate an interval of initial inventory levels that satisfy the required service levels.

Contrary to Raviv and Kolka (2013) and Schuijbroek et al. (2013) who model the inventory level in a single station, Datner et al. (2015) propose a system-wide method to calculate the desired initial inventory levels. This approach is motivated by the interactions between neighboring stations due to demand spillovers and also by the dependencies between demand for bicycles at the origin stations and demand for lockers at the destination stations. A simulation based guided local search is developed in order to determine simultaneously the initial inventory levels at all stations. Numerical results demonstrate that neglecting the interaction between stations leads to inferior solutions.

2.4. Repositioning activities
In this section, we review studies regarding the planning of repositioning activities. This topic, has so far received most of the attention in the Operations Research literature. It can be divided into two main streams, namely, static repositioning and dynamic repositioning. Static repositioning operations are carried out during off-peak periods when demands are negligible (at night) in order to prepare for the demands of the following day. Dynamic repositioning operations are carried out during peak periods, while bicycles are on the move.

The following studies deal with the static case. Though essentially the goal in all of them is to rebalance the inventory levels, these studies differ significantly due to a variety of modeling assumptions. Regarding, for example, routing time limitations, existence of inventory target levels and single/multi vehicle models.
Nair and Miller Hooks (2011) use chance constraint modeling to generate least cost repositioning plans for vehicle sharing systems. The model assumes linear repositioning costs between two given stations. The routing of repositioning staff between the stations is abstracted out.

Benchimol et al. (2011) introduce a single vehicle rebalancing problem. In this problem each station has a given target inventory level. Given the initial inventory levels, the goal is to find a minimum cost route for the single vehicle so as to set inventory levels to their targets. The vehicle may visit any station more than once, and there is no restriction regarding the total travel time of the vehicle. Benchimol et al. (2011) prove that the problem is NP-Hard and provide lower bounds for it. Chemla et al. (2013a) construct a branch-and-cut algorithm to solve a relaxation of the problem and use a Tabu search to generate feasible solutions of the problem. Erdoğan et al. (2015) solve the problem using a Benders decomposition based branch-and-cut algorithm. The presented algorithm is able of solving instances with up to 60 stations.

Erdoğan et al. (2014) extend the single vehicle problem to allow the final inventory at each node to be within a prescribed interval instead of at a unique level. They present a branch-and-cut algorithm and a Benders decomposition that allows solving instances with up to 50 stations.

Raviv et al. (2013) study a multi-vehicle version of the problem where: bicycle loading and unloading times are taken into account and the total route time of each vehicle is limited. Inventory levels are not restricted to given targets, but instead each inventory level is associated with a cost according to the UDF (user dissatisfaction function), as presented in Raviv and Kolka (2013). The objective function is a weighted sum of the routing costs and the UDF values in all stations. Two mixed integer linear program formulations (time-indexed and arc-indexed) are presented and are strengthened by several valid inequalities and dominance rules. In Appendix D, we extend their arc-indexed formulation to accommodate for maintenance aspects, namely, the existence of unusable bicycles and decisions regarding their collection.

Forma et al. (2015) propose a 3-step mathematical programming based heuristic for the multi vehicle model formulated in Raviv et al. (2013). In the first step, stations are clustered and in the second step, repositioning vehicles are routed through the clusters. In the third step, the original repositioning problem is solved under the restriction that vehicles may only travel between stations that belong to consecutive clusters, as determined in the second stage. The heuristic is tested on instances with up to 200 stations and three repositioning vehicles.

Ho and Szeto (2014) reformulate the model of Raviv et al. (2013) to a single vehicle model. The objective function is simplified to take into account only the service level penalties, i.e., the UDF. A tabu search heuristic is devised and is shown to produce solutions for instances with up to 400 stations in several seconds with optimality gaps smaller than 1%.

Rainer-Harbach et al. (2013) formulate a multi vehicle problem where the objective is to minimize the deviation of the final inventory levels from given target levels subject to a total route time limit. To solve the problem, they propose a Variable Neighborhood Search (VNS) algorithm that generates candidate routes and loading/unloading
decisions are determined through three alternative methods: a greedy heuristic, a maximum flow calculation and linear programming. Instances with up to 90 stations and 5 vehicles are solved. In Raidl et al. (2013) the same VNS framework is used and a new method for the loading/unloading decisions is presented. The method is based on two sequential maximum-flow calculations and is shown to outperform the previously proposed methods.

Dell’Amico et al. (2014) solve a variant of the multi-vehicle static problem where each station has to be visited exactly once with a goal of minimizing the total routing costs. Four alternative mixed integer linear mathematical formulations are presented and solved using a standard branch-and-cut algorithm.

O’Mahony and Shmoys (2015) state that in the Citibike system they have studied, it is desirable to move only full trucks of bikes between stations. They formulate a routing problem on a bi-partite graph (representing pickup and delivery stations) with the objective of visiting as many stations as possible using a given number of trucks and restrictions on the path length of each truck.

The following studies deal with the dynamic case: Contardo et al. (2012) formulate a multi-vehicle problem in which the demands vary over time, with the goal of minimizing unmet demand for bicycles and lockers. Their solution approach is based on two decomposition schemes, Dantzing-Wolfe decomposition and Benders decomposition.

Kloimüller et al. (2014) extend the model presented in Rainer-Harbach et al. (2013) and Raidl et al. (2013) to consider the dynamic case. A greedy construction heuristic is used and two metaheuristic algorithms: VNS and greedy randomized adaptive search (GRASP).

Pessach et al. (2015) tackle a stochastic and dynamic multi-vehicle repositioning problem. They use a model similar to the time-indexed model of Raviv et al. (2013) within a rolling horizon framework. Their approach is shown to produce better results as compared to several methods based on dispatching rules.

O’Mahony and Shmoys (2015) formulate an ILP model to pair between relatively close stations with opposite demand patterns (pickup and delivery). During rush hours, each pair of stations is assigned with a vehicle with relatively small capacity (3-5 bicycles) that continuously travels between the stations and repositions bicycles from the pickup station to the delivery station.

2.5. Passive regulations
Passive regulations are mechanisms used for redirecting demand in order to improve the performance of the bike sharing system. Such mechanisms do not affect the true demand for journeys but may instead shift users to rent (return) vehicles at stations different from their true origin (destination) station or to use alternative modes of transportation.

Fricker and Gast (2015) study a system regulation under which each user declares two optional destination stations and the system directs her to the less congested one. They formulate a Markovian model of a homogenous bike sharing system and use Mean-field techniques to derive the steady state behavior of the system. Using the
results of this analysis, they optimize the fleet size so as to minimize the proportion of stations that become full/empty in steady state.

Several studies focus on pricing regulations as means of self-balancing vehicle sharing systems. Chemla et al. (2013b) and Pfommer et al (2014) propose dynamic repositioning models which combine repositioning activities and pricing mechanisms. Specifically, users are incentivized to change their destination stations to nearby stations with more vacant parking spaces. The tradeoff between the repositioning costs and user incentives is studied.

Waserhole et al. (2013) and Waserhole and Jost (2015) model the vehicle sharing system as a closed queuing network model with infinite buffer capacity and Markovian demands. In Waserhole et al. (2013) a fluid approximation of the systems is used to derive a static pricing policy and an upper bound on the performance of the system in terms of the number executed trips. In Waserhole and Jost (2015) it is further assumed that traveling times are negligible. A heuristic combining Maximum Circulation and a greedy algorithm is used to generate a static policy whose relative gap from the optimal solution is bounded. We note that both studies do not fall within our definition of passive regulations since they assume that it is possible to generate any demand by setting, possibly negative, prices. This is equivalent to using the users as repositioning workers. In this sense, the regulation proposed in these two studies, are both active and passive.

The parking reservation policies studied in Part I of this dissertation are also modes of passive regulations. Their advantage is not only in reducing user’s uncertainty about the total travel time, but more importantly, in regulating the system by redirecting users to less congested stations near their true destinations.
Part I
Parking Reservation Policies
3. Complete parking reservation policies


In this chapter, we consider the following policy: when a user rents a vehicle, she declares her destination station and a vacant parking space in that station is reserved for her, if one is available. That is, the reserved vacant parking space at the destination becomes unavailable for other users from the moment the reservation is made, until the user reaches her destination. This assures that when she reaches her destination, she will be able to return her vehicle. If there is no available parking space at the destination, the transaction is denied, so that the user is unable to rent the vehicle. We refer to this policy as the Complete-Parking-Reservation (CPR) policy. It is complete in the sense that all users are obliged to reserve their parking space. By implementing this parking reservation policy, the system can secure an ideal ride for some users. On the other hand, it holds parking spaces without utilizing them for a certain amount of time, and by doing so, possibly rejecting other potential users. Due to this tradeoff, the effectiveness of using reservations is not obvious.

The contribution of this chapter is as follows: for the first time, we study a parking reservation policy in one-way vehicle sharing systems and compare its performance to a base policy, entitled No-Reservation (NR) policy. We propose measuring the performance of the system by the total excess time users spend in the system due to unfulfilled renting requests or delays in returning the vehicles. We show that implementing parking reservations as suggested in this study will improve the performance of one-way vehicle sharing systems in most realistic systems. In particular, using Markovian models, we prove our main theoretic result, stating that if the demand faced by the system is not extremely high, the CPR policy outperforms the NR policy. In other words, under the CPR policy users will spend less excess time in the system. We further demonstrate through a small example that in some cases the CPR is superior for any demand rate. Finally, the main theoretic result is reinforced by a numerical study based on a discrete event simulation of a real-world vehicle sharing system with an enhanced user behavior model. The bike sharing system in Tel-Aviv, Tel-O-Fun, was used as our case study. The behavior model assumes users are strategic, i.e., in case of an unfulfilled renting request or unavailability of a parking space, the user determines her alternative route so as to minimize the expected time she spends in the system. Through this numerical study, we also demonstrate that the CPR policy outperforms the NR policy under several additional performance measures.

The structure of this chapter is as follows. In Section 3.1 we discuss service oriented performance measures used in vehicle sharing systems and in public transportation in general. In Section 3.2, a Markovian model of a vehicle sharing system is described. In Section 3.3, the NR and CPR policies are compared analytically and numerically. A simulation model of a vehicle sharing system is presented in Section 3.4.
Using the model, the superiority of the CPR policy is demonstrated using data from a real world system. In Section 3.5, concluding remarks and some directions for future research are given.

3.1. Performance measures

Most of the one-way vehicle sharing systems are service oriented; accordingly, these systems are typically measured by the quality of service given to the users. The most common measure used in practice is based on the percentage of time in which stations are empty or full. Specifically, it is obtained by averaging these percentage values over all stations. We refer to this measure as station-availability. Since the vehicle inventory levels in the stations are updated on-line in the information systems, it is quite easy for the operators or other interested parties to monitor this performance measure. This measure has also been at the focus of some previous research studies. However, we claim that this measure is a biased reflection of the quality of service because the state of the stations should be weighted by their demand rates. For example, the adverse effect of an empty station that faces a low rate of demand for rentals is smaller compared with an empty station that faces a higher rate. Moreover, due to changes in demand rates along the day, it may even be desirable for certain stations to be full at certain times, and for other stations, to be empty. For example, during the mornings, many systems experience high demand for journeys from residential areas to business areas. Thus, in these hours it may be beneficial to fill up stations in residential areas and to empty stations in business areas. Other considerations that should be taken into account in a performance measure are the length of the trip, which may affect the inconvenience caused to a user due to an unfulfilled request, the time of the day, the proximity to other stations, available alternative modes of transportation, etc.

In contrast to the station-availability performance measure, it is common in studies of other public transportation systems to represent the service level by the total time users spend in the system. See, for example, road selection (Campbell 1992), buses and trams (Borndörfer et al. 2007), and passenger trains (Schöbel and Scholl 2006). A related measure to the total time is the total excess time users spend in the system. As presented in the Section 1.2, the excess time of a user is the difference between her actual and her ideal (shortest) travel time, see Ceder and Wilson (1986). In this study, we suggest using the total excess times of all users as the performance measure in a vehicle sharing system. The excess time is caused to a user due to an unfulfilled renting request or delays in returning the vehicle. We use this measure to evaluate and compare our suggested reservation policy to the current practice of one-way vehicle sharing systems, in which parking space reservation is not implemented. Note that focusing on the total excess time allows us to remove from the analysis a fixed constant that in any case cannot be reduced, that is, the total ideal time.

Currently, information systems of vehicle sharing systems log the actual journey times and itinerary, i.e., origins, destinations and journey durations. The true preferences of the users are unknown and there are also no records about abandonments. Based on this information, it is impossible to calculate the excess time of the users.
However, the information needed to calculate the excess time can be quite easily obtained. For example, by incorporating reservation policies, the true destinations of the users would be revealed. In addition, operators can encourage users to declare their requested origin and destination and to report if they decide to abandon, in order to improve their own (possibly future) service. Technically, this would be done using smartphone apps or station information kiosks.

We view the excess time as a better performance than the one currently used (station-availability), therefore in this paper we focus on this measure. However, to make the case for parking space reservation stronger, we compare (in Section 3.4) the CPR policy with the NR policy using both measures. Our simulation study demonstrates the advantage of the CPR with respect to both of these measures.

3.2. A Markovian model
In this section, the vehicle sharing system is modeled as a continuous time Markov chain, see for example Ross (2009). This is a simplistic model created in order to derive some general insights into the performance of the system under the NR (No-Reservation) and CPR (Complete-Parking-Reservation) policies. These insights are verified using actual data and a detailed simulation model in Section 3.4.

We use the following general notation to describe the vehicle sharing system configuration:

- **S**: The number of stations in the system
- **C_i**: The number of parking spaces in station *i* (station capacity)
- **V**: The number of vehicles dispersed in the system
- **T_{ij}(t)**: The expected travel time between any two stations *i*, *j* of users who depart at time period *t*
- **λ_{ij}(t)**: The arrival rate of users who wish to travel from station *i* to station *j* at time period *t*

The travel times reflect structural characteristics of the system, such as the distance between the stations, traffic density within the different segments, the topography of the city, and so on. Note that in general, **T_{ij}(t) ≠ T_{ji}(t)**. For example, it is clear that riding a bicycle downhill is much faster than riding uphill. The expected travel times represent the duration of journeys that users actually make, which may not necessarily be the shortest possible. For example, **T_{ii}(t) ≠ 0**, that is, roundtrip journeys are possible. As for the users, the demand for journeys between each pair of stations is given as a stochastic process. Furthermore, user behavioral rules are set in order to define how users react to shortages in either vehicles or parking spaces. Finally, this model is service oriented; the performance of the system is measured by the total excess time spent by users due to shortages of vehicles or parking spaces.

3.2.1. The NR policy
In this section, we model the system under the NR policy and Markovian assumptions. The arrival of renters to the system follows a Poisson process, and the travel time of journeys is exponentially distributed. For the sake of simplicity, the distributions are set to be homogenous in time. Therefore, we denote **λ_{ij}(t) = λ_{ij}** and **T_{ij}(t) = T_{ij}** for all *t*. 

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Finally, to be able to assume a steady state, repositioning operations are not included in the model.

The following user behavior is assumed: (1) a user that faces a shortage of vehicles will abandon the system immediately. That is, she will choose an alternative mode of transportation and therefore will spend excess time on her journey. (2) A user that faces a shortage of vacant parking spaces will wait at the destination until a parking space becomes available, as a result of an arrival of a renter. Namely, the user will enter a waiting queue and will spend excess time waiting for her turn to return her vehicle. This assumption is made in order to simplify the Markovian model. In Section 3.4.1 the user behavior is extended so that roaming between stations is allowed.

The excess time due to abandoning is modeled as proportional to the travel time. Specifically, the excess time of an abandoning user who wishes to travel from station $i$ to $j$ is denoted by $\alpha_{ij}T_{ij}$ where $\alpha_{ij}$ is the penalty ratio.

Given the assumptions above, the state of the system is described by the following vector:

$$x = (x_{01}, x_{11}, ..., x_{S1}, ..., x_{0S}, x_{1S}, ..., x_{SS})$$

$$x_{ij} \geq 0 \ \forall \ i = 0, ..., S \ \forall \ j = 1, ..., S$$

where $x_{0j}$ denotes the number of vehicles in stations $j$ and $x_{ij}$ denotes the number of vehicles traveling from station $i$ to station $j$. In particular, if $x_{0j} > C_j$ then all the parking spaces in station $j$ are occupied and there are $x_{0j} - C_j$ users waiting in station $j$ for parking spaces to become available. Furthermore, a possible state satisfies the following:

$$\sum_{j=1}^{S} \sum_{i=0}^{S} x_{ij} = V$$

Note that any given vehicle can be either at one of the $S$ stations or traveling between one of the $S^2$ pairs of stations. In addition, there can be up to $V$ vehicles in any given station or traveling between the same given pair of stations. We denote the set of all possible states by $X$. Recall that the number of parking spaces in a station is not binding because the formation of queues is possible. Therefore, the number of states is the number of ways to place $V$ identical vehicles in $S(S + 1)$ "bins". Hence, the number of possible states in the system is $\binom{S \cdot (S + 1) + V - 1}{S \cdot (S + 1) - 1}$. (Since the number of possible ways to distribute $K$ identical items into $N$ bins is $\binom{N + K - 1}{N - 1}$).

A transition between the system states occurs either when a vehicle is rented or when a user arrives at her destination. A rent transition between two possible states $x, x'$ is denoted by the following indicator function:

$$\gamma_{ij}(x, x') = \begin{cases} 1, & \text{if } x' = \left( x_{01}, ..., x_{S1}, ..., x_{0t} - 1, ..., x_{ij} + 1, ..., x_{0S}, ..., x_{SS} \right) \\ 0, & \text{otherwise} \end{cases}$$
where $i$ is the origin station and $j$ is the destination station. A return transition between two possible states $x, x'$ is denoted by the following indicator function:

$$
\delta_{ij}(x, x') = \begin{cases} 1, & \text{if } x' = (x_{01}, ..., x_{S1}, ..., x_{0j} + 1, ..., x_{ij} - 1, ..., x_{0S}, ..., x_{SS}) \\
0, & \text{otherwise}
\end{cases}
$$

where $i$ is the station in which the vehicle was rented and $j$ is the destination station.

The transition rates between any two possible states $x, x'$ are given by:

$$
\sum_{i,j} \lambda_{ij} Y_{ij}(x, x') + \sum_{i,j} \frac{x_{ij}}{T_{ij}} \delta_{ij}(x, x')
$$

(3.2)

Note that while the arrival rates of users do not depend on the number of vehicles in the station, the returning rates are linear functions of the number of traveling vehicles.

As discussed in Section 3.1, we measure the performance of the system by the total excess time users spend in the system. According to our model this is due to waiting at queues for a vacant parking space or due to abandoning the system and use of alternative mode of transportation. In the analysis of the Markovian models, we focus on the total excess time added per time period in steady state, referred to as the expected excess time rate.

We denote the total number of users waiting to return a vehicle in state $x$ by $Q(x)$, where:

$$
Q(x) = \sum_{j: x_{0j} \geq c_j} (x_{0j} - c_j)
$$

The expected number of users waiting to return a vehicle in steady state is given in Equation (3.3):

$$
\sum_{x \in X} Q(x) \pi(x)
$$

(3.3)

where $\pi(x)$ is the limiting probability of state $x$. Note that the expected number of users waiting to return a vehicle is equivalent to the expected excess time rate due to waiting in queues. This is because for every time period a user waits in a queue, one period of excess time is accumulated. The steady state excess time rate due to abandonments in state $x$ is given by:

$$
\sum_{i: x_{0i} = 0} \sum_{j=1}^{S} \lambda_{ij} a_{ij} T_{ij}
$$

(3.4)

In the internal sum of (3.4), the excess time rate due to abandonments is calculated for each empty station $i$, and the external sum goes over all empty stations. For the entire system, the expected excess time rate due to abandonments is obtained by summing (3.4) over all states, multiplied by their respective limiting probabilities, and is given by:
\[
\sum_{x \in X} \sum_{i:i_{0i}=0} \sum_{j=1}^{S} \pi(x) \cdot \lambda_{ij} \alpha_{ij} T_{ij}
\] (3.5)

In conclusion, the expected excess time rate is obtained by summing (3.3) and (3.5).

It may be argued that when abandoning the system the quality of service is adversely affected not only by the travel time using an alternative mode of transportation but also by the mere fact that the user has to seek such an alternative service. For example, if an alternative mode of transportation is a bus, the user has to wait for it to arrive and to pay for a ticket. The waiting time and the cost of the ticket are typically unrelated to the travel time. Hence, the penalty incurred by the system for each passenger who uses a different mode of transportation may consist of a fixed component, in addition to the variable component which is proportional to the travel time. The fixed component should be expressed in units that are equivalent to travel time. The models presented in this section neglect this fixed component, but we revisit this issue and justify our simplifying assumption in Section 3.4, where we analyze a real world system using discrete event simulation.

### 3.2.2. The CPR policy

We now specify how the Markov chain for the NR policy is adapted to the CPR policy. Recall that a user will rent a vehicle only if there is one available in her origin station and there is an available parking space at the destination, namely, empty and non-reserved parking. Therefore, the total number of vehicles that travel to station \(i\) or are parked at station \(i\) cannot exceed the number of parking spaces in the station. In addition to (3.1), a state in a system managed by a CPR policy satisfies:

\[
\sum_{i=0}^{S} x_{ij} \leq C_j \quad \forall j = 1 \ldots S
\] (3.6)

We denote the set of all possible states in the CPR policy by \(\tilde{X}\) and note that \(\tilde{X} \subset X\), that is, the set of possible states in the CPR policy is a subset of the NR set. In particular, \(\tilde{X}\) only includes states that satisfy (3.6). Since the formation of queues is impossible, excess time under this policy is only due to abandoning. The expected excess time rate due to abandoning when in state \(x\) is given by:

\[
\sum_{(i,j): x_{0i}=0}^{(i,j): x_{0i}=0} \sum_{\sum_{k=0}^{S} x_{kj}=C_j} \lambda_{ij} \alpha_{ij} T_{ij}
\]

For the entire system, the expected excess time rate due to abandoning is given by:

\[
\sum_{x \in \tilde{X}} \sum_{(i,j): x_{0i}=0} x \sum_{\sum_{k=0}^{S} x_{kj}=C_j} \tilde{\pi}(x) \lambda_{ij} \alpha_{ij} T_{ij}
\] (3.7)

where \(\tilde{\pi}(x)\) is the limiting probability of state \(x\).
3.3. Comparison between the CPR and NR policies
In this section the vehicle sharing system under the Markovian assumptions and the user behavior assumptions described above is referred to as the M-VSS model (Markovian Vehicle Sharing System). In Section 3.3.1 we define the notion of offered load in the context of the M-VSS model, and demonstrate that the expected excess time rate is a function of it. In Section 3.3.2, using the M-VSS model, we compare the CPR and NR policies under a range of offered loads. The comparison is illustrated through a small example in Section 3.3.3.

3.3.1. Offered load
The following definitions refer to the ideal situation in which all requested origin-destination journeys are satisfied. Denote $T$ as the expected travel time per arriving user, specifically:

$$T = \frac{\sum_{i=1}^{S} \sum_{j=1}^{S} \lambda_{ij} T_{ij}}{\sum_{i=1}^{S} \sum_{j=1}^{S} \lambda_{ij}}$$

In the same manner, let $\lambda$ denote the average arrival rate at a station, which can be written as:

$$\lambda = \frac{\sum_{i=1}^{S} \sum_{j=1}^{S} \lambda_{ij}}{S}$$

A user who rents a vehicle at station $i$ and travels to station $j$ will use the vehicle for an expected $T_{ij}$ time units. That is, the expected work added to the system by this user is $T_{ij}$ time units of vehicle usage. We define the offered load as the expected work to be added to an average station per time unit if all demands could be satisfied. The offered load is the product of $T$ and $\lambda$:

$$\lambda T = \frac{\sum_{i=1}^{S} \sum_{j=1}^{S} \lambda_{ij} T_{ij}}{S}$$

Note that $S \lambda T$ represents the offered load in the entire system. That is, the total travel time added to the entire system per time unit. In other words, it represents the expected number of vehicles that would be in use at any given moment if the system could meet all the demand.

In the following section, we will measure the performance of the system under various loads by varying the values of $T$ and $\lambda$. In order to facilitate a fair comparison, the relations between the stations are kept fixed. This is done by noting that all travel times $T_{ij}$ can be written as the proportion $\tau_{ij}$ of the expected travel time $T$, that is $T_{ij} = \tau_{ij} T \ \forall i, j$ and similarly, all arrival rates $\lambda_{ij}$ can be written as the proportion $\nu_{ij}$ of the expected arrival rate, that is $\lambda_{ij} = \nu_{ij} \lambda$. Then, when $T$ is changed, all travel times are adjusted proportionally and similarly when $\lambda$ is changed, the arrival rates at all stations are adjusted proportionally. In other words, while $T$ and $\lambda$ may be altered, the proportions $\tau_{ij} \ \forall i, j$ and $\nu_{ij} \ \forall i, j$ are kept fixed.

**Lemma 3.1:** For the M-VSS model, when $\nu_{ij}$ and $\tau_{ij}$ are fixed, the limiting probabilities are functions of the offered load $\lambda T$. 

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Proof: the coefficients in the steady state equations are generally in the form prescribed in (3.2), that is, either proportional to \( \lambda \) or to \( \frac{1}{T} \). By multiplying all equations by \( T \) an equivalent system of equations is received where all the coefficients are either proportional to \( \lambda T \) or constant. This set of equations is given in (3.8).

\[
\lambda T \sum_{i,j} \sum_{x \in X} v_{ij} \pi(x) \gamma_{ij}(x, x') + \sum_{i,j} x_{ij} \tau_{ij} \pi(x) \delta_{ij}(x, x') = \lambda T \sum_{i,j} \sum_{x \in X} v_{ij} \pi(x') \gamma_{ij}(x', x) + \sum_{i,j} x_{ij}' \tau_{ij} \pi(x') \delta_{ij}(x', x) \quad \forall x
\]

(3.8)

In addition, the coefficients in (3.9) are all constants.

\[\sum_{x \in X} \pi(x) = 1\]

(3.9)

The limiting probabilities are the solution of a system of equations where all coefficients are either proportional to \( \lambda T \) or constant. Hence, the limiting probabilities are functions of the two parameters (\( T \) and \( \lambda \)) only through their product, regardless of their separate values. \( \blacksquare \)

Lemma 3.2: For the M-VSS model, when \( v_{ij} \) and \( \tau_{ij} \) are fixed, the expected excess time rate can be stated as a function of \( \lambda T \).

Proof: this is straightforward from (3.3), (3.5) and Lemma 3.1. Consequently, the expected excess time rate is sensitive to changes in the offered load, rather than to changes in the expected travel time per user and the expected arrival rate separately. For instance, if we double the expected arrival rate, \( \lambda^* = 2\lambda \), and divide the expected traveling time by two, \( T^* = \frac{T}{2} \), the excess time rate will not change.

3.3.2. Conditions for dominance of the CPR policy

In this section we compare the performance of the M-VSS model under NR and CPR policies over a range of offered loads. In particular, our focus is on the range \( \lambda T \in \left( 0, \frac{V}{S} \right) \). The upper bound is received through the following observation: there can be at most \( V \) vehicles in use at any given moment. The expected offered load in the entire system is given by \( S\lambda T \). Hence \( S\lambda T = V \) represents the 100% utilization of the system. This bound is equivalent to the \( \rho < 1 \) bound in classic queueing systems. In the long run, it is safe to assume that the system will adapt itself to this range. In the case of higher loads, either the system will need to be further developed or the rate of user arrivals will drop due to constant lack of service. We note that in an unlikely case where \( V \leq \min_i \{ C_i \} \), users will always be able to return their vehicles in any station, that is, the system will perform the same under both policies. Since in practice the number of vehicles is much larger than the capacity of any station, we assume that \( V > \min_i \{ C_i \} \).

We begin with the extreme case of \( \lambda T = 0 \), namely the case when the expected travel time is negligible.
Lemma 3.3: For the M-VSS model, when $\lambda T = 0$, the expected excess time rate due to abandoning is zero under both policies.

Proof: To see this we rewrite Equations (3.5) and (3.7) as (3.10) and (3.11), respectively:

\[
\lambda T \sum_{x \in \mathcal{X}} \pi(x) \sum_{i: x_{0i} = 0}^{S} \sum_{j=1}^{S} u_{ij} \alpha_{ij} \tau_{ij} \quad (3.10)
\]

\[
\lambda T \sum_{x \in \mathcal{X}} \tilde{\pi}(x) \sum_{(i,j): x_{0i} = 0} \sum_{\sum_{k=0}^{S} x_{kj} = C_j} u_{ij} \alpha_{ij} \tau_{ij} \quad (3.11)
\]

Since the expressions in the internal sums are fixed and the sum of limiting probabilities is bound from above by 1, both sums equal zero when the offered load is zero.

It remains to examine the excess time rate due to queueing in the NR policy.

Lemma 3.4: For the M-VSS model, in the NR policy, when $\lambda T = 0$, the expected excess time rate due to queueing is a positive constant, independent of $\lambda$ and $T$.

Proof: See Appendix A.

In conclusion, when $\lambda T = 0$, the excess time in the CPR policy tends to zero while in the NR policy it tends to a positive value. Therefore, when $\lambda T$ is zero, the CPR policy is preferable. It turns out that there is a discontinuity point at $\lambda = 0$ since clearly there is no excess time when no customers arrive at the system. We are now ready to state an important result.

Theorem 3.1: For the M-VSS model, there exists some $\beta > 0$ for which the CPR policy delivers smaller excess time (better service) compared to the NR policy under any workload that satisfies $\lambda T < \beta$.

Proof: Both excess time rate expressions are continuous functions of $\lambda T$ (both are sums of continuous functions). By Lemma 3.3 and Lemma 3.4, when $\lambda T = 0$ the CPR policy is preferable. If the functions intersect, the first intersection point $\beta$ defines a range $(0, \beta]$ in which the CPR policy is preferable. Otherwise, the CPR policy is preferable for any $\lambda T$. ■

3.3.3. An illustrative example

Theorem 3.1 presented above is general in the sense that no limitations were imposed on the number of stations in the system or their capacities. To demonstrate this result, we use a small system, configured as follows: the system is comprised of two stations ($S = 2$). The capacity of both stations is $C_1 = C_2 = 2$, the number of vehicles in the system is $V = 3$. The expected travel time between the two stations is identical in both directions, $T_{12} = T_{21} = T$. The arrival rates of renters at the two stations are denoted by $\lambda_{12}$ and $\lambda_{21}$. For the sake of simplicity there is no demand for round-trip journeys, that is $\lambda_{11} = \lambda_{22} = 0$. This allows us to reduce the state representation to $x = (x_{01}, x_{21}, x_{02}, x_{12})$. Recall that $x_{0i}$ represents the number of vehicles in station $i$ and $x_{ji}$ represents the number of vehicles traveling from station $j$ to station $i$. For example, the state $(20, 01)$ indicates that two vehicles are parked in station 1, no vehicle is travelling to station 1, there are no vehicles parked in station 2 and one vehicle is travelling to
station 2. Finally, the excess time due to abandonment is $\alpha T$ ($\alpha_{12} = \alpha_{21} = \alpha$). In Figure 3.1 we depict the resulting Markov chains for the NR and CPR policies. The entire figure represents the NR policy chain and the sub-graph which consists of solid arcs and nodes represents the CPR policy chain. Notice that even for a small and simple system, the resulting Markov chains are quite intricate.

Figure 3.1: Graph representation of the Markov chains for the NR and CPR policies of the illustrative example

The limiting probabilities of the two Markov chains were calculated by solving the system of equations given by (3.8) and (3.9). The expected excess time rate for each policy was calculated according to (3.3), (3.5) and (3.7). We compare the two policies by subtracting the excess time rate of the CPR policy from the excess time rate of the NR policy. The function that describes this difference, as a function of the offered load $\lambda T$, is termed the difference function. A positive difference means that the CPR policy performs better and a negative value means that the NR policy performs better for the corresponding offered load. From Theorem 1 we know that there exists $\beta > 0$ such that for all $\lambda T \in (0, \beta]$ the difference function is positive.

In Figure 3.2, we present six difference function graphs for various settings. In the top graphs we set demand rates in both stations to be the same, that is $\lambda_{12} = \lambda_{21} = \lambda$. In the bottom graphs the total demand rate is kept constant but demands are unequal with $\lambda_{12} = 0.5\lambda$, $\lambda_{21} = 1.5\lambda$. Namely, the demand rate from station 2 to station 1 is three times the demand rate in the opposite direction. Recall that $\alpha$ is the penalty ratio in the case of abandonment. We set the penalty ratio to three different levels, $\alpha = 0.5, 1, 10$, as presented in the left, middle and right graphs, respectively. To illustrate the range of $\alpha$ values that are likely to be used, we note, for example, that in a bike sharing system, an abandoning user will probably walk to her destination. Assuming that the walking time is twice the riding time, the excess time due to
abandoning is equal to the riding time, i.e. $\alpha = 1$. Notice that only in the upper right graph, Figure 3.2(e), the curve intersects the axis, that is, there exists a range of offered loads in which the NR policy is preferable. This graph describes a case where the demands are balanced and the penalty for abandonment is extremely high.

In order to examine the effect of imbalanced demand on the difference function, we analyze the above system by assigning $\lambda_{12} = \lambda \theta$, $\lambda_{21} = \lambda (2 - \theta)$, $\theta \in [0,2]$, and study the resulting difference function over $\theta$. Indeed, the results show that for any $\lambda$, there exists a unique minimum point of the difference function at $\theta = 1$, which corresponds to the case $\lambda_{12} = \lambda_{21}$. Therefore, in this small example, the worst performance of the CPR policy compared to the NR policy is when the system is completely symmetric in terms of travel times and arrival rates. Note that in this case, rebalancing is least needed.

Recall that the bound for the system’s offered loads is $S\lambda T \leq V$. In this example, $S = 2$, $V = 3$ and so $\lambda T = 1.5$ represents 100% utilization. To further study the range in which the CPR policy is preferable over NR, we present in (3.12) and (3.13) the resulting excess time rate functions when $\lambda_{12} = \lambda_{21} = \lambda$, for the NR and CPR policies, respectively. The resulting expressions are relatively compact compared to the general expressions. Here, it is easy to see that both are functions of the offered load $\lambda T$. The difference function is given in (3.14).

### NR Policy:

$$\text{NR Policy:} \quad \frac{3 + 3\alpha \lambda T + 6\alpha (\lambda T)^2 + 6\alpha (\lambda T)^3 + 4\alpha (\lambda T)^4}{6 + 9\lambda T + 6(\lambda T)^2 + 2(\lambda T)^3} \quad (3.12)$$

### CPR Policy:

$$\text{CPR Policy:} \quad \frac{2\alpha \lambda T(6 + 10\lambda T + 7(\lambda T)^2 + 9(\lambda T)^3 + 3(\lambda T)^4)}{12 + 32\lambda T + 28(\lambda T)^2 + 14(\lambda T)^3 + 3(\lambda T)^4} \quad (3.13)$$

$$\frac{36 + 12(8 - 3\alpha)\lambda T + 12(7 - 5\alpha)(\lambda T)^2 + 6(7 + 2\alpha)(\lambda T)^3 + 9(1 + 8\alpha)(\lambda T)^4 + 67\alpha (\lambda T)^5 + 24\alpha (\lambda T)^6 + 2\alpha (\lambda T)^7}{72 + 300\lambda T + 528(\lambda T)^2 + 552(\lambda T)^3 + 376(\lambda T)^4 + 167(\lambda T)^5 + 46(\lambda T)^6 + 6(\lambda T)^7} \quad (3.14)$$

One can see that when the penalty parameter is in the range $0 \leq \alpha \leq 1.4$, (3.14) is positive for any $\lambda T$, that is, the CPR policy is always preferable. Recall that for bike-sharing systems a reasonable value of $\alpha$ is 1. Also, for $\lambda T \geq 0.82$, (3.14) is positive for
any $\alpha$, that is, the CPR policy performs better than NR, no matter how high the penalty for abandoning is. For large $\alpha$ values, the excess time due to waiting in a queue is likely to be smaller than the excess time when abandoning due to the inability to make a reservation. Therefore as $\alpha$ increases the range in which the CPR policy is preferable, narrows. Note that when the offered load is extremely high, the difference function converges to a constant, which means that the difference between the policies in terms of excess time per user is negligible. Indeed, if the offered load is extremely high, all vehicles will be constantly traveling, that is, in both policies most users will abandon due to shortage of vehicles, and therefore the system will perform the same under both policies.

Recall that the excess time rate function of the NR policy is composed of two components, the excess time rate due to abandoning and the excess time rate due to waiting. Next, we examine the tradeoff between the two components and compare them to the excess time rate function of the CPR policy, which is due to abandoning only. In Figure 3.3 we further examine the case where in the illustrative example, the demand rates in both directions are identical ($\lambda_{12} = \lambda_{21} = \lambda$) and the penalty ratio is set to $\alpha = 1$. We present the functions of the two components of the NR policy and their sum in black dotted, dashed and solid lines, respectively, and the excess time rate function of the CPR policy in a solid gray line. As the offered load increases, the excess time due to waiting decreases while the excess time due to abandoning increases. For this instance, the CPR policy is superior (lower excess time) over the entire examined range of offered loads. While for low loads the dominance is mainly due to the waiting times, for higher loads ($\lambda T > 0.8$), the excess time due to abandoning in the NR policy is higher by itself than in the CPR policy. This means that for relatively high loads, fewer users will abandon under the CPR policy.

![Figure 3.3: Excess time rate functions: NR policy (abandoning, waiting and total) and CPR policy](image)

$\lambda_{12} = \lambda_{21} = \lambda$

$\alpha = 1$
3.4. Simulation model

To further examine the effectiveness of the CPR policy, real world systems should be examined. Unfortunately, it is intractable to evaluate the excess time rate function for large size vehicle sharing systems. Moreover, in the Markovian models, a simple user behavior was assumed and the demand process was assumed to be homogenous throughout the day. Both assumptions do not fit real systems. In order to better model the complexities of real vehicle sharing systems we use discrete event simulation in which we relax the homogenousity assumption and present extensions made to the user behavior model. In Section 3.4.2 we describe a real-world bike sharing system that was used as a case study. For a clearer presentation we use throughout this section bike-sharing terminology. Specifically, parking spaces are referred to as lockers and the vehicles are simply bicycles. The alternative mode of transportation selected by the users is assumed to be walking.

3.4.1. Enhancing the user behavior model

The movement of users within the bike-sharing system is a complicated process. In particular, users may react in different ways to shortages of bicycles or lockers. Each reaction may project on a different group of users in the system. We assume that the users are strategic and that they make use of information available in the stations’ kiosks or accessed by their smartphones through the internet. Specifically, users have full knowledge of the travel times between each pair of stations, arrival rates of renters and real-time inventory levels. Each user is interested in reaching her destination in the shortest time. At decision points, due to an unfulfilled demand, we assume that the users choose the alternative that minimizes the expected remaining time in the system. The users are myopic, in the sense that they do not take into account the implications of possible changes in the inventory levels of intermediate stations during their journey.

The behavior models under the NR and the CPR policies are described in Figure 3.4(a) and 3.4(b), respectively. In Table 3.1 we further elaborate on the user decision processes, presenting the exact calculation made by a user at a decision point. Three types of decision points exist, denoted in Figure 3.4 and in Table 3.1 by I, II and III.

Under the NR policy, a user that does not find an available bicycle may choose to roam to a nearby station or walk directly to her destination (I). When a bicycle is rented the user rides to her destination. If upon arrival at the destination she finds an available locker, she returns the bicycle there and leaves the system. Otherwise, the user may either enter a waiting queue in that station or ride to a nearby station (II). If the bicycle is not returned at the originally desired destination, the user walks from the actual
returning station to the desired destination. The main difference from the NR policy is that once a bicycle is rented, the returning is ensured.

**NR Policy – User Behavior Model**

**CPR Policy – User Behavior Model**

![Figure 3.4: User behavior models in the NR and CPR policies](image)

Under the CPR policy, a user that does not find an available bicycle may choose to roam to a nearby station or walk directly to her destination (I). When a bicycle is available, if the user is unable to reserve a locker at her desired destination, she may choose either to reserve a locker in a station near her destination or to waive the service of the system altogether and walk directly to her destination (III). Again, if the bicycle is not returned at the originally desired destination, the user walks from the actual returning station to the desired destination. The main difference from the NR policy is that once a bicycle is rented, the returning is ensured.

We use the following notation to describe the user decision processes in Table 3.1:

- \( T_{ij} \) Expected riding time from station \( i \) to station \( j \) (as defined in Section 3.2)
- \( \lambda_i(t) \) Arrival rate of renters to station \( i \) at decision time \( t \) \( (\lambda_i(t) = \sum_{j=1}^{S} \lambda_{ij}(t)) \)
- \( W_{ij} \) Expected walking time from station \( i \) to station \( j \) \( (W_{ij} = T_{ij} + aT_{ij} \ \forall i \neq j, W_{ii} = 0 \ \forall i) \)
- \( Q_i \) Queue length in station \( i \) at the decision time
- \( B_i \) Number of bicycles in station \( i \) at the decision time
- \( L_i \) Number of available lockers (not occupied and not reserved) in station \( i \) at the decision time

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We assume that the riding times \((T_{ij})\) and walking times \((W_{ij})\) do not change along the day. For simplicity, we omit the index \(t\) from the state variables \(Q_i, B_i, L_i\).

### Table 3.1: Decision processes of users who face shortages of bicycles or lockers

<table>
<thead>
<tr>
<th>Situation</th>
<th>Question</th>
<th>Decision Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>User with destination (j) arrives at station (i) in which there are no available bicycles (NR, CPR)</td>
<td>Roam to a nearby station? (I)</td>
<td>Set (k^* = \arg \min_{k:B_k&gt;0} (W_{i,k} + T_{k,j})) If (W_{i,k^<em>} + T_{k^</em>,j} &lt; W_{i,j}) roam to station (k^*) (Yes) Else, walk to (j) (No)</td>
</tr>
<tr>
<td>User with destination (j) arrives with a bicycle to station (i), in which there are no vacant lockers (possibly (i = j)) (NR)</td>
<td>Wait for a vacant locker? (II)</td>
<td>Set (k^* = \arg \min_{k:L_k&gt;0} (T_{i,k} + W_{k,j})) If (T_{i,k^<em>} + W_{k^</em>,j} \geq (Q_i + 1)/\lambda_i(t) + W_{i,j}) wait in station (i) (Yes) Else, ride to station (k^*) (No)</td>
</tr>
<tr>
<td>User finds an available bicycle at station (i) but cannot make a reservation at her destination (j) (CPR)</td>
<td>Vacant locker in station near destination? (III)</td>
<td>Set (k^* = \arg \min_{k:L_k&gt;0} (T_{i,k} + W_{k,j})) If (T_{i,k^<em>} + W_{k^</em>,j} &lt; W_{i,j}) reserve a locker and ride to station (k^*), from there walk to (j) (Yes) Else, walk to (j) (No)</td>
</tr>
</tbody>
</table>

The total time a user spends in the system is taken as the difference between the time the user reaches her destination and her arrival time to the system. The excess time is obtained by subtracting the ideal time, the net riding time from the desired orig to the desired destination. We refer to the net riding time as the ideal time because this is the time the user spends in the system if she does not experience shortage of bicycles or lockers, the ideal situation.

#### 3.4.2. A real world system

As a case study we take the bike sharing system in Tel-Aviv, Tel-O-Fun, with 130 stations scattered in an area of about 50 square kilometers. A total of 2,500 lockers and 900 bicycles are dispersed in the system. Weekday rent transactions were collected over two months. Daily demand patterns did not change significantly throughout this period. The average number of daily trips was about 4,200.

By aggregating the transactions, we estimated the arrival rate of renters during 30 minute periods throughout the day. As may be expected, in most stations the demand process was not homogenous over time. For example, the demand for bicycles in stations located near working areas was low at the beginning of the day and increased significantly towards the end of the working day.

Riding times were estimated using the Google Maps API. For regular trips, it is safe to assume that most users ride directly from the origin to the destination. Indeed, the average time of all transactions was about 12 minutes. This is not the case for round-trips, where the average duration was approximately 30 minutes. Such trips were about 8% of all rent transactions. The penalty for an abandoning user was set to \(a = 1\), assuming that walking time is twice the riding time.
We note that in its current state, the information system of Tel-O-Fun cannot document information regarding abandonments. This is mainly due to the fact that when a user arrives at an empty station, she does not attempt to rent a bicycle and therefore is not identified by the system. Moreover, the system cannot tell apart users who returned bicycles at their desired destination and those who had to roam to a nearby station. To deal with this issue, we estimated the proportion of time a station was empty or full and inflated the demand rates accordingly.

3.4.3. Numerical results
The discrete event simulation, together with the user behavior model logic was coded in MathWorks Matlab™. The average daily bicycle usage (the total rent durations), was about 920 hours. In a system with 900 bicycles there are 21,600 available bicycle hours a day. That is, the average daily utilization was 4.2%. In addition, the average utilization at peak hours was 8.3%.

In order to test the capabilities of the system under various offered loads, we multiplied the measured offered load by the following factors: 0.5,1,2,4,8. This was done by multiplying uniformly all arrival rates by these factors (load multipliers). In addition, we used two starting points for the initial inventory level of bicycles at the beginning of the working day: (1) the actual initial station inventories on a randomly chosen day, after the operators executed repositioning activities. (2) the inventory levels prescribed by the method of Raviv and Kolka (2013).

For each load factor, we randomly generated 50 daily demand realizations, including renters’ arrival times to each station and their destinations. In order to reduce variation, we used the same realizations for each combination of policy and initial inventory (Common random numbers).

The simulation results are presented in Table 3.2 and Table 3.3 for the two starting inventory levels. In the first column we present the load multiplier and in the second we present the policy that was used. In the third and fifth columns we present the average total time spent by users in the system, over 50 replications, and the total ideal time, respectively. The absolute and relative excess times in each configuration are reported in the sixth and seventh columns, respectively. In the fourth and eighth columns we present, respectively, the relative reduction of the total time and of the excess time in the CPR policy as compared to the NR policy. In the ninth and tenth columns we present the percentage of users who received an ideal ride and the percentage of users who did not rent a bicycle at all, respectively.
Table 3.2: Simulation results for various system loads with initial inventory taken from actual random day

<table>
<thead>
<tr>
<th>Load Multiplier</th>
<th>Policy</th>
<th>Total Time In System (hr/day)</th>
<th>% Total Time In System Reduced</th>
<th>Total Ideal Time (hr/day)</th>
<th>Absolute Excess Time (hr/day)</th>
<th>%Excess</th>
<th>% Excess Reduced</th>
<th>Ideal Rides</th>
<th>Unserved</th>
<th>Fixed Penalty (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>NR</td>
<td>492.40</td>
<td>0.93%</td>
<td>459.29</td>
<td>33.11</td>
<td>7.21%</td>
<td>13.89%</td>
<td>86.23%</td>
<td>2.72%</td>
<td>45.98</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>487.80</td>
<td></td>
<td></td>
<td>28.51</td>
<td>6.21%</td>
<td></td>
<td>86.43%</td>
<td>3.00%</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>NR</td>
<td>1,009.76</td>
<td>1.33%</td>
<td>919.87</td>
<td>89.90</td>
<td>9.77%</td>
<td>14.99%</td>
<td>82.36%</td>
<td>3.84%</td>
<td>52.45</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>996.28</td>
<td></td>
<td></td>
<td>76.42</td>
<td>8.31%</td>
<td></td>
<td>82.78%</td>
<td>4.21%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>NR</td>
<td>2,108.86</td>
<td>2.58%</td>
<td>1,831.42</td>
<td>277.44</td>
<td>15.15%</td>
<td>19.62%</td>
<td>75.45%</td>
<td>6.28%</td>
<td>477.53</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>2,054.43</td>
<td></td>
<td></td>
<td>223.01</td>
<td>12.18%</td>
<td></td>
<td>77.18%</td>
<td>6.36%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>NR</td>
<td>4,554.72</td>
<td>3.91%</td>
<td>3,675.82</td>
<td>878.90</td>
<td>23.91%</td>
<td>20.27%</td>
<td>65.11%</td>
<td>11.68%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>4,376.54</td>
<td></td>
<td></td>
<td>700.71</td>
<td>19.06%</td>
<td></td>
<td>68.83%</td>
<td>10.58%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>NR</td>
<td>10,172.35</td>
<td>4.39%</td>
<td>7,335.99</td>
<td>2,836.37</td>
<td>38.66%</td>
<td>15.74%</td>
<td>51.83%</td>
<td>22.72%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>9,725.77</td>
<td></td>
<td></td>
<td>2,389.79</td>
<td>32.58%</td>
<td></td>
<td>55.93%</td>
<td>19.82%</td>
<td></td>
</tr>
</tbody>
</table>

It can be observed in both tables that for all the tested values of offered loads, the CPR policy outperformed the NR policy in terms of the total excess time (hence also in terms of total time in the system) and in terms of the percentage of ideal rides. This is accompanied by a slight decrease in the percentage of served users in low load configurations (0.5,1) and a significant increase in the percentage of served users in high load configurations (8). That is, the operator can achieve a significant improvement in the quality of service at a small loss of rent revenue when the offered load is relatively low, and even increase revenue when the offered load is high. Note that unserved users that are not able to make a reservation under the CPR policy, are likely to spend significant excess time looking for a vacant locker had the system allowed

Table 3.3: Simulation results for various system loads with initial inventory calculated according to the method of Raviv and Kolka (2013)

<table>
<thead>
<tr>
<th>Load Multiplier</th>
<th>Policy</th>
<th>Total Time In System (hr/day)</th>
<th>% Total Time In System Reduced</th>
<th>Total Ideal Time (hr/day)</th>
<th>Absolute Excess Time (hr/day)</th>
<th>%Excess</th>
<th>% Excess Reduced</th>
<th>Ideal Rides</th>
<th>Unserved</th>
<th>Fixed Penalty (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>NR</td>
<td>477.20</td>
<td>1.26%</td>
<td>459.29</td>
<td>17.91</td>
<td>3.90%</td>
<td>33.56%</td>
<td>93.00%</td>
<td>0.49%</td>
<td>28.22</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>471.19</td>
<td></td>
<td></td>
<td>11.90</td>
<td>2.59%</td>
<td></td>
<td>92.79%</td>
<td>1.11%</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>NR</td>
<td>979.91</td>
<td>1.93%</td>
<td>919.87</td>
<td>60.05</td>
<td>6.53%</td>
<td>31.46%</td>
<td>88.69%</td>
<td>1.39%</td>
<td>36.58</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>961.02</td>
<td></td>
<td></td>
<td>41.16</td>
<td>4.47%</td>
<td></td>
<td>88.92%</td>
<td>2.13%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>NR</td>
<td>2055.57</td>
<td>3.73%</td>
<td>1,831.42</td>
<td>224.16</td>
<td>12.24%</td>
<td>34.18%</td>
<td>80.51%</td>
<td>3.31%</td>
<td>78.04</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>1978.96</td>
<td></td>
<td></td>
<td>147.54</td>
<td>8.06%</td>
<td></td>
<td>82.42%</td>
<td>4.02%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>NR</td>
<td>4383.17</td>
<td>5.05%</td>
<td>3,675.82</td>
<td>707.35</td>
<td>19.24%</td>
<td>31.30%</td>
<td>71.19%</td>
<td>6.98%</td>
<td>559.14</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>4161.77</td>
<td></td>
<td></td>
<td>485.95</td>
<td>13.22%</td>
<td></td>
<td>75.00%</td>
<td>6.92%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>NR</td>
<td>9586.93</td>
<td>7.26%</td>
<td>7,335.99</td>
<td>2250.94</td>
<td>30.68%</td>
<td>30.93%</td>
<td>59.03%</td>
<td>15.09%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>8890.63</td>
<td></td>
<td></td>
<td>1554.65</td>
<td>21.19%</td>
<td></td>
<td>65.89%</td>
<td>11.75%</td>
<td></td>
</tr>
</tbody>
</table>
them to rent a bicycle. In the long run, better service is essential to retaining users and attracting new ones and hence both the users and the operator are likely to benefit from the CPR policy.

Next, we evaluate how the addition of a fixed penalty component per abandoning user (see discussion at the end of Section 3.2.1) affects the superiority of the CPR policy. To this end we calculate, for every configuration, the value of the fixed penalty for which the two policies breakeven. These values are presented in the last column of Table 3.2 and Table 3.3. The lowest calculated breakeven value is about 40 minutes, and in some configurations the value is infinity, i.e., for any fixed penalty the CPR policy outperforms the NR policy. This happens when the percentage of unserved users is smaller under the CPR policy. Given that an average duration of a rent is about 12 minutes, the superiority of the CPR policy continues to hold for any reasonable fixed penalty.

Another noticeable result is that excess times in Table 3.3 are lower per offered load and policy as compared to the figures reported in Table 3.3. That is, better results are obtained by setting the initial inventories according to the method of Raviv and Kolka (2013). Though this result is not a part of the contribution of this study, it addresses the issue of interaction between repositioning and reservation policies. As can be seen in Table 3.3, even if system operators carry out effective static repositioning, implementation of the CPR policy can further improve performance.

We note that when using the method of Raviv and Kolka (2013), the total initial inventory of bicycles was 1,268 bicycles, about half the total number of lockers in the system. Clearly, during the reviewed period the operators had not dispersed a sufficient number of bicycles in the system. To bridge this gap, we leveled the total inventory by uniformly adding bicycles to the actual initial stations’ inventories on the randomly chosen day. We ran the simulation with these initial inventories as well. The resulting system performance measures were similar to those of Table 3.2. Hence, the difference from Table 3.3 can be attributed to proper distribution of the bicycles at the beginning of the day.

Next we compare the two policies in terms of the station-availability performance measure. Recall that while under the NR policy a user cannot return a bicycle if the station is full, under the CPR policy a user cannot make a reservation if all the lockers are either occupied or reserved, i.e., the station is blocked. In Table 3.4 we present the average percentage of time a station is empty or full/blocke. In addition, we present the percentage of users who could not rent a bicycle upon their first attempt and the percentage of users who could not return a bicycle or make a reservation upon their first attempt. In the third to sixth columns we present these figures for the case where the initial inventories were taken from a random day. In the last four columns we give the figures for the case where the initial inventories were set according to Raviv and Kolka (2013).
Table 3.4: Percentages of unfulfilled first attempts to rent or return a bicycle

<table>
<thead>
<tr>
<th>Load Multiplier</th>
<th>Policy</th>
<th>Initial Inventory</th>
<th>Actual day</th>
<th>Raviv and Kolka (2013)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Empty</td>
<td>Full/Blocked</td>
<td>Unfulfilled in rent/return/reserve</td>
</tr>
<tr>
<td>0.5</td>
<td>NR</td>
<td>6.29%</td>
<td>1.47%</td>
<td>12.17%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>6.19%</td>
<td>1.39%</td>
<td>11.80%</td>
</tr>
<tr>
<td>1</td>
<td>NR</td>
<td>9.06%</td>
<td>2.25%</td>
<td>16.59%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>8.85%</td>
<td>2.06%</td>
<td>15.87%</td>
</tr>
<tr>
<td>2</td>
<td>NR</td>
<td>12.92%</td>
<td>3.76%</td>
<td>25.57%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>12.28%</td>
<td>3.03%</td>
<td>23.55%</td>
</tr>
<tr>
<td>4</td>
<td>NR</td>
<td>19.40%</td>
<td>5.20%</td>
<td>44.55%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>17.98%</td>
<td>4.10%</td>
<td>38.76%</td>
</tr>
<tr>
<td>8</td>
<td>NR</td>
<td>27.63%</td>
<td>6.14%</td>
<td>78.05%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>25.70%</td>
<td>5.00%</td>
<td>68.59%</td>
</tr>
</tbody>
</table>

For all simulated configurations the percentage of empty and full/blocked stations are lower under the CPR policy, that is, the CPR policy is preferable also under the station-availability performance measure. As discussed in Section 3.1, this performance measure does not truly reflect the quality of service given to the users because, among other flaws, it does not take into account the number of arriving users while the stations are empty or full. This is easily noticed in Table 3.4 when comparing the percentages that appear in the “Empty” columns with the “Unfulfilled in rent” columns as well as when comparing the percentages that appear in the “Full/Blocked” columns with the “Unfulfilled in return/reserve” columns. Moreover, the differences can be quite significant, to either direction.

Focusing now on unfulfilled requests, the percentage of users who cannot receive service due to lack of bicycles (Unfulfilled in rent) is lower under the CPR policy for all configurations. As may be expected, in some configurations the percentage of users who failed to make a reservation was higher than the percentage of users who failed to return a bicycle (unfulfilled in return/reserve). If the two columns are summed, we see that the percentage of unfulfilled requests is higher in the NR policy (except for when the load multiplier is 0.5 and the initial inventory is set according to the method of Raviv and Kolka (2013)). The situations in which users cannot return a bicycle are typically perceived as ones that cause more frustration to the users since they are "trapped" in the system. If indeed a higher priority is assigned to securing the ability of users to return the bicycles, the advantage of the CPR policy increases.

Lastly, we demonstrate that the above results are not sensitive to the penalty ratio. The above experiment was repeated using higher penalty ratios, namely $\alpha = 2, 4, 8$. Under these settings, the traveling times using alternative modes of transportation (e.g., walking) are three, five and nine times the riding times, respectively. The results of this experiment are reported in Table 3.5. In the third to sixth columns we present the
percentage of excess time spent by users in the system for the case where the initial inventories were taken from a random day. In the last four columns we present these values for the case where the initial inventories were set according to Raviv and Kolka (2013). Note that the total ideal times do not depend on the penalty ratio, and therefore they are identical, for each load multiplier, to those reported in Table 3.3. Observe that as the penalty ratio grows, the absolute and relative difference between the NR and CPR policy grows, that is, the superiority of the CPR policy increases. As the penalty ratio grows, more users will enter the system, that is, the effective load on the system will grow. This aligns with the results obtained by increasing the load multipliers.

Table 3.5: Percentage of excess time spent in the system for various penalty ratios

<table>
<thead>
<tr>
<th>Load Multiplier</th>
<th>Policy</th>
<th>Actual day</th>
<th>Raviv and Kolka (2013)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>α = 1</td>
<td>α = 2</td>
</tr>
<tr>
<td>0.5</td>
<td>NR</td>
<td>7.21%</td>
<td>13.04%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>6.21%</td>
<td>12.10%</td>
</tr>
<tr>
<td>1</td>
<td>NR</td>
<td>9.77%</td>
<td>18.06%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>8.31%</td>
<td>16.49%</td>
</tr>
<tr>
<td>2</td>
<td>NR</td>
<td>15.15%</td>
<td>28.20%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>12.18%</td>
<td>24.50%</td>
</tr>
<tr>
<td>4</td>
<td>NR</td>
<td>23.91%</td>
<td>46.42%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>19.06%</td>
<td>38.31%</td>
</tr>
<tr>
<td>8</td>
<td>NR</td>
<td>38.66%</td>
<td>79.40%</td>
</tr>
<tr>
<td></td>
<td>CPR</td>
<td>32.58%</td>
<td>67.59%</td>
</tr>
</tbody>
</table>

3.5. Discussion

This is the first study on parking space reservation policies in vehicle sharing systems. We view such policies as a tool to passively redirect demands and balance inventory levels. Specifically, we show that the excess time spent by users in a system managed under the CPR policy is lower compared to the excess time in the base policy, for a large range of offered loads. This is demonstrated by both an analytical analysis of Markovian models and by a simulation of a real world system.

Reservation policies also reduce the uncertainty related to the usage of vehicle sharing systems. The guarantee that a parking space will be available upon the return of the vehicle at the destination can save the time and anxiety associated with the possibility of having to search for a parking space.

Throughout this study, we compared CPR and NR policies. It may be worth examining “smarter” policies. For example: time limited reservation policies, station specific reservation policies or even user-based policies that involve reservations. In the CPR policy, a reserved parking space is unavailable until the vehicle is returned. It may be better not to allow the seizing of resources for too long. In addition, some users may prefer not to declare their destination in advance. If only some of the users reserve in
advance a parking space, “revenue management” questions rise. For example, how many parking spaces should be kept for users who happen to arrive at the station without pre-reservation? What kind of incentives should be awarded to users who reserved?

In addition to the advantages discussed in this chapter, we note that by placing or trying to place reservations, the users reveal information that is currently not available to the operators of vehicle sharing systems. Such information may be useful both for operational and strategic decisions. For example, information received via reservations can assist the operators in predicting the near future state of the system. This may allow better short term planning of repositioning activities. In addition, information on journeys that cannot be realized is collected. Such information is vital when planning capacity expansion of stations in the system.

In this study we have assumed that users act according to the dictated policy regulations. However, a user may decide to return the vehicle at a station different from the one in which the parking space was reserved. Furthermore, strategic users may declare a different destination than their true desired destination, simply to be able to rent a vehicle. It is interesting to examine the effects of such behavior on the performance of the system and to determine whether the system should allow such vehicle returns. These questions require further research.

To conclude, the findings of this paper suggest that incorporation of parking reservation policies in vehicle sharing systems will improve the quality of service given to the users. Technologically, incorporation of parking reservation policies merely requires minor software and, in some systems, hardware updates.
4. Performance bounds for parking reservation policies


In this chapter we extend our analysis of parking reservation policies beyond the complete policies (CPR, NR). Under the CPR policy, a reserved parking space remains empty until the user returns her vehicle. In the meantime, other users cannot use this resource, i.e., it is blocked. The tradeoff in implementing such a policy is that while some users are guaranteed an ideal service (since they will certainly be able to return their vehicle at their desired destination) other users may receive poorer service due to the blocking of parking spaces. That is, it might be beneficial not to require that all users make parking reservations.

Here, we examine whether and to what extent a further reduction in the total excess time may be achieved by any other passive regulation, and especially by any other parking reservation policy. Towards that, we use mathematical programming models to devise lower bounds on the total excess time that users spend in the system under any passive regulation and under any parking reservation policy. We consider the benefit of limiting the requirement to make a reservation to only some of the journeys. We refer to these policies as partial reservation policies, which combine the two extreme (complete) policies in different ways. We evaluate the performance of all policies and compare them to the lower bounds.

The contribution of this chapter is as follows. First, using mathematical programming models, we provide for the first time lower bounds on the performance of a vehicle sharing system, measured by the total excess time, under any passive regulation and under any parking reservation policy. Second, we introduce the concept of partial reservation policies. We examine three different partial policies, each is based on a simple sound principle, which is easy to control by the system’s managers and communicate to the users. We define the user behavior under these policies and examine their performance using discrete event simulation of real world systems. Third, we examine the potential benefit of parking space overbooking.

This chapter is organized as follows: in Section 4.1 a generic description of the VSS is presented and mathematical models are formulated to bound its performance under passive regulations and in particular under parking reservation policies. In Section 4.2 a behavior model of users in VSSs is presented and the proposed partial parking reservation policies are described. A utopian overbooking policy is presented at the end of this section. A description of two real world vehicle sharing systems and numerical results for their performance are presented and discussed in Section 4.3. Concluding remarks are provided in Section 4.4.

4.1. Lower bounds on the total excess time in a VSS

As mentioned in the introduction, the operational actions that a VSS operator can take in order to provide a high quality service can be of two forms, namely, active
regulations and passive regulations. In this study, we focus on passive regulations, i.e., mechanisms used for redirecting demand.

VSSs are decentralized systems, that is, each user makes decisions regarding her planned itinerary so as to minimize her own expected excess time. Such decisions depend on the availability of vehicles or parking spaces at the system’s stations at the renting time as well as on the user’s expectations regarding future availability. In addition, the user’s decisions are subject to the passive regulation prescribed by the system. Under passive regulations, the system may influence the users’ decisions by limiting their choices or by incentivizing them to prefer certain itineraries. However, the system does not assign itineraries to the users. For example, under the CPR policy, if a user cannot make a parking reservation at a certain station, she is not allowed to travel with a shared vehicle to that station, but the choice of her actual alternative itinerary is hers. From the operator’s point of view the question is: how to design a passive regulation so that the outcome of all users’ decisions minimizes the expected total excess time?

A passive regulation can be formally defined as a mapping of the state of the system and the demand for journeys to a set of itineraries allowed for each journey. The set of possible passive regulations is enormous. However, a major share of these regulations may be hard to implement or hard to communicate to the users. In this study we introduce and analyze regulations in the form of parking reservation policies, which are based on simple principles and are easy to communicate to the users. In order to assess the potential improvement that may be achieved by passive regulations in terms of the expected total excess time, we formulate mathematical programs that provide lower bounds. First, we devise a lower bound on the expected total excess time under any passive regulation. Second, since this study focuses on parking reservation policies, we devise a tighter bound specifically for any parking reservation policy.

The rest of this section is organized as follows: Section 4.1.1 includes a description of a VSS, and assumptions concerning the demand. Section 4.1.2 contains a mixed integer program (MILP) whose optimal value is a lower bound on the excess time that may be achieved by any passive regulation. Section 4.1.3 modifies the MILP formulation to account for passive regulations that involve only parking reservations, thus obtaining a tighter bound. A formal proof for the validity of this lower bound is presented.

4.1.1. Description of the VSS
In this section we discuss the information needed in order to model a VSS. Such information is used both in the mathematical models that are presented in this section, and in the user behavior model presented in Section 4.2. The information needed in order to describe the system is as follows:

- The number of stations in the system
- The number of parking spaces in each station (referred to as the capacity of the station)
- The initial inventory level (number of vehicles) at each station
- The expected travel time between any pair of stations using the shared vehicles.
The expected travel time between any pair of stations, using an alternative mode of transportation.

Note that information about the locations of the stations is not needed. In order to describe the relations between the stations, it is enough to specify the traveling time between each pair of stations. The distance between the stations, the topography of the city, congested roads and other considerations are all taken into account in the traveling times. In some situations, due to shortages in vehicles or parking spaces, users may roam to nearby stations (using an alternative mode of transportation) or may decide to abandon the system altogether and make their entire journey using an alternative mode of transportation. Therefore, the traveling times between any pair of stations using an alternative mode of transportation are also needed. The travelling time can represent any additional cost or inconvenience incurred by the user in her journey, in addition to the actual value of the time spent in the system. In bike sharing systems the alternative mode for most of the potential journeys is walking. This is also the case in car sharing systems for roaming between neighboring stations.

Each demand for a journey is defined by the desired origin and destination stations and by the desired starting time. An underlying assumption of the models introduced in this paper is that all the journeys in the system start and end at stations of the VSS. In reality, users’ journeys start and end at general locations (GPS points) in the city, however, such fine spatial granularity is not required for strategic decisions on reservation policies. Moreover, information regarding exact origins/destinations is currently not available for the operators of VSS. Finally, we assume that an alternative mode of transportation is always available to the users while the shared vehicle (resp., parking space) may or may not be available at the origin (resp., destination).

4.1.2. A lower bound on the total excess time under any passive system regulation

Our goal in this section is to establish a lower bound on the total excess time that results from the users’ decisions under any passive regulation. Given the system's characteristics and a demand realization for journeys during a predetermined planning horizon (typically a day), we formulate an optimization problem that centrally selects the itineraries of the VSS's users so as to minimize the total excess time.

The solution value of this optimization problem is a lower bound on the total excess time that may be achieved by any passive regulation, due to the following two assumptions on which the optimization problem is based:

1. All demands for journeys are known in advance.
2. A central planner determines the itinerary of each user, in a way that benefits the entire system. The justification of this assumption is that any solution selected by the central planner may be selected by the users under some passive regulation.

In practice, each user determines her own itinerary, according to her individual objective and given the information she has. Thus, the excess time of the optimal assignment obtained by a central planner with full information is a lower bound on the excess time that results from any passive regulation policy, i.e., a policy that affects the itinerary selection of the users in some restricted way. We note that due to the system's
limited resources, this bound is typically strictly positive, thus it is better than the trivial bound of zero excess time (no shortages of any type).

In practice the demand for journeys is a stochastic process. Therefore, the average solution value of the optimization problem for numerous demand realizations, drawn from a given stochastic process, provides an estimator for a lower bound on the expected total excess time under any passive regulation.

A demand realization is described by a set of journeys where each journey is characterized by an “origin-destination-time” tuple. Each journey can be materialized by one of several possible itineraries. We assume that a possible itinerary is in one of the following forms:

1. Use a shared vehicle all the way from the origin to the desired destination.
2. Use a shared vehicle from the origin station to another station with an available parking space and then use an alternative mode to get to the desired destination.
3. Use an alternative mode of transportation to get to a station with an available vehicle and then use a shared vehicle from this station to the desired destination.
4. Use an alternative mode of transportation to get to a station with available vehicle and then use a shared vehicle from this station to another station with an available parking space. Then, from this station use an alternative mode to get to the desired destination.
5. Use an alternative mode all the way from the origin to the desired destination.

We refer to the station where the vehicle is actually rented (resp., returned) as the renting (resp., returning) station. Upon attempting to return a shared vehicle, the user may be required to wait at the returning station until a parking space becomes available, and then proceed with her itinerary (leave the system or continue with an alternative mode of transport). We assume that the user will not wait for a vehicle to become available in a renting station, since information regarding the number of vehicles in each station is available to her in real time. Instead, she would roam to a nearby station or use an alternative mode of transportation for the entire journey.

In Figure 4.1 we present an example with several possible itineraries that materialize the journey of a user who wishes to travel from station A to station B. The travel time of each segment of the journey is depicted on the corresponding arc and the itinerary's excess time (denoted by X) is presented below each graph. For example, in Figure 4.1(b) the excess time is 5 since the travel time is 12, as compared to a travel time of 7 in the ideal journey, see Figure 4.1(a). In Figures 4.1(a) – 4.1(e), we present examples for each of the five forms of itineraries presented above, respectively. Note that since the excess time associated with using an alternative mode of transportation for the entire journey, as in 4.1(e), is 14, itineraries with larger excess time, such as 4.1(f), will never be selected by a user and thus can be excluded from consideration of the central planner.

Each possible itinerary can be defined by its renting station, returning station and renting time. The returning time is determined by the renting time and the traveling time between the two stations. Waiting times at the returning stations are not considered as part of the itinerary times, they are calculated separately. In addition, a journey can be
materialized by an itinerary that includes only an alternative mode of transportation. Clearly, such an itinerary is not associated with renting and returning stations.

We define a set of possible events, where each event is a time-station tuple that refers to a renting or returning time and location of a possible itinerary. We assume without loss of generality that at most one event can occur at each station at a given time. At the time of each event the state of the corresponding station is defined by the number of vehicles parking at the station and the number of users that are (possibly) waiting to return their vehicle at the station.

The assignment of itineraries to users, carried out by the central planner, is constrained by several considerations that are related to the availability of vehicles and parking spaces at the stations. In Figure 4.2, we use a network flow graph to present the movement of vehicles in the system over time. We use the possible itineraries of the journey depicted in Figure 4.1 as an example, assuming that the journey starts at time 20. Each possible itinerary that includes a movement of a vehicle [itineraries (a)-(d) of our example] is depicted by a black solid arc from a node that represents the renting time and location to a node that represents the returning time and location. The costs of these arcs are the excess times associated with their itinerary and their capacities are 1. The use of an alternative mode of transportation is not reflected directly by arcs in the network. However, the times of the nodes and the costs of the arcs are affected by the use of the alternative modes of transportation. For example, in itinerary (c), depicted in both Figure 4.1 and Figure 4.2, the use of a shared vehicle begins at station D at time 26.
although the itinerary starts at station A at time 20. The cost of this arc is 5, which represents the excess time of the itinerary.

To represent an entire demand realization we construct a network as in Figure 4.2, with a set of nodes and arcs for all possible itineraries of all demanded journeys. The nodes in this network correspond to events. Each pair of consecutive nodes on the time axis, that are associated with the same station, is connected by two “horizontal” parallel arcs. The solid gray arc represents the number of vehicles parked in the station between the two events and the dashed arc represents the number of vehicles (and drivers) waiting in the station for a vacant parking space, during this time interval. Since the pair of nodes is consecutive, the number of vehicles parking and waiting in the station does not change during this time interval. The costs of the parking arcs are zero and their capacity equals the capacity of the station. The per unit cost of the waiting arcs equal to the time difference between their end nodes and their capacity is not limited. For example, if the flow on the waiting arc that connects nodes (B,27) and (B,32) is 3 then the excess time due to waiting in station B between time periods 27 and 32 would be 15.

The network also includes one source node for each station, with a supply that represents the initial inventory in the station and one sink node. The net demand of the rest of the nodes is zero. A feasible assignment of itineraries to journeys is obtained as a feasible integer flow on this network with additional side constraints. These constraints limit the total flow on all the itinerary arcs associated with each journey to at most 1. A solution where the total flow on the arcs associated with a certain journey is zero represents selection of an itinerary that uses an alternative mode only, e.g. Figure 4.1(e). The excess time incurred in such a solution is the sum of the flow costs plus the costs due to the journeys that use alternative modes only. Thus, our lower bound is obtained by minimizing this excess time. We solve this optimization problem using an MILP that is formulated below. Next, the notation required to formulate this model is presented.
Indices:
\[ s \] Station
\[ t \] Time
\[ j \] Journey
\[ i, k \] Itinerary

Parameters:
\[ S \] Set of stations
\[ J \] Set of journeys
\[ C_s \] Capacity of station \( s \)
\[ L_s^0 \] Initial vehicle inventory in station \( s \)
\[ E \] Set of possible events \( [(s, t) \text{ tuples}] \)
\[ I_j \] Set of possible itineraries of journey \( j \), we also use \( I \equiv \bigcup_{j \in J} I_j \)
\[ X_i \] The excess time of itinerary \( i \) (not including waiting time)
\[ D_{s,t} \] The time difference between event \((s, t)\) and the next event at station \( s \)
\[ B(s, t) \] Set of itineraries in which a vehicle is rented at station \( s \) at time \( t \)
\[ F(s, t) \] Set of itineraries in which a vehicle is returned at station \( s \) at time \( t \)
\[ (s, t)' \] The event that precedes event \((s, t)\) at station \( s \)

In addition, we define two artificial events \((s, 0)\) and \((s, H)\) for each station \( s \), that denote the beginning and the end of the planning horizon, respectively. Note that \( X_i \) represents the excess time associated with selecting itinerary \( i \). This excess time includes the additional time incurred by using alternative modes of transportation to materialize the entire journey or part of it. It does not include additional excess time that the user may experience due to waiting for a vacant parking space at the returning station. This waiting time is reflected by the parameter \( D_{s,t} \). Without loss of generality the set \( I_j \) consists only of journeys with excess times that are not greater the excess time of using alternative mode for the entire journey.

Decision variables:
\[ r_i \] 1 if itinerary \( i \) is selected, 0 otherwise
\[ p_{s,t} \] Number of vehicles parking at station \( s \) right after event \((s, t)\)
\[ w_{s,t} \] Number of users waiting to return a vehicle at station \( s \) right after event \((s, t)\)

With respect to the network flow model, the \( r_i \) variables represent the flows on the itinerary arcs. The \( p_{s,t} \) variables represent the flows on the parking arcs and the \( w_{s,t} \) variables represent the flows on the waiting arcs. The problem can now be formulated as an MILP model. We refer to this model as the Passive Regulation Lower Bound (PR-LB).
\[
\text{minimize} \sum_{i \in I} X_i \cdot r_i + \sum_{(s,t) \in E} D_{s,t} \cdot w_{s,t}
\]

Subject to

\[
\sum_{i \in I_j} r_i = 1 \quad \forall j \in J
\] (4.2)

\[
p_{(s,t)} + w_{(s,t)} + \sum_{i \in F(s,t)} r_i = p_{s,t} + w_{s,t} + \sum_{i \in B(s,t)} r_i \quad \forall (s,t) \in E
\] (4.3)

\[
p_{s,0} = 1^0_s \quad \forall s \in S
\] (4.4)

\[
p_{s,t} \leq C_s \quad \forall (s,t) \in E
\] (4.5)

\[
w_{s,0} = 0 \quad \forall s \in S
\] (4.6)

\[
w_{s,H} = 0 \quad \forall s \in S
\] (4.7)

\[
r_i \in \{0,1\} \quad \forall i \in I
\] (4.8)

\[
p_{s,t} \geq 0 \quad \forall (s,t) \in E
\] (4.9)

\[
w_{s,t} \geq 0 \quad \forall (s,t) \in E
\] (4.10)

The objective function (4.1) sums over the excess time of the selected itineraries and the waiting times of all the users who wait to return their vehicle at their returning station. These are the two components of the total excess time of all the users in the system. Constraints (4.2) assure that for each journey exactly one itinerary is selected. Constraints (4.3) are vehicle inventory balance equations. Constraints (4.4) set the initial vehicle inventory in each station. Constraints (4.5) limit the number of parked vehicles in a station to the capacity of the station. Constraints (4.6) and (4.7) state that no user is waiting to return a vehicle at the beginning or at the end of the planning horizon. Constraints (4.8) stipulate that the itinerary decision variables are binary. Constraints (4.9) and (4.10) are non-negativity constraints on the number of parked vehicles and waiting users after each event.

In this model the central planner may assign a user with any of its given potential itineraries. In some cases the users may be referred to relatively distant rent or return stations, merely in order to balance the vehicle inventories, for the system’s benefit, and not necessarily because the system cannot satisfy their demand by better itineraries. In the next section we extend the model to limit such occurrences.

Theoretically, a user may begin her ride and return the vehicle at any station. Therefore, the number of potential itineraries of a journey is the square of the number of stations. However, most of these potential itineraries would take longer than simply using the alternative mode of transportation for the entire journey (i.e., abandoning the system). Under many regulations it is safe to assume that users will not accept such itineraries. In the numerical experiment reported in Section 4.3, we let the central planner consider only potential itineraries that are not longer than using the alternative mode of transportation for the entire journey. Moreover, in order to reduce the
computational effort required for solving the PR-LB model (4.1)-(4.10), we relaxed the integrality constraints (4.8) and replaced them with non-negativity constraints. This clearly preserves the result as a lower bound. In our numerical experiment we observed that the effect of this relaxation on the obtained lower bound is negligible, see the discussion in Section 4.3.

When restricting ourselves to itineraries that are not longer than using the alternative mode of transportation, an alternative lower bound on the total excess time could be obtained by including the possible waiting times at the destination within the itineraries and removing the waiting variables \( w_{s,t} \) from the model. In such a formulation we could allow only sequences of travel and waiting times that \( together \) are shorter than using the alternative mode of transportation for the entire journey. This further limits the decision space of the central planner and thus may result in a tighter lower bound. However, such an approach would result in a significantly larger number of possible itineraries (and thus decision variables). In the instances that we have solved, we observed that the total waiting time was negligible, as compared to the total excess time. Therefore, we believe, that the potential improvement of the lower bound is not significant.

Though this study focuses on parking reservation policies, the above model serves as a lower bound on the excess time under any passive regulation. In particular, since the input for this model consists of all the demand for journeys it can also serve as a lower bound for vehicle reservation policies, trip reservation policies, the best of two regulation proposed by Fricker and Gast (2014), and the pricing regulations proposed by Chemla et al (2013b) Pfrommer et al. (2014).

4.1.3. A lower bound on the total excess time under any parking reservation policy

In this section we focus on a subset of all possible passive regulations, namely, parking reservation policies. A parking reservation is a process in which, when attempting to rent a vehicle, the user declares on her destination and the trip is either allowed or denied by the system. If allowed, a parking space is reserved for the user at the desired destination. If denied, the user may try to place reservations to other destinations until one is allowed. A parking reservation policy is a set of rules which determine: in which subset of trips the user is required to place reservations, whether a reservation request is allowed or denied and when allowed, whether the parking space is reserved for the user temporarily or permanently (until her arrival to the destination). The operator is allowed to overbook parking spaces that are currently not available. However, for a parking reservation policy to be enforceable and sustainable over time, the operator must not deny parking reservation requests unjustifiably. Next, we formally define such a set of parking reservation policies, to be studied in this paper.

**Definition 4.1** (A parking reservation policy). A passive regulation in which the operator can deny renting a vehicle only if there are no reservable parking spaces at the destination at the renting time.

Recall that under a parking reservation policy, reservation is not always required, but when it is required, the condition of definition 4.1 must hold. For example, the CPR
and NR policies are both legitimate parking reservation policies. In the CPR policy, reservation of a reservable space is always required. The NR policy satisfies the requirement of definition 4.1 trivially, since under this policy no reservation is required and thus reservations are never denied.

Under any parking reservation policy, the set of possible itineraries that can materialize journey $j$, $I_j$, can be partitioned into three subsets given the state of the system when the journey begins: (I) Itineraries that cannot be denied by a parking reservation policy. This set includes any itinerary with an available vehicle at its renting station at its renting time and a reservable parking space at its returning station at the renting time. In addition, the itinerary that consists of the alternative mode only is always included in this set, since it is also an itinerary that cannot be denied. (II) Itineraries that can be either denied or allowed by a parking reservation policy. This set includes any itinerary with available vehicle in its renting station at its renting time but no reservable parking space at its returning station at the renting time. Assignment of itineraries from this set may be materialized in overbooking policies or in partial reservation policies where some users start their journey without reservations at all. (III) Itineraries that cannot be allowed by a parking reservation policy. This set includes any itinerary with no available vehicles at its renting station at its renting time. The parking reservation policy dictates which of the itineraries in (II) are available to the user. The user, from her side, selects the itinerary from (I) or from the allowed itineraries in (II) that minimizes her excess time.

Note that under a general passive regulation, the system may offer to the user any subset of itineraries from the union of (I) and (II), as long as this subset contains the itinerary that uses the alternative mode only. However, under a parking reservation policy the offered subset must include all the itineraries in (I) and possibly some of the itineraries in (II). Thus, in these policies the system has less control over decisions of the users.

The PR-LB model (4.1)-(4.10) is modified such that the central planner may assign to each journey either the shortest itinerary in (I) or a shorter itinerary from (II). Recall that in the original model, any itinerary from the union of (I) and (II) could be assigned. The partition of potential itineraries to the sets (I), (II), and (III) cannot be pre-defined as an input to the model. This is because the selection of the itineraries that are included in these subsets depends on the system’s state at the decision time and on all the decisions that were made for journeys that begin prior to that journey. Instead, we modify the PR-LB model (4.1)-(4.10) by adding sets of decision variables and constraints so as to properly assign itineraries that users who minimize their excess time would actually choose under some parking reservation policy. We refer to this extended model as the Parking Reservation Policy Lower Bound model (abbreviated PRP-LB). We use the same notation as in the PR-LB model (4.1)-(4.10) and add the following parameters and decision variables:
Parameters:

- \( O(i) \): A \((s, t)\) tuple that represents the renting station and renting time of itinerary \( i \)
- \( D(i) \): A \((s, t)\) tuple that represents the returning station and returning time of itinerary \( i \)
- \( J(i) \): The journey that can be materialized by itinerary \( i \)
- \( T(s, t) \): Time of node \((s, t)\)
- \( S(s, t) \): Station of node \((s, t)\)
- \( \mathcal{R}_i \): A set of itineraries for which a parking space may be reserved at the returning station of itinerary \( i \) at the renting time of itinerary \( i \). That is, an itinerary \( k \) is in the set if:
  - It is of a different journey, \( J(k) \neq J(i) \)
  - It has the same returning station as itinerary \( i \), \( S(D(k)) = S(D(i)) \).
  - The renting time of itinerary \( k \) is earlier than the renting time of itinerary \( i \), \( T(O(k)) < T(O(i)) \)
  - The returning time of itinerary \( k \) is later than the renting time of itinerary \( i \), \( T(D(k)) > T(O(i)) \)
- \( M \): A very large number (for example, twice the capacity of the largest station)

Auxiliary decision variables:

- \( e_{s, t} \): 0 if a vehicle is available at station \( s \) at time \( t \), otherwise it can either be 0 or 1.
- \( f_i \): 0 if at renting time \( T(O(i)) \) there are some reservable parking spaces at station \( S(D(i)) \), otherwise it can either be 0 or 1.

The PRP-LB model can be stated now as (1)-(10) with the following additional constraints:

\[
\begin{align*}
 e_{O(i)} + f_i &\geq r_k \quad \forall i, k \in I_j : X_i < X_k \quad \forall j \in J \\
 M \cdot (1 - e_{s,t}) &\geq p_{s,t} + w_{s,t} \quad \forall (s, t) \in E \\
 C_{S(D(i))} \cdot f_i &\leq p_{D(i)} + w_{D(i)} + \sum_{k \in \mathcal{R}_i} r_k \quad \forall i \in I \\
 e_{s,t} &\in \{0,1\} \quad \forall (s, t) \in E \\
 f_i &\in \{0,1\} \quad \forall i \in I
\end{align*}
\]

Constraints (4.11) stipulate that each journey is materialized by the shortest possible itinerary, i.e., the one with the shortest excess time that is allowed by some parking reservation policy. For any itinerary \( k \), if an itinerary \( i \) of the same journey with shorter excess time that belongs to (I) exists then itinerary \( k \) cannot be selected. Recall that if
itinerary \( i \) is in (I) then a vehicle is available at its renting station \( (e_{0(i)}=0) \) and a parking space is available at its returning station \( (f_i=0) \). In this case the left hand side of (4.11) is zero and thus \( r_k \) is forced to be zero. Note that if itinerary \( i \) is in (II) the right hand side of (4.11) is greater than zero. In this case, the model may or may not assign itinerary \( k \) to the journey. By constraints (4.12), a station can be considered "empty" in a given time, only if there are no vehicles parked or waiting in it at that time. Constraints (4.13) assure that the \( f_i \) variables are set to zero if there are reservable parking spaces at the returning station of itinerary \( i \) at the renting time. The decision variable \( w_{s,t} \) is added to the right hand side of constraints (4.12) and (4.13) in order the make sure that the central planner will not “leave” vehicles waiting outside a non-full station in order to gain more flexibility in selecting possible itineraries. Constraints (4.14) and (4.15) stipulate that the variables \( e_{s,t} \) and \( f_i \) are binary.

The value of the solution of the PRP-LB model (4.1)-(4.15) provides a tighter bound on the total excess time as compared to the PR-LB model (4.1)-(4.10). This is because it is based on a super-set of its constraints and since parking reservation policies are a subset of any passive regulation. As in the case of PR-LB, this model was solved while relaxing the binary variables \( r_i \). The binary variables \( e_{s,t} \) and \( f_i \) were not relaxed since without imposing their integrality the resulting relaxation is very weak. This is due to the effect of the big-M terms in constraints (4.12) and (4.13). Indeed, this model is more difficult to solve (see Section 4.3).

In the PRP-LB model, unlike the PR-LB, if vehicles are available at the station at the arrival time of a renter, the system must offer one to the user. Therefore, this model cannot provide a lower bound for the performance of vehicle or trip reservation policies. Next, we formally prove the validity of the optimal solution value of PRP-LB as a lower bound on the total excess time under any parking reservation policy.

**Proposition 4.1**: For any demand realization, the total excess time associated with the optimal assignment of itineraries to journeys under PRP-LB is not greater than the excess time under any parking reservation policy.

**Proof**: Consider the assignment of itineraries to journeys obtained by some parking reservation policy (that satisfies the condition of Definition 4.1). We refer to this assignment as \( A^* \). We claim that such an assignment can be mapped to a feasible solution of PRP-LB and therefore the optimal solution of PRP-LB is a lower bound on the excess time that results from any parking reservation policy. First note that since \( A^* \) is a feasible assignment of itineraries to journeys, it must satisfy constraints (2)-(10) when setting the \( r_i \) variables to represent the actual itineraries that were selected by the users under policy \( A^* \) and setting the values of the variables \( p_{s,t} \) and \( w_{s,t} \) to represent the number of vehicles that are parking and waiting at the stations after each event \( (s,t) \), respectively. Next, we will show that the values of the binary variables \( e_{s,t} \) and \( f_i \) can be set such that the rest of the constraints of PRP-LB can be satisfied. First we set the value of \( e_{s,t} \) as follow

\[
e_{st} = \begin{cases} 
0 & p_{s,t} + w_{s,t} > 0 \\
1 & \text{otherwise}
\end{cases}
\]
Such an assignment would immediately satisfy constraints (12) for each event \((s, t)\). Similarly we set

\[
f_i = \begin{cases} 
0 & p_{D(i)} + w_{D(i)} + \sum_{k \in R_i} r_k < c_{S(D(i))} \\
1 & \text{otherwise}
\end{cases}
\]

which immediately satisfy constraints (4.13) for each itinerary \(i\). Now, it is left to show that with this assignments constraint (4.11) is satisfied for each pair of itineraries of the same journey \((i, k)\) such that \(k\) is selected under policy \(A^*\) and \(X_i < X_k\), that is, itinerary \(i\) has a shorter excess time than itinerary \(k\). Recall that if \(k\) was selected, then \(r_k = 1\). Assume by contradiction, that constraint (4.11) is violated, which implies \(e_{0(i)} = 0\) and \(f_i = 0\). This means that for itinerary \(i\) a vehicle was available and a reservable parking space was available at the renting time. By Definition 4.1, such an itinerary could not be denied by a parking reservation policy. Finally, since it is shorter than itinerary \(k\), it must have been selected by the user, which is a contradiction. ■

By proposition 4.1, the assignment of itineraries that can result from any parking reservation policy under any demand realization is a feasible solution of PRP-LP. Thus, the excess time that can be achieved by any parking reservation policy is bounded from below by the optimal solution of the model.

### 4.2. Additional parking reservation policies

The lower bounds developed in the previous section may be used to evaluate the effectiveness of any regulation or parking reservation policy. In this section, we introduce several parking reservation policies. The performance of a VSS under these policies or under any other regulation can be evaluated only with respect to the response of the users to the rules prescribed by the regulation. We base our analysis, with respect to users’ response, on an axiomatic approach and model the users as rational independent agents whose goal is to minimize their own excess time. However, achieving this goal may be too complex for many users due to the stochastic nature of the VSS. Therefore, we postulate a user behavior model that heuristically approximates this minimization problem and, in fact, provides an optimal solution in most of the cases.

In Section 4.2.1 we present this user behavior model. The model describes the decisions taken by the users at different decision points. These decisions are affected by the state of the system and the settings of the regulation. In Section 4.2.2 we present three partial reservation policies, discuss the motivation for using them and explain how they are reflected in the user behavior model. In Section 4.2.3 we present a utopian parking overbooking policy, whose performance is used to gauge the potential benefit of parking overbooking policies.

#### 4.2.1. User behavior model

The movement of users in the system depend both on its regulation and on the state of the system (availability of vehicles and parking spaces). A user who enters the system acts as follows: If there are no available vehicles at her origin station, she may either
decide to go to a nearby station, using an alternative mode of transportation, in search for an available vehicle, or she may decide to abandon the system. An abandoning user is assumed to travel to her destination using an alternative mode of transportation. Note that in modern VSS, the user can make this decision based on real time information about the availability of vehicles in the stations of the system. Once a user finds an available vehicle, there are two options: (1) A parking reservation is not required and (2) a parking reservation is required. In option (1), the user rents a vehicle and travels to her destination. When the user reaches her destination (with a vehicle), if she finds an available parking space she returns the vehicle and exits the system. If there is no available parking space at the destination station, the user may decide to wait at the station until a parking space becomes available (i.e. she enters a waiting queue). Alternatively, the user may decide to roam to a nearby station in search for an available parking space. Again, this decision is based on real time information on the availability of parking spaces in the stations. In option (2), the user attempts to make a parking reservation in her destination station. If the reservation is approved, the user makes a rent-and-reserve transaction and travels to her destination station. If the parking reservation is guaranteed, the user can immediately exit the system upon reaching her destination. If the reservation is not guaranteed, the user travels to the returning station and proceeds as in option (1). If the parking reservation is not approved, the user can either try to make a reservation at another station near her destination or she may decide to abandon the system. Finally, if, for some of the above reasons, the vehicle is returned to a station different than the user’s destination, the user uses an alternative mode of transportation to reach her destination station and then exits the system.

This behavior model is described in Figure 4.3. At decision points, we assume that users have full knowledge of the system state, including inventory levels at each station and the arrival rate of renters to each station (for example, the operator, or a third party, can provide this information via a smartphone application). Users are assumed to be strategic so that at decision points they will choose the alternative that minimizes their expected remaining traveling time. An alternative user behavior model could be based on the maximum utility theory, introducing randomness in the itinerary selection decisions and reflecting factors that are not included in the current model. However, we use a deterministic itinerary selection model that is based solely on excess time, because it is based on data that is readily available to the operators. We believe that such a model is sufficiently accurate to provide insights on the effect of various parking reservation polices.
We further elaborate on the user decision processes, denoted in Figure 4.3 by I, II and III:

I. A renter who arrives at a station with no available vehicles would consider a nearby station such that the total time spent using an alternative mode of transportation.
transportation to get to that station and the traveling time from that station to the
destination, is the shortest among all stations with available vehicles. The user
would choose an alternative mode of transportation for the entire journey, if it is
shorter than the above alternative.

II. A renter who arrives with a shared vehicle to a station with no available parking
spaces would consider a nearby station such that the total time spent traveling
with the shared vehicle to that station and using an alternative mode of
transportation from there to the destination is the shortest among all stations
with available parking spaces. The user would choose to wait in the station until
a parking space becomes available, if the expected time until this happens is
shorter than the above alternative.

III. A renter who cannot make a parking reservation at the destination station would
consider making reservation at a nearby station such that the total time spent
traveling with the shared vehicle to the chosen returning station and using an
alternative mode of transportation from there to the destination is the shortest
among all stations for which it is possible to make parking reservation. The user
would choose using an alternative mode of transportation for the entire journey
if it was shorter than the above alternative.

In the user behavior model, there are three junctions that represent the settings of the
policy:

- Parking reservation required?
- Reservation approved?
- Vacant parking space guaranteed?

To highlight these junctions, we plot them in Figure 4.3 as trapezoids. The NR and CPR
policies, are complete in the sense that under each of these policies the answer to each
of the above three questions is identical for all the users of the system. For example,
under the CPR policy, all users are required to make a parking reservation, the
reservation is approved if at the renting time there is an available parking space at the
returning station and a vacant parking space is guaranteed to all users who are able to
make a parking reservation.

4.2.2. Partial parking reservations policies
In this section we present three types of partial parking reservation policies. Each type is
based on a simple, yet reasonable principle. The common motivation for these policies
is to enforce parking reservations only when they are likely to make a positive effect on
the performance of the system. In the descriptions below, a trip is defined as a direct
ride between a pair of origin-destination stations.

Trip based partial reservation policy
Under this policy, parking reservations are required only for trips with expected
traveling times shorter than a given threshold. At rent, a user will declare her destination
and if the expected traveling time is shorter than the given threshold she will be required
to reserve a parking space in her destination. As in the CPR policy, if at the renting time
there is no vacant parking space at the destination, the transaction is denied and the user
may try to make a reservation at a different station. A user with expected riding time longer than the threshold time, will not be required to make a parking space reservation. If such a user finds an available vehicle at her origin, she will be able to rent it and ride to her destination, as in the NR policy. The rationale behind this policy can be stated as follow: since a parking space is a valuable resource in a VSS and a reservation practically blocks it for the duration of the trip, the parking space should be reserved only for short trips. Moreover, users with short traveling times may be more sensitive to excess time due to shortage of parking space at the destination.

Note that if the threshold time is set to zero, this policy coincides with the NR policy. On the other hand, if the threshold is set to a large enough value this policy coincides with the CPR policy. Different partial polices of this type can be obtained by setting the value of the threshold parameter between the two extremes.

**Station based partial reservation policy**

Under this policy a parking reservation is required only if the difference between the expected returning and renting rates at the destination station during a certain time interval is higher than a pre-specified value, referred to as the *difference threshold*. Otherwise, no reservation is required. Expected renting and returning rates can be estimated using past transactions. The difference is calculated for each station during predefined time intervals of each day. If the calculated difference is lower than the *difference threshold* the user will behave as under the NR policy.

The rationale behind this policy can be stated as follows: the probability of parking space shortages in a station grows as the imbalance (difference) between the demand rates for parking spaces and for vehicles grows. Such imbalances may be consistent, for example, in bike sharing stations at relatively low altitude locations, where bicycles are more likely to be returned than rented. Or, the imbalance may change during the day, for example, in stations located at working areas where returning in the morning is much more prevalent than renting. When the demand rate for parking spaces (returning) is higher than the demand rate for vehicles (renting), there is a greater chance that users will find the station full. By enforcing parking reservations at such stations, the system prevents users from traveling to stations with no available parking spaces. This will shift some of the users to nearby, less congested stations. Such a shift would have probably happened anyway, since users who find a full station typically roam to a nearby station in order to return their vehicle. With parking reservations, the change in the retuning station is determined in advance, which is likely to reduce the users’ excess time. On the other hand, it seems less effective to enforce parking reservations in stations that are likely to be empty anyway.

Note that the higher the difference threshold is, the fewer the cases in which reservations are required. For extremely high threshold values the policy coincides with the NR policy, while for extremely low (negative) values it coincides with the CPR policy.
Time limited partial reservation policy
Under this policy all users are required to make a parking reservation, as in the CPR policy, but this reservation is valid for a limited time. After the reservation expires, the reserved parking space becomes available to other users, and a vacant parking space is no longer guaranteed to the user. If the reservation expires and no parking space is available by the time the user arrives at the destination station, she will have to either wait by the station or roam to a nearby station (as in the NR policy).

The rationale behind this policy can be stated as follows: by making a reservation, a user with a long traveling time that reaches her destination only after her reservation expires, still affects the system because as long as her reservation is valid, she may block other users from making a reservation. That is, the reservation may divert subsequent demand, which may increase the probability of the user to find a vacant parking space, even if her reservation expired.

Note that if the time limit is set to a large enough value, this policy coincides with the CPR policy. However, if the time limit is set to zero, the resulting policy is still different from the NR policy. This is because users still must make a reservation, and they cannot begin traveling to a station that is full at the renting time. In Section 4.3 we compare the performance of this specific setting (in which the time limit is zero) to the performance of the NR policy and discuss the differences and their implications.

4.2.3. Utopian parking space overbooking policy
In many service systems in which reservations are carried out, it is a common practice to allow overbooking. It means accepting reservations for resources that, based on previous reservations, will not be available at the required time. Overbooking may be an effective policy in the presence of stochasticity in the arrival or service process that expects no-shows of customers who made reservations. In a VSS that practices parking reservations, no-shows are not an issue because the reservations are made at the renting time and the users must return the vehicle at the stated destination. Nevertheless, it might be beneficial to allow in some cases for users to travel to a station even if it has no available parking spaces, or in other words, to allow overbooking. This is because a parking space may become available by the time the user reaches her destination station. A good overbooking policy is based on reliable forecast that is capable of predicting such occurrences.

In order to evaluate the potential benefit of overbooking policies, we envision a system that has full information regarding the demand for vehicles at the station for which it considers allowing overbooking. The overbooking decisions will be based on this information, thus it is referred as a utopian overbooking policy. Note however, that this policy optimizes the service provided to each user individually rather than taking the system point of view as in the lower bounds presented in Section 4.1.

Under this policy, upon renting the user is required to declare her destination, and then the system decides whether a reservation can be made or not. The system’s decision is made based on knowledge of the current state of the destination station, including users who are on their way to that station with a vehicle, and of all future arrivals of renters to that station (including their exact arrival times). We refer to the
system decision process as *look-ahead*, since the system’s decision is based on anticipating whether there will be an available parking space in the destination station upon arriving to it. The look-ahead algorithm is presented in Table 4.1. We use the following notation to describe it:

- $E$: A list of future events in the returning station, including return events of reservations that were already approved and all future rent events. The list is sorted in ascending order of time, where each event is of the form (time, type)
- $x$.time: The time of event $x$
- $x$.type: The type of event $x$
- $rt$: Return time of the user who is attempting to make a reservation.
- $lt$: The latest returning time of a reservation that was previously made in the return station.
- $O$: Occupancy at the return station (parked and waiting vehicles)
- $C$: Capacity of the return station

### Table 4.1: Look-ahead algorithm

<table>
<thead>
<tr>
<th>Input: $(E, rt, lt, O, C)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. While $x$.time &lt; $rt$</td>
</tr>
<tr>
<td>If $x$.type = $rent$ and $O &gt; 0$, set $O = O - 1$.</td>
</tr>
<tr>
<td>If $x$.type = $return$, set $O = O + 1$.</td>
</tr>
<tr>
<td>Set $x$ to the following event in the list</td>
</tr>
<tr>
<td>2. Set $O = O + 1$ (<em>for current user</em>)</td>
</tr>
<tr>
<td>If $O &gt; C$, go to 5.</td>
</tr>
<tr>
<td>3. If $lt &gt; rt$</td>
</tr>
<tr>
<td>While $x$.time ≤ $lt$</td>
</tr>
<tr>
<td>If $x$.type = $rent$ and $O &gt; 0$, set $O = O - 1$.</td>
</tr>
<tr>
<td>If $x$.type = $return$, set $O = O + 1$.</td>
</tr>
<tr>
<td>If $O &gt; C$, go to 5.</td>
</tr>
<tr>
<td>Set $x$ to the following event in the list</td>
</tr>
<tr>
<td>4. Return &quot;Reservation Allowed&quot;.</td>
</tr>
<tr>
<td>5. Return &quot;Reservation Denied&quot;.</td>
</tr>
</tbody>
</table>

Interestingly, in some cases under this utopian overbooking policy, users may arrive at their returning station and find that there is no vacant parking space to return the vehicle to. This may occur because in the look-ahead algorithm, it is assumed that all future demand for journeys outgoing from the destination station will reduce the occupancy of the station. But, some renters may decide to abandon the system due to their inability to make a reservation at their destination and therefore the occupancy of the station may be higher than anticipated. Here, the system is not penalized for shortages of parking spaces. Instead, the users are assumed to leave the system at their destination as if they were allowed to park their vehicles near the station. In other words, the station capacity is increased artificially until a parking space becomes available. Therefore, all users who made reservations are guaranteed that they will be able to return their vehicles upon arrival at their destination.

We note that in a real stochastic setting, overbooking is likely to lead to more shortage events than in this utopian policy since the demand forecast is less accurate. Moreover, in reality, when shortage events occur, users are not allowed to leave their
vehicles near the stations. Instead, they will have to waste more time in search for a vacant parking space or to wait for a parking space to become available. Therefore, under an actual overbooking policy, the total excess time is likely to be higher than in our utopian overbooking policy.

In Table 4.2 we summarize the answers to each of the three questions that appear in the user behavior model, which characterize the settings of the parking reservation policies described above.

**Table 4.2: Settings of the various parking reservation policies**

<table>
<thead>
<tr>
<th>Parking reservation policy</th>
<th>Parking reservation requirement</th>
<th>Conditions to approve a parking reservation</th>
<th>Vacant parking space guaranteed</th>
</tr>
</thead>
<tbody>
<tr>
<td>NR</td>
<td>For none of the users</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>CPR</td>
<td>For all users</td>
<td>A vacant parking space at the destination at the renting time</td>
<td>Yes</td>
</tr>
<tr>
<td>Partial: trip based</td>
<td>For users with trip time shorter than a given threshold</td>
<td>A vacant parking space at the destination at the renting time</td>
<td>Yes</td>
</tr>
<tr>
<td>Partial: station based</td>
<td>For users with a destination station in which the difference between the returning rate and the renting rate is higher than a given threshold</td>
<td>A vacant parking space at the destination at the renting time</td>
<td>Yes</td>
</tr>
<tr>
<td>Partial: time limited</td>
<td>For all users</td>
<td>A vacant parking space at the destination at the renting time</td>
<td>Only for users with trip time shorter than the time limit</td>
</tr>
<tr>
<td>Utopian overbooking</td>
<td>For all users</td>
<td>The system anticipates that there will be a vacant parking space at the destination at the returning time</td>
<td>Yes, in this hypothetical utopian setting the user is allowed to return a vehicle even when no parking space is available</td>
</tr>
</tbody>
</table>

4.3. Numerical study

In this section, we evaluate the proposed partial reservation policies with various threshold parameters and demand characteristics, using discrete event simulation of VSSs. The simulation is based on the user behavior model presented in Section 4.2.1. The results are compared to the lower bounds that were devised in Section 4.1. The numerical study is based on data from two real world bike sharing systems, Capital Bikeshare and Tel-O-Fun. In Section 4.3.1 we describe the two bike sharing systems and the trip data that was used to generate the input to our models. In Section 4.3.2 we present the results of the numerical experiments and discuss their implications.

4.3.1. Case studies

The Capital Bikeshare system was launched on September 2010. The system operates in Washington D.C., Arlington County and Alexandria, Virginia and Montgomery County, Maryland. The operating Company, Alta Bicycle Share, provides full trip history data, which can be downloaded from the following link: [http://capitalbikeshare.com/trip-history-data](http://capitalbikeshare.com/trip-history-data). In this study we use trip data from the second quarter of 2013. In that period there were 232 operative stations with 3860 parking spaces and about 1750 bicycles in the system. The average number of daily trips on weekdays was about 7800.
In Figure 4.4 we present a map with the stations that were operative in that period. On the map we mark three clusters of stations: Arlington, Alexandria and Crystal City. As can be observed, in these clusters the stations are scattered densely while they are relatively distant from other stations in the system. Indeed, most of the trips that originated or ended in these clusters were within the cluster. In Alexandria, about 90% (resp., 88%) of the journeys that originated (resp., ended) in the cluster ended (resp., originated) in the cluster. In Crystal City, the figures are 77% and 74% and in Arlington, 70% and 76%, respectively. In the following section, we present results for the entire system and for each of the three clusters separately. While generating the data for each of the clusters, we neglected trips from/to other stations in the system. Though the resulting data does not fully reflect the occurrences in these stations, it allows us to analyze small systems in varying sizes which are “close to real”.

![Figure 4.4: Map of Capital Bikeshare stations (2nd quarter 2013)](image)

The second system that was studied is the Tel-O-Fun bike sharing system in Tel-Aviv. The system was launched on April 2011 and the trip data was collected during a period of two months in the beginning of 2012. At that time, the system consisted of 130 stations scattered in an area of about 50 square kilometers, a total of 2500 parking spaces and about 900 bicycles. During this period, the average number of daily trips (on weekdays) was about 4200.

The input for the simulation for both systems was generated as follows. We assume that the alternative mode of transportation is walking, which we believe is typically the
case for bike sharing systems. The riding times and walking times were estimated using the Google Maps API. The capacities of the stations were retrieved from the systems’ websites. The arrival rates of renters during 30 minutes periods throughout the day were estimated by aggregating the weekday trips. Assuming Poisson demand processes, for each system we randomly generated 50 daily demand realizations including renters’ arrival times to each station and their destinations. In order to reduce variation, we used the same realizations for all examined policies (Common random numbers). In addition, for each demand realization, we generated the input for the PR-LB and PRP-LB models, namely the set of potential itineraries per realized journey.

Two methods for setting the initial inventory level of vehicles at the stations were used: (1) the actual initial station inventories on a randomly chosen day, after the operators executed repositioning activities; (2) the initial inventory levels prescribed by the method of Raviv and Kolka (2013). The purpose of using two different initial inventory levels is merely to check the sensitivity of our results and insights to these parameters. Clearly, we could use other methods known in the literature to determine the initial inventory, as discussed in the introduction.

4.3.2. Results
The discrete event simulation, together with the user behavior model and the preprocessing of the input for the mathematical models, were coded in MathWorks Matlab™. The PR-LB and PRP-LB models were solved using IBM ILOG CPLEX Optimization Studio 12.5.1. The codes and data are available upon request from the authors.

We begin by discussing the results of the lower bounds and the utopian overbooking policy. The results and discussion regarding the partial reservation policies are presented at the end of this section. In Table 4.3 we present results for the Capital Bikeshare and Tel-O-Fun systems. In the first and second columns, the name of the system and the number of stations in the system are given. In the third column we present the method according to which the initial inventory levels were set. In the fourth to seventh columns we present the average total excess time, over 50 realizations, for the NR policy, the CPR policy, the utopian overbooking policy and the PR-LB model. In the last column we present the average total ideal times, over 50 realizations. Recall that the ideal time is the total traveling time if all the journeys could be served ideally by a shared vehicle from the desired origin to the desired destination. The problem instances for the PRP-LB model could not be solved using the available computational resources and thus this lower bound is not presented here. We revisit this model when analyzing the smaller sub-systems below.
Table 4.3: Results for the two real-world systems

<table>
<thead>
<tr>
<th>System</th>
<th>Stations</th>
<th>Initial Inventory</th>
<th>Total Excess Time (hours/day)</th>
<th>Total Travel Time (hours/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>NR</td>
<td>CPR</td>
</tr>
<tr>
<td>Capital Bikeshare</td>
<td>232</td>
<td>Actual Day</td>
<td>346.9</td>
<td>282.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Raviv &amp; Kolka</td>
<td>183.9</td>
<td>141.1</td>
</tr>
<tr>
<td>Tel-O-Fun</td>
<td>130</td>
<td>Actual Day</td>
<td>89.9</td>
<td>76.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Raviv &amp; Kolka</td>
<td>59.5</td>
<td>41.2</td>
</tr>
</tbody>
</table>

We observe from Table 4.3 that the lower bound on the total excess time provided by the PR-LB model is significantly tighter than the trivial lower bound obtained by assuming that all the journeys are materialized by their ideal itineraries, i.e., no excess time, as in Kaspi et al. (2014). For example, in Capital Bikeshare, about 40% of the gap between the CPR policy and the trivial lower bound (zero excess time) is explained by the PR-LB model. That is, at least 40% of the excess time under the CPR policy, cannot be reduced by any passive system regulation. Furthermore, recall that in the PR-LB model we assume that all demands for journeys are known in advance and that a central planner determines the itinerary of each user. As this setting is unrealistic, we can expect that the excess time under any real policy would be much higher. In other words, a major part of the remaining gap can be explained by these assumptions. Recall that each of the figures in Table 4.3 is an estimation of the excess time under a certain reservation policy based on an average of 50 demand realizations. The differences between the values in each row of the table were tested by a one-sided sign test and were found to be significant at $p-value < 0.000012$.

The results presented for the PR-LB model are based on the LP relaxation of the model. In addition, we solved the original MILP model for the smaller instances that are based on the Tel-O-Fun data. In 97 out of these 100 instances, the value of the LP relaxation solution was identical to the one obtained by the MILP model, while the latter were obtained at a substantially longer processing times. In the remaining three instances, the lower bound obtained by the MILP model was slightly higher but the difference was negligible, less than 0.002%.

Using the initial inventories as prescribed by the method of Raviv and Kolka (2013), a significant reduction in excess time is obtained, as can be observed in Table 4.3 for all policies. Indeed, proper planning of static repositioning results with a major improvement in the service level. Nevertheless, the results for the CPR policy and the PR-LB model suggest that an additional substantial reduction in the total excess time can be achieved by integrating repositioning activities with an efficient passive regulation.

As can be observed in Table 4.3, the utopian overbooking policy produced only slightly better results as compared to the CPR policy. This is quite surprising, when recalling the assumptions on which the utopian overbooking policy is based on. That is, even with full knowledge of the demand realizations and the use of overbooking, a significant improvement cannot be obtained. This implies that, realistic overbooking policies are not likely to be significantly (or at all) beneficial in terms of reducing the
excess time in VSS. This unexpected finding can be explained by the fact that in VSS, a positive side-effect of parking space reservations is the diversion of the demand toward less congested stations. This in turn may have a positive effect on future users of the system who are less likely to face shortages of vehicles and parking spaces. Allowing overbooking reduces this positive side-effect. Given the fact that a good overbooking policy is much more complicated to implement than the CPR policy, and that it also introduces additional uncertainty and thus reduces the trust of the users in the system, we believe that this type of policy should not be practiced in VSSs.

In Table 4.4, we present some statistics on the dimension of the PR-LB instances that we solved and the solution times. We present the number of stations in each system, the average number of users (over the 50 demand realizations), the average number of itineraries per user, the number of variables in the linear programming model and the average solution time of both the LP relaxation and the MILP model, where the itinerary variables are defined as binary ones. Note that the MILP model could be solved in a reasonable time only for the smaller instances of the Tel-O-Fun network. The solution time of the PR-LB model is not of a particular interest in our study since such a model is not supposed to be solved very often. It is observable that the solution times are reasonable for most of the strategic and operational scenarios. That is, a similar formulation can be used for other purposes, where time consideration is more important.

<table>
<thead>
<tr>
<th>System</th>
<th>Stations</th>
<th>Number of users</th>
<th>Average number of itineraries per user</th>
<th>Initial Inventory</th>
<th>PR-LB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Number of variables</td>
<td>Solution time LP (sec)</td>
</tr>
<tr>
<td>Capital Bikeshare</td>
<td>232</td>
<td>7,826.4</td>
<td>204.5</td>
<td>Actual Day</td>
<td>4,993,194</td>
</tr>
<tr>
<td>Raviv &amp; Kolka</td>
<td></td>
<td></td>
<td></td>
<td>Raviv &amp; Kolka</td>
<td>1,605.15</td>
</tr>
<tr>
<td>Tel-O-Fun</td>
<td>130</td>
<td>4,154.9</td>
<td>62.7</td>
<td>Actual Day</td>
<td>765,050</td>
</tr>
<tr>
<td>Raviv &amp; Kolka</td>
<td></td>
<td></td>
<td></td>
<td>Raviv &amp; Kolka</td>
<td>54.37</td>
</tr>
</tbody>
</table>

Solving the PRP-LB model that was presented in Section 4.1.3 is impractical for large real-world systems, due to the large number of binary variables. To obtain the insights provided by the solution of the PRP-LB model, we generated three small sized systems based on three clusters of stations in the Capital Bikeshare system, namely, Alexandria with 8 stations, Crystal City with 15 stations and Arlington with 30 stations. In Table 4.5, we present the results for these systems. The table is supplemented with an additional column (the seventh), presenting the lower bound on the expected total excess time produced by the PRP-LB model. We observe from the table, that for the three small sized systems, the value obtained from the PR-LB model explained about 56-66% of the gap from the trivial (zero) lower bound. However, a larger portion of this gap, namely 67%-81% was explained by the PRP-LB value. This result further strengthens our belief that no parking reservation policy is likely to result in a dramatic (if at all) improvement over the CPR policy. We also note that for these systems, the excess time for the utopian overbooking policy is sometimes slightly higher than that of the CPR policy. Recall that each of the figures in Table 4.5 is an estimation of the
excess time under a certain reservation policy based on an average of 50 demand realizations. The differences between the values in each row of the table were tested by a one-sided sign test and were found to be significant at $p-value < 10^{-7}$.

Table 4.5: Results for the three clusters in Capital Bikeshare

<table>
<thead>
<tr>
<th>System</th>
<th>Stations</th>
<th>Initial Inventory</th>
<th>Total Excess Time (hours/day)</th>
<th>Total Travel Time (hours/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>NR</td>
<td>CPR</td>
</tr>
<tr>
<td>Arlington</td>
<td>30</td>
<td>Actual Day</td>
<td>2.907</td>
<td>2.262</td>
</tr>
<tr>
<td>Raviv &amp; Kolka</td>
<td></td>
<td></td>
<td>1.600</td>
<td>1.129</td>
</tr>
<tr>
<td>Crystal City</td>
<td>15</td>
<td>Actual Day</td>
<td>1.314</td>
<td>1.120</td>
</tr>
<tr>
<td>Raviv &amp; Kolka</td>
<td></td>
<td></td>
<td>0.656</td>
<td>0.564</td>
</tr>
<tr>
<td>Alexandria</td>
<td>8</td>
<td>Actual Day</td>
<td>0.589</td>
<td>0.352</td>
</tr>
<tr>
<td>Raviv &amp; Kolka</td>
<td></td>
<td></td>
<td>0.225</td>
<td>0.184</td>
</tr>
</tbody>
</table>

In Table 4.6, we present statistics of the instances for the three clusters of Capital Bikeshare and of mathematical model that were used to create the lower bounds. The table is in the same format as Table 4.4. Interestingly, it is observed that the initial inventory has a significant effect on the solution time of PRP-LB. The optimized inventory levels obtained by the method of Raviv and Kolka (2013) results with models that can be solved much more quickly, although the dimension of the mathematical models are identical.

Table 4.6: Statistics for the three clusters in Capital Bikeshare

<table>
<thead>
<tr>
<th>System</th>
<th>Stations</th>
<th>Number of users</th>
<th>Average number of itineraries per user</th>
<th>Initial Inventory</th>
<th>PR-LB</th>
<th>PRP-LB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Number of continuous variables</td>
<td>Solution time (sec)</td>
<td>Number of Auxiliary Binary variables</td>
</tr>
<tr>
<td>Arlington</td>
<td>30</td>
<td>255.6</td>
<td>42.2</td>
<td>Actual Day</td>
<td>36,223</td>
<td>0.66</td>
</tr>
<tr>
<td>Raviv &amp; Kolka</td>
<td></td>
<td></td>
<td></td>
<td>Raviv &amp; Kolka</td>
<td>0.72</td>
<td>23,501</td>
</tr>
<tr>
<td>Crystal City</td>
<td>15</td>
<td>128.5</td>
<td>17.0</td>
<td>Actual Day</td>
<td>8,052</td>
<td>0.11</td>
</tr>
<tr>
<td>Raviv &amp; Kolka</td>
<td></td>
<td></td>
<td></td>
<td>Raviv &amp; Kolka</td>
<td>0.11</td>
<td>5,120</td>
</tr>
<tr>
<td>Alexandria</td>
<td>8</td>
<td>68.6</td>
<td>5.9</td>
<td>Actual Day</td>
<td>1,601</td>
<td>0.03</td>
</tr>
<tr>
<td>Raviv &amp; Kolka</td>
<td></td>
<td></td>
<td></td>
<td>Raviv &amp; Kolka</td>
<td>0.03</td>
<td>1,003</td>
</tr>
</tbody>
</table>

Next, we consider the partial reservation policies that were presented in Section 4.2.2., and examine whether they can improve the performance obtained by the CPR policy. In Figure 4.5, we present the simulation results for these policies. The figure contains six graphs, one for each combination of the two studied real-world systems and the three partial policies. In each graph, two curves are displayed, representing the percentage of excess time obtained using the two methods for setting the initial inventories. Namely, an actual day, displayed in black, and the method of Raviv and Kolka (2013), displayed in gray.

For each partial policy, we plot the percentage of excess time (relative to the ideal time) under various settings. For the trip based partial policy, we tested 31 time thresholds in an interval of three minutes. For the time limited partial policy we tested
31 time limits, again with three minutes intervals. For the station based partial policy, we tested 11 difference thresholds, of 0%, 10%,..., 100%; the thresholds were calculated for one hour time intervals during the day. In order to use the same scale on the horizontal axis for the two systems, we present the percentage of stations in which a parking space reservation is required, instead of presenting the difference thresholds.

Recall that extreme settings of such partial policies result with the complete policies (except for the lower extreme of the time limited partial policy). The figure demonstrates that as the time threshold increases, the same trend appears in all six graphs, i.e. when more reservations are required, the excess time decreases. The best performance is achieved when parking reservations are required from all the users, i.e., under the CPR policy.

These results demonstrate that using a simple rule to define a partial parking reservation policy is not likely to produce better results than the CPR policy. Another outcome of this analysis is that the more users are required to reserve parking spaces the better the performance of the system is. However, in situations where it is possible to require reservations only from some users, it is better to apply a partial reservation policy rather than not require reservations at all.
Recall that in the time limited partial reservation policy, all users must make a reservation, but the reservation expires after a given time. If the time limit is set to zero, users would only be able to travel to stations that are not full at the renting time, but a parking space will not be guaranteed in the destination in any case. In Figure 4.3, the graphs of the time limited policy begin at lower points, compared with the other two partial policies. That is, as compared to the NR policy, it is observable that a significant improvement may be obtained simply by redirecting users to returning stations that are not full at the renting time. In fact, most of the improvement accomplished by the CPR policy may be attributed to the redirection of users to stations with vacant parking spaces.

Next, we examine whether the above insights are relevant to systems with other characteristics. In particular, we consider systems with the same geography and similar demand patterns, but with different congestion, i.e., offered load, and with different station capacities. For each of the systems (Tel-O-Fun and Captial Bikeshare) we generated new instances by multiplying the demand rates in all stations by several load multipliers, where 1 represents the original systems. Fifty demand realizations were generated based on each of these load multipliers.

In Figure 4.6, we present the performance of the trip based partial reservation policy with various time thresholds and load multipliers. It is observable that in both systems and under various congestion levels the excess time is reduced as more reservations are required. This implies that the effect observed under the original demand load is, qualitatively, not affected by the congestion level. However, as the congestion increases the benefit obtained from the reservation is increasing. This can be attributed to the fact that in a more congested systems, shortage events are more likely to occur. For the sake of brevity, we present the results only for the trip base partial reservation policy. Very similar trends were observed under the station based and the time limited partial reservation policies.

![Figure 4.6: Trip based reservation policy- percentage excess time under various load factors with initial inventory obtained from an actual day](image)
Further on, we examined the effect of the capacities of the stations on the performance of the system under the same 50 demand realizations. To this end, we have conducted the following test: the capacities of all the stations in the system were decreased/increased by 25% (and rounded to the closest integer). In Figure 4.7 we present the results for the trip based partial reservation policy. Similar to previous results, the excess time reduces as the time threshold increases. That is, the same trends are observed regardless of the capacity of the stations. For a given demand rate, as the capacity of the stations are increased, the number of parking space shortages are reduced. It is observable that the excess time under the NR policy, the various settings of the partial reservation policies and the CPR almost converge to the same value as the station capacities are increased. As may be expected, the benefit of implementing parking reservations increases when the parking spaces are scarcer. Again, similar trends are observed under the station based and the time limited partial policies.

Figure 4.7: Trip based reservation policy - percentage excess time under various capacities with initial inventory obtained from an actual day

4.4. Discussion
The main message of this study is reinforcing the effectiveness of parking reservations in VSS as a method to improve the service provided to its users. It is shown that the simplest possible parking reservation policy (namely, CPR) appears to be the most effective one in terms of reducing the total excess time. This was observed through empirical tests under numerous settings that are based on the geography and demand trends of two real-world systems, diverse offered loads, station capacities and initial inventories. Our case studies, presented in Section 4.3.1, are based on data received from bike sharing systems. However, we believe that the concept of parking space reservation, and other passive regulations, is even more relevant to car sharing systems where the cost of active regulation (i.e., relocation of vehicles) is prohibitive.

Using a lower bound calculated by the PR-LB model, we have demonstrated that, in our case studies, a significant share of the excess time that could be theoretically saved by any passive system regulation, is already saved by the CPR policy. Our extended
PRP-LB model shows that other parking reservation policies are not likely to be able to save substantially (if at all) more excess time.

We also studied several partial reservation policies and demonstrated that while these policies are slightly inferior to the CPR, they may also be a good alternative to the basic NR policy in cases where CPR cannot be implemented for some reasons. Finally, we precluded reservation policies that are based on overbooking as a parking reservation approach that is likely to outperform the CPR policy. This was achieved by showing that even under a utopian scenario in which the system looks ahead into future demand, such policies cannot significantly reduce the excess time obtained by CPR.

The PR-LB based lower bound, introduced in Section 4.1.2, can be used to evaluate the effectiveness of various other VSS related policies. This model reflects the fact that each journey may be assigned to one of several itineraries. This adds a lot of flexibility to VSSs and affects its dynamics in a way that should not be ignored by a strategic planner. Though we have focused on reducing the excess time of users, our model can be extended to accommodate other objectives of the users. That is, each potential itinerary can be assigned with a measure that reflects a combination of several objectives. We also suggest using our model in the future to incorporate considerations of the operator. For example, if a car sharing operator faces profit losses due to some possible itinerary choices of the users, these values can be weighted and added to the excess time. It would be interesting to examine the effect of parking reservation policies on the obtained profit under various pricing schemes.
Part II

Maintenance Operations
5. Detection of unusable bicycles in bike-sharing systems

A paper based on the research presented in this chapter was published in: M. Kaspi, T. Raviv and M. Tzur, Detection of Unusable Bicycles in Bike-Sharing Systems. Omega, forthcoming.

In this chapter we propose using data that is already collected by existing bike sharing systems to estimate the probability that each bicycle is usable. In Section 5.1 we formulate a Bayesian model that makes use of on-line transactions data to constantly update these probabilities. In Section 5.2 we propose a method to approximate these probabilities in real-time. Numerical results based on the data received from CitiBike are presented in Section 5.3. In Section 5.4, we present some possible extensions of the model and explain how additional information such as user complaints can be incorporated in the model. In addition, we discuss how an equivalent model can be used for detection of locker (dock) failures. The chapter is concluded with a short discussion in Section 5.5.

5.1. A Bayesian model

The goal of this study is to estimate the number of unusable bicycles in a station and to continuously update this estimation in real-time. We begin by focusing on each bicycle independently. We assign a Probability of Unusability (PoU) to each bicycle in the system and update it continuously. A good indication for unusability of a bicycle is the fact that it was not rented for a long period. However, this probability also depends on other factors such as the number of renting transactions since the bicycle arrived at the station and the availability of other bicycles in the station when these transactions occurred. The model that will be presented next makes use of the transaction data in order to estimate the PoU of each bicycle in a single station.

We use the following notation:

- $i$: Bicycle ID
- $e$: Rent event
- $C$: Set of all lockers in the station, $|C|$ is the capacity of the station
- $p_i$: Prior probability that bicycle $i$ is returned to the station unusable
- $S^e$: Set of bicycles that are parked in the station right before rent event $e$
- $q^e(m,S)$: The probability that right before rent event $e$ there are $m$ usable bicycles in the set $S$
- $P^e(x)$: The probability of scenario $x$ at rent event $e$
- $P^e(x,y)$: The joint probability of scenarios $x$ and $y$ at rent event $e$
- $p_i^e$: The PoU of bicycle $i$ right after the occurrence of rent event $e$

We assume that when a bicycle is rented, it is usable, that is a user never rents an unusable bicycle. Formally, we assume $P^e(i \text{ usable}|i \text{ rented}) = 1$ and $P^e(i \text{ rented}|i \text{ unusable}) = 0$. However, bicycle $i$ may turn unusable during a ride, and therefore there is a probability $p_i$ that the bicycle will be returned to the station unusable. See discussion in Section 5.4.3. regarding the calculation of this probability.
For simplicity of the presentation, we initially assume that the users have no preferences regarding the locker from which the bicycle will be rented. That is, a user uniformly selects a bicycle from the usable bicycles that are parked in the station. In Section 5.4.1, we discuss how user preferences regarding the lockers can be incorporated in the model.

Our goal is to update the PoU of bicycles that are parked in the station. Given that at rent event \( e \) bicycle \( j \) was rented, we use Bayes’ rule to calculate the probability that bicycle \( i \) \((i \neq j)\) is unusable. This calculation is carried out for any bicycle that is left parked at the station:

\[
p_i^e = P^e(i \text{ unusable} \mid j \text{ rented}) = \frac{P^e(i \text{ unusable}, j \text{ rented})}{P^e(j \text{ rented})} \quad \forall i \in S^e \setminus \{j\} \tag{5.1}
\]

Figure 5.1 depicts the notation used right before, at and right after rent event \( e \). The updating of the PoU is carried out for bicycles that are left parked in the station right after each rent.

To calculate (5.1), let’s consider first the denominator. Given that bicycle \( j \) is usable, the probability that it will be rented at rent event \( e \) is given by:

\[
P^e(j \text{ rented} \mid j \text{ usable}) = \sum_{m=0}^{\left|S^e\right|-1} \frac{1}{1 + m} \cdot q^e(m, S^e \setminus \{j\}) \tag{5.2}
\]

This expression is obtained by conditioning on the number of usable bicycles in the station \((m)\), excluding bicycle \( j \), and multiplying the probability of having this number, \( q^e(m, S^e \setminus \{j\}) \), by the uniform probability that \( j \) will be selected from within \( m + 1 \) usable bicycles.

Then, by definition: \( P^e(j \text{ usable} \mid j \text{ rented}) = 1 \), and equivalently: \( P^e(j \text{ rented}, j \text{ usable}) = P^e(j \text{ rented}) \). Using Bayes’ rule, we obtain the probability that bicycle \( j \) will be rented at event \( e \):

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\[ P^e(j \text{ rented}) = P^e(j \text{ rented}, j \text{ usable}) \]
\[ = P^e(j \text{ usable}) \cdot P^e(j \text{ rented}| j \text{ usable}) \]
\[ = (1 - p_j^{e-1}) \cdot \sum_{m=0}^{\left| S^e \right| - 1} \frac{1}{1 + m} \cdot q^e(m, S^e \setminus \{j\}) \]  
\[(5.3)\]

Where \( p_j^{e-1} \) denotes the PoU of bicycle \( j \) right before rent event \( e \). Similarly to calculate nominator of (5.1) we condition in addition over bicycle \( i \) and obtain the joint probability that bicycle \( i \) is unusable and bicycle \( j \) is rented at rent event \( e \):

\[ P^e(i \text{ unusable}, j \text{ rented}) \]
\[ = p_i^{e-1} \cdot (1 - p_j^{e-1}) \cdot \sum_{m=0}^{\left| S^e \right| - 2} \frac{1}{1 + m} \cdot q^e(m, S^e \setminus \{i, j\}) \]  
\[(5.4)\]

Equations (5.3) and (5.4) contain an expression for the probability of the number of usable bicycles \( m \), excluding \( i \) and \( j \), that are parked in the station right before rent event \( e \). Note that each bicycle that is parked in the station has a different probability of being usable. Therefore, this expression is the sum of Bernoulli variables with different success probabilities, which is a Poisson Binomial distribution (see, for example, Johnson and Kotz, 1992), given by the following probability mass function:

\[ q^e(m, S^e) = \sum_{S \in F_m(S^e)} \prod_{k \in S} (1 - p_k^{e-1}) \prod_{k \in S^e \setminus S} p_k^{e-1} \]

Where \( F_m(S) \) denotes the collection of all subsets of set \( S \) with cardinality \( m \). Note that in Equations (5.3) and (5.4) this probability is calculated for all possible values of \( m \) and therefore the calculation effort for evaluating (5.3) and (5.4) grows exponentially in \( S^e \). Thus, the exact on-line updating of the PoU is impractical for large bike sharing stations. However, we observe that a related quantity, the expected number of usable bicycles, is easier to calculate as it is merely the sum of the probabilities of usability of the bicycles in the station:

\[ \mathbb{E}(\text{usables in } S^e) = \sum_{i \in S^e} (1 - p_i^{e-1}) \]

In the following section we propose a method to approximate the PoU, based on the expected number of usable bicycles in the station.

5.2. Approximating the probability of unusability

Henceforth, we denote the approximated PoU of bicycle \( i \) after rent event \( e \) by \( \tilde{p}_i^e \). Recall that we assume that bicycles are selected uniformly from the set of available usable bicycles in the station. We approximate the probability that bicycle \( j \) is rented, given that it is usable, by assuming that the number of usable bicycles in the station is known and equals its expectation. Specifically, given that bicycle \( j \) is usable, the
expected number of usable bicycles in the station is one plus the expected number of bicycles in the remaining set of available bicycles. Thus, Equation (5.2) is approximated as follows:

\[
P^e(j \text{ rented} | j \text{ usable}) = \frac{1}{\mathbb{E}(\text{usables in } S^e | j \text{ usable})} = \frac{1}{1 + \sum_{k \in S^e \setminus \{j\}} (1 - \tilde{p}_k^{e-1})}
\]  

(5.5)

where the expected number of usable bicycles in the station right before rent event \(e\) is approximated by:

\[
\mathbb{E}(\text{usables in } S^e \setminus \{j\}) = \sum_{k \in S^e \setminus \{j\}} (1 - \tilde{p}_k^{e-1})
\]  

(5.6)

Similarly to the calculation of Equation (5.3), we multiply Equation (5.5) by the probability that bicycle \(j\) is usable, to obtain the approximated probability that bicycle \(j\) is rented at rent event \(e\):

\[
\tilde{p}^e(j \text{ rented}) = \tilde{p}^e(j \text{ rented}, j \text{ usable}) = \tilde{p}^e(j \text{ usable}) \cdot \tilde{p}^e(j \text{ rented} | j \text{ usable})
\]

\[
= \frac{1 - \tilde{p}_j^{e-1}}{1 + \sum_{k \in S^e \setminus \{j\}} (1 - \tilde{p}_k^{e-1})}
\]  

(5.7)

Next, given that bicycle \(i\) is unusable, the expected number of usable bicycles in the station is updated to exclude \(i\) and thus we obtain an approximation of the following conditional probability:

\[
P^e(j \text{ rented} | i \text{ unusable}) = \frac{1 - \tilde{p}_j^{e-1}}{1 + \sum_{k \in S^e \setminus \{i,j\}} (1 - \tilde{p}_k^{e-1})}
\]

And again, using Bayes’ rule, we obtain an approximation of the joint probability:

\[
\tilde{p}^e(j \text{ rented}, i \text{ unusable}) = \frac{(1 - \tilde{p}_j^{e-1}) \cdot \tilde{p}_i^{e-1}}{1 + \sum_{k \in S^e \setminus \{i,j\}} (1 - \tilde{p}_k^{e-1})}
\]  

(5.8)

Finally, by dividing Equation (5.8) by Equation (5.7) we obtain an approximation of the updated PoU:

\[
\tilde{p}_l^e = \tilde{p}^e(i \text{ unusable} | j \text{ rented}) = \tilde{p}_l^{e-1} \cdot \frac{1 + \sum_{k \in S^e \setminus \{j\}} (1 - \tilde{p}_k^{e-1})}{1 + \sum_{k \in S^e \setminus \{i,j\}} (1 - \tilde{p}_k^{e-1})}
\]  

(5.9)

Using Equation (5.6), we can rewrite Equation (5.9) as:

\[
\tilde{p}_l^e \equiv \tilde{p}^e(i \text{ unusable} | j \text{ rented}) = \tilde{p}_l^{e-1} \cdot \frac{1 + \mathbb{E}(\text{usables in } S^e \setminus \{j\})}{\tilde{p}_l^{e-1} + \mathbb{E}(\text{usables in } S^e \setminus \{j\})}
\]
We observe that the PoU of a bicycle increases after every rent event in which it is not selected. In addition, as the initial PoU of a bicycle is its prior probability, it is easy to see that the PoU of a bicycle increases also as its prior probability increases.

So far, our focus was on calculating the PoU for each bicycle separately. However, it is typically more interesting for the operators and the users to view all the bicycles in a station aggregately. Similar to the above, our analysis also provides the expected number of unusable bicycles in the station, given by:

$$
\mathbb{E}(\text{unusables in } S^e) = \sum_{i \in S^e} p_i^{e-1}
$$

(5.10)

We note that the value calculated in (5.10) can be used as a reliable estimator for the actual number of unusable bicycles after a sufficient number of rent events. In the next section this will be demonstrated numerically. The expected number of unusable bicycles is a more concise measure, compared to the PoU of each bicycle in the station. It can be used and understood by the operators and by the users.

The approximation of the PoU and the expected number of unusable bicycles in a station can be carried out after each rent event in $O(|C|)$ time, i.e., linearly in the station capacity. At each rent event these values can be updated in a fraction of a second. Therefore, the estimated number of unusable bicycles can be displayed on-line to the operators and the users. In Appendix B, we show that this is a very accurate approximation by comparing it to the result of the exact calculation for small stations with up to 15 lockers. Note that the complexity of the exact method is $O(|C| \cdot 2^{|C|})$ for each rent event, which is impractical for on-line usage.

5.3. Numerical results

In this section we present the results of a numerical experiment carried out to test our proposed detection method. To simulate the on-line calculation of the PoU, we have used CitiBike trip history transactions data from July-August 2014. Using this data, we estimated the renting/returning rates on weekdays in each station during 30 minute periods along the day. We generated 100 demand realizations per station. Each demand realization consists of a set of renting and returning events and their times of occurrence along a 2-day period. Each return event is supplemented with a binary parameter that indicates whether the bicycle is usable or not. For the experiment, we set the failure probability of all bicycles to 0.01. That is, the unusability indicator value was drawn from a Bernoulli distribution with parameter 0.01. In addition, we assume that at the initial state of the station all bicycles parked at the station are usable, as if replenishment activities and collection of unusables were just executed. The initial inventory level is set to the optimal level according to the method of Kaspi et al. 2015a (as will be presented in Chapter 6). The demand realizations data used in the simulation can be downloaded from http://www.eng.tau.ac.il/~morkaspi/publications.html.

At a rent event, if there are available usable bicycles in the station, one is selected uniformly and is removed from the set of available bicycles. If there are no available usable bicycles in the station the user is assumed to abandon the station. At return
events, if there are available lockers in the station, one is selected uniformly and the bicycle is returned to that locker. If there are no available lockers in the station the user is assumed to abandon the station. If an unusable bicycle is returned to the station, it “occupies” a locker, but is not entered to the set of available usable bicycles in a station (and therefore will never be selected at a rent event).

We compare the approximated expected number of unusable bicycles (Equation (5.10)) to a naïve approach for assessing the expected number of unusable bicycles in a station. The naïve expectation is obtained by summing the prior probabilities of the bicycles returned to a station in a given time period. For example, assume that the prior probability of all bicycles is 0.01 and that in a given time period 200 bicycles were returned to the station. The naïve estimation of the number of unusable bicycles that were returned to the station would be 2. We note that this unbiased estimator of the expected number of unusable bicycles by itself may provide a relatively good picture regarding the amount of unusable bicycles in a station. Given that in some bike sharing systems the number of unusable bicycles is not assessed at all, using even this naïve method would be valuable.

In Table 5.1 we present simulation results for 20 arbitrarily selected stations. Simulation results of another 80 stations are available online, as an electronic supplementary, at http://www.eng.tau.ac.il/~morkaspi/publications.html. In the first and second columns of Table 5.1, the station ID and capacity are presented, respectively. The average (over 100 realizations) of the number of bicycles that were rented and returned to the station in the simulation are presented in the third and fourth column. Note that the realized number of rent/return events was in most cases a bit larger, but not all bicycles could be rented/returned due to bicycle/locker shortages. In the fifth column, the average number of unusable bicycles that were returned to the station is presented. For each demand realization, we calculate at the end of the 2-days period the difference between the actual number of unusable bicycles and its estimation obtained by the naïve approach and by the PoU approach. The mean absolute deviation (and the standard deviation) of these differences are presented in the sixth and seventh columns, respectively. In addition, we count the number of times in which the PoU estimation was closer to the actual value as compared to the estimation of the naïve approach. This number is presented in the eighth column. The P-value of the sign-test used to determine whether the PoU approach generates closer estimation as compared to the naïve approach is presented in the last column.

As can be observed, both the mean absolute deviation and the standard deviation of the PoU approach are significantly smaller than those of the naïve approach. In particular, as more rent events occur in the station (the table is sorted in decreasing order of the number of rent events), more information is accumulated by the PoU approach and can be used to better estimate the number unusable bicycles. The figures in the eighth and last columns demonstrate the superiority of the PoU approach as compared to the naïve approach.
Table 5.1: Simulation results – average over 100 realizations for 20 stations in CitiBike

<table>
<thead>
<tr>
<th>Station ID</th>
<th>Capacity</th>
<th>Number of Rents</th>
<th>Number of Returns</th>
<th>Unusable Bicycles</th>
<th>Naïve MAD(Stdev)</th>
<th>PoU MAD(Stdev)</th>
<th>PoU better (out of 100)</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>134</td>
<td>35</td>
<td>393.67</td>
<td>403.61</td>
<td>4.34</td>
<td>1.56 (1.92)</td>
<td>0.39 (0.61)</td>
<td>85</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>145</td>
<td>36</td>
<td>392.96</td>
<td>404.82</td>
<td>4.09</td>
<td>1.38 (1.67)</td>
<td>0.50 (0.67)</td>
<td>80</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>132</td>
<td>35</td>
<td>366.35</td>
<td>364.71</td>
<td>3.85</td>
<td>1.42 (1.77)</td>
<td>0.37 (0.58)</td>
<td>87</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>135</td>
<td>42</td>
<td>251.89</td>
<td>256.65</td>
<td>2.54</td>
<td>1.22 (1.50)</td>
<td>0.14 (0.30)</td>
<td>94</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>139</td>
<td>35</td>
<td>208.78</td>
<td>206.55</td>
<td>1.90</td>
<td>1.13 (1.38)</td>
<td>0.15 (0.31)</td>
<td>88</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>144</td>
<td>27</td>
<td>179.65</td>
<td>177.77</td>
<td>1.52</td>
<td>0.98 (1.20)</td>
<td>0.31 (0.48)</td>
<td>92</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>142</td>
<td>42</td>
<td>176.33</td>
<td>182.24</td>
<td>1.89</td>
<td>1.11 (1.40)</td>
<td>0.32 (0.51)</td>
<td>87</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>133</td>
<td>27</td>
<td>163.04</td>
<td>175.27</td>
<td>1.93</td>
<td>1.04 (1.31)</td>
<td>0.54 (0.66)</td>
<td>73</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>138</td>
<td>39</td>
<td>108.53</td>
<td>110.37</td>
<td>1.30</td>
<td>0.84 (1.11)</td>
<td>0.21 (0.42)</td>
<td>80</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>137</td>
<td>28</td>
<td>100.81</td>
<td>112.66</td>
<td>1.13</td>
<td>0.84 (1.06)</td>
<td>0.60 (0.74)</td>
<td>59</td>
<td>0.0284</td>
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<tr>
<td>136</td>
<td>23</td>
<td>82.74</td>
<td>94.71</td>
<td>0.87</td>
<td>0.62 (0.84)</td>
<td>0.44 (0.51)</td>
<td>54</td>
<td>0.1841</td>
</tr>
<tr>
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<td>27</td>
<td>76.70</td>
<td>90.85</td>
<td>0.89</td>
<td>0.69 (0.94)</td>
<td>0.46 (0.61)</td>
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<td>0.0666</td>
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<tr>
<td>128</td>
<td>19</td>
<td>67.45</td>
<td>77.97</td>
<td>0.78</td>
<td>0.69 (0.86)</td>
<td>0.41 (0.50)</td>
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<td>&lt; 0.0001</td>
</tr>
<tr>
<td>129</td>
<td>29</td>
<td>65.71</td>
<td>82.16</td>
<td>0.74</td>
<td>0.66 (0.81)</td>
<td>0.45 (0.56)</td>
<td>62</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>143</td>
<td>23</td>
<td>56.26</td>
<td>69.22</td>
<td>0.68</td>
<td>0.69 (0.81)</td>
<td>0.40 (0.53)</td>
<td>78</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>140</td>
<td>27</td>
<td>38.30</td>
<td>45.36</td>
<td>0.43</td>
<td>0.56 (0.67)</td>
<td>0.38 (0.54)</td>
<td>71</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>130</td>
<td>23</td>
<td>33.33</td>
<td>36.09</td>
<td>0.31</td>
<td>0.47 (0.54)</td>
<td>0.45 (0.53)</td>
<td>53</td>
<td>0.2421</td>
</tr>
<tr>
<td>126</td>
<td>24</td>
<td>32.51</td>
<td>33.60</td>
<td>0.31</td>
<td>0.46 (0.51)</td>
<td>0.31 (0.43)</td>
<td>83</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>141</td>
<td>31</td>
<td>31.81</td>
<td>40.88</td>
<td>0.48</td>
<td>0.56 (0.69)</td>
<td>0.46 (0.61)</td>
<td>73</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>131</td>
<td>23</td>
<td>24.82</td>
<td>34.03</td>
<td>0.35</td>
<td>0.48 (0.55)</td>
<td>0.34 (0.48)</td>
<td>82</td>
<td>&lt; 0.0001</td>
</tr>
</tbody>
</table>

Recall that in the simulation, we set the actual failure probability to 0.01. The estimations presented in Table 5.1 are based on the assumption that indeed the prior probability is 0.01. In reality, the operator may not have an exact knowledge of the prior probabilities. Next, we examine whether the PoU approach results with better estimations as compared to the naïve approach even if the exact prior probabilities are unknown exactly. We conducted the following analysis: we used the same demand realizations as in Table 5.1 (using a failure probability of 0.01) but assumed different levels of prior probabilities in the calculation of the PoU and naïve based estimations.

In Table 5.2, we present the number of times (out of 100 realizations) that the PoU based estimation was closer to the actual values as compared to the naïve approach. The first and second columns of Table 5.2 are identical to those of Table 5.1. In the third to seventh columns, we present these values for the following assumed prior probabilities: 0.001, 0.005, 0.01, 0.02, and 0.05, respectively.

Noticeably, as the assumed prior increases (or decreases) relative to the actual prior, the number of times the PoU approach delivers better estimations increases. This demonstrates that the PoU approach is more robust with respect to the estimation of the prior probability as compared to the naïve approach.
Table 5.2: Sensitivity analysis – Actual failure probability 0.01

<table>
<thead>
<tr>
<th>Station ID</th>
<th>Capacity</th>
<th>PoU better (out of 100)</th>
<th>Assumed Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.001 0.005 0.01 0.02 0.05</td>
<td></td>
</tr>
<tr>
<td>134</td>
<td>35</td>
<td>98     85     85     96     100</td>
<td></td>
</tr>
<tr>
<td>145</td>
<td>36</td>
<td>99     87     80     99     100</td>
<td></td>
</tr>
<tr>
<td>132</td>
<td>35</td>
<td>100    92     87     97     100</td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>42</td>
<td>100    98     94     98     100</td>
<td></td>
</tr>
<tr>
<td>139</td>
<td>35</td>
<td>98     86     88     96     100</td>
<td></td>
</tr>
<tr>
<td>144</td>
<td>27</td>
<td>94     83     92     93     100</td>
<td></td>
</tr>
<tr>
<td>142</td>
<td>42</td>
<td>93     79     87     92     100</td>
<td></td>
</tr>
<tr>
<td>133</td>
<td>27</td>
<td>90     73     73     90     100</td>
<td></td>
</tr>
<tr>
<td>138</td>
<td>39</td>
<td>95     95     80     86     100</td>
<td></td>
</tr>
<tr>
<td>137</td>
<td>28</td>
<td>76     75     59     80     100</td>
<td></td>
</tr>
<tr>
<td>136</td>
<td>23</td>
<td>81     81     54     86     99</td>
<td></td>
</tr>
<tr>
<td>127</td>
<td>27</td>
<td>77     75     57     80     99</td>
<td></td>
</tr>
<tr>
<td>128</td>
<td>19</td>
<td>80     80     72     82     98</td>
<td></td>
</tr>
<tr>
<td>129</td>
<td>29</td>
<td>78     79     62     77     99</td>
<td></td>
</tr>
<tr>
<td>143</td>
<td>23</td>
<td>87     87     78     68     98</td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>27</td>
<td>71     71     71     63     92</td>
<td></td>
</tr>
<tr>
<td>130</td>
<td>23</td>
<td>54     53     53     51     45</td>
<td></td>
</tr>
<tr>
<td>126</td>
<td>24</td>
<td>82     82     83     82     89</td>
<td></td>
</tr>
<tr>
<td>141</td>
<td>31</td>
<td>73     73     73     63     79</td>
<td></td>
</tr>
<tr>
<td>131</td>
<td>23</td>
<td>80     81     82     81     84</td>
<td></td>
</tr>
</tbody>
</table>

5.4. Extensions
In previous sections, we have made some simplifying assumptions regarding the available data and the user preferences, mainly for ease of the presentation. However, additional available information can be used to fine-tune the estimation of the PoU of each bicycle in the system. We now discuss some enhancements of the model.

5.4.1. User preferences
So far, we have assumed for simplicity that a renter selects uniformly a bicycle from within the set of usable bicycles in the station. However, in some stations, we observe that some lockers are much busier than others, probably due to their distance from the station’s kiosk or due to their accessibility to pedestrians. Gathering information about users’ preferences of lockers can improve the estimations of the PoU. For example, if a locker is less likely to be selected due to its distance from the station’s kiosk, there is a larger probability that a usable bicycle will be parked there for a long period of time. On the other hand, if a preferred locker in which a bicycle is parked is not selected, it is more likely that the bicycle may be unusable.

Next, we introduce additional notation needed in order to incorporate user locker preferences in the model:
\[ l \quad \text{Locker id, } l \in \mathcal{C} \\
\mathbb{L}(i) \quad \text{The locker in which bicycle } i \text{ is parked} \\
\mathbb{L}(S) \quad \text{The set of lockers in which the set of bicycles } S \text{ are parked} \\
a(l, L) \quad \text{The probability that locker } l \text{ will be selected from within the set of lockers } L \subseteq \mathcal{C} \\
\]

The values of locker selection probabilities \( a(l, L) \) satisfy \( \sum_{l \in \mathbb{L}} a(l, L) = 1 \forall L \) and \( a(l, L) = 0 \forall l \in \mathcal{C} \setminus L \). The function \( a(l, L) \) may be defined explicitly for each subset of lockers \( L \) or implicitly by some oracle that is capable of calculating or estimating it. Equations (5.3) and (5.4) can be re-written to accommodate users’ locker preferences as follows, respectively:

\[
P^e(j \text{ rented}) = \left(1 - p_j^{e-1}\right) \cdot \sum_{S \in F(S^e)} a(\mathbb{L}(j), \mathbb{L}(S \cup \{j\})) \cdot \prod_{k \in S} \left(1 - p_k^{e-1}\right) \prod_{k \in S \setminus (S \cup \{j\})} p_k^{e-1}
\]

\[
P^e(i \text{ unusable, } j \text{ rented}) = p_i^{e-1} \cdot \left(1 - p_j^{e-1}\right) \cdot \sum_{S \in F(S^e \setminus \{i, j\})} a(\mathbb{L}(j), \mathbb{L}(S \cup \{j\})) \cdot \prod_{k \in S} \left(1 - p_k^{e-1}\right) \prod_{k \in S \setminus (S \cup \{i, j\})} p_k^{e-1}
\]

Where \( F(S) \) denotes the collection of all subsets of set \( S \). Note that these equations are more complex as compared to (5.3) and (5.4), since not only the number of usable bicycles that are parked in the station is taken into account, but also the location of these bicycles.

Similarly, the approximated conditional probability (5.5) can be updated as follows:

\[
\hat{p}^e_j(j \text{ rented}|j \text{ usable}) = \frac{a(\mathbb{L}(j), \mathbb{L}(S^e))}{a(\mathbb{L}(j), \mathbb{L}(S^e)) + \sum_{k \in S^e \setminus \{j\}} a(\mathbb{L}(k), \mathbb{L}(S^e)) \cdot (1 - \hat{p}_k^{e-1})}
\]

(5.11)

Note that if \( a(\mathbb{L}(i), \mathbb{L}(S^e)) = \frac{1}{|S^e|} \forall i \in S^e \), i.e. all lockers have the same probability to be selected, Equation (5.11) is reduced back to Equation (5.5). In addition, given that all the bicycles in the station are usable, the probability in Equation (5.11) equals \( a(\mathbb{L}(j), L^e) \), namely, the probability that locker \( \mathbb{L}(j) \) will be selected.

Due the same mathematical arguments as in Section 5.2, we obtain the following iterative equation for updating the PoU of bicycle \( i \) after event \( e \):

\[
\hat{p}_i^e = \hat{p}^e(i \text{ unusable}|j \text{ rented})
= \hat{p}_i^{e-1} \cdot \frac{a(\mathbb{L}(j), \mathbb{L}(S^e)) + \sum_{k \in S^e \setminus \{j\}} a(\mathbb{L}(k), \mathbb{L}(S^e)) \cdot (1 - \hat{p}_k^{e-1})}{a(\mathbb{L}(j), \mathbb{L}(S^e \setminus \{i\})) + \sum_{k \in S^e \setminus \{i, j\}} a(\mathbb{L}(k), \mathbb{L}(S^e \setminus \{i\})) \cdot (1 - \hat{p}_k^{e-1})}
\]

Note that if \( a(\mathbb{L}(i), \mathbb{L}(S^e)) \) equals 0, we obtain \( \hat{p}_i^e = \hat{p}_i^{e-1} \). In other words, if the probability that a locker \( \mathbb{L}(i) \) will be selected equals zero or is close to zero, the fact that bicycle \( i \) was not selected in event \( e \) does not provide information regarding the usability of bicycle \( i \).
5.4.2. Station idle time

Until now we considered methods to update the PoU only at rent events. However, there may be situations in which for relatively long periods of time no rent event occurs in a station, even though bicycles are parked in it. This may be explained by one of the following: (1) no renters have arrived at the station (2) renters have arrived at the station but none of the bicycles were in a usable condition and no rent transaction has occurred (3) the station is malfunctioning. Recall that in the information system, there is no direct evidence for any of these occurrences. Here we present a method to update the PoU between rent events in order to call the attention of the operators to situations (2) or (3).

The arrival rates of renters during different time periods along the day can be estimated using trip history transaction data. If no rent event occurs for a long period of time in a non-empty station even though the estimated arrival rate of renters is high, the probability that the parked bicycles in the station are unusable (or cannot be rented due to station failure) increases.

Here we assume that the demand for bicycles is a time heterogeneous Poisson process. We denote by $T$ the elapsed time since the last rent event in a station, and let $t_1, t_2, ..., t_m$ ($T = \sum_{r=1}^{m} t_r$) be the lengths of consecutive time intervals. The expected number of arrivals of renters at each of these time intervals is denoted by $\mu_1, \mu_2, ..., \mu_m$, then the probability that no renter arrived until time $T$ is:

$$\prod_{r=1}^{m} \exp(-\mu_r t_r) = \exp\left(-\sum_{r=1}^{m} \mu_r t_r\right)$$

Given that the elapsed time since the last rent event ($e$) is $T$, the PoU of bicycle $i$ is recalculated as follows. If no renter arrived at the station, the PoU of bicycle $i$ is $\tilde{p}_i^T$. However, if one or more renters arrived at the station but no rent event occurred, then bicycle $i$ is unusable. By conditioning over these two complementary events and multiplying by their corresponding probabilities we can update the PoU of any bicycle $i$ in the station to:

$$\tilde{p}_i^e \cdot \exp\left(-\sum_{r=1}^{m} \mu_r t_r\right) + 1 \cdot \left(1 - \exp\left(-\sum_{r=1}^{m} \mu_r t_r\right)\right)$$

Note that this updating expression depends on the time in which it is performed due to the dependency of $\mu_1, \mu_2, ..., \mu_m$ on this time. Noticeably, the PoU increases with the arrival rates in the given time intervals and the length of these time intervals. This update is effective until the next rent event at the station occurs. Once a rent event occurs the PoU is updated as discussed above in Section 5.2 or as in Section 5.4.1.

5.4.3. Enhancing the estimation of the prior probabilities

In this section, we discuss additional available information that can be used to estimate the prior probabilities. A generic estimator for the prior probability may be obtained by dividing the total number of bicycles repaired in a given period by the total number of the trips taken in the same period. However, the prior of each specific bicycle can be
better estimated given data features such as: elapsed time since its last repair, accumulated riding time, mileage, usage areas, users’ characteristics, etc. Specifically, it might be reasonable to assume that the prior probability of a bicycle that is returned from maintenance is close to zero. For a discussion on classes of life distributions based on notions of aging, see Barlow and Proschan (1975).

In addition to user’s trips and maintenance activities, bicycles may also be removed from a station for the purpose of rebalancing the stations’ bicycle inventory levels (repositioning activities). If the repositioning worker is instructed to check the condition of each loaded/unloaded bicycle, we can assume that when the bicycle is returned to a station at the end of the repositioning its prior probability to be unusable is close to zero. Alternatively, the calculation of the conditional probability may continue from the calculated value right before the repositioning.

Other aspects that can be taken into account when estimating the prior probability are the transactions’ characteristics. For example, a short time (less than two minutes) round-trip (identical start and end stations) transaction suggests that a user unlocked a bicycle from a locker and almost immediately returned it to the same station. This may indicate that the bicycle is unusable. This kind of transaction is not rare; one may evaluate the percentage of times this kind of transaction was followed by a maintenance activity. This can be done by cross-checking transaction history and maintenance data.

Failure reports provided by users, i.e., by complaint calls or by a maintenance button, installed on the locker, can also be incorporated into the model. In particular, if user complaints are considered highly reliable, the reported bicycles can be flagged as unusable, i.e. $p_i^e = 1$. Given such information, the unusable bicycle can be removed from the set of available bicycles in the station, and the PoU of the other bicycles can then be updated accordingly.

5.4.4. Locker failure detection

Another failure type that may decrease the quality of service is locker failures. Specifically, the electro-mechanical locking system may sometimes fail to work properly. When this occurs, the users cannot rent or return the bicycles at such lockers. If the locker is occupied with a bicycle, this bicycle will eventually be flagged as unusable using our method. However, if the faulty locker is vacant it will be left empty until the locking mechanism is repaired. Such type of failure is not reported in the information system, and so the on-line state of the stations presented to the users may not be accurate. Currently, the operators cannot remotely detect such failures.

A complementary model equivalent to the one presented in Sections 5.1 and 5.2 may be formulated in order to assess the usability of vacant lockers. As a mirror scenario, the data to be used are the returning transactions. On each return of a bicycle to a station, we calculate the conditional probability that a locker is unusable given that bicycles were not returned to it. And again, as more return events occur in a station there is a greater probability that a locker that is left empty is unusable.
5.5. Discussion
In this chapter, we presented a method to detect unusable resources (bicycles and lockers). The negative implication of the presence of unusable resources is the reduction of the station capacity and the presentation of misleading information to the users. Unusable bicycles/lockers may have different effect on the quality of service in different stations, depending on the capacity of the station and the demand patterns. Evaluating this effect may assist in prioritizing the stations that should be visited by maintenance and repositioning workers. Once this is determined, the next planning stage is to determine the routes of the maintenance workers.
6. User dissatisfaction in the presence of unusable bicycles

A paper based on the research presented in this chapter was submitted for publication under the title: M. Kaspi, T. Raviv and M. Tzur, Bike Sharing Systems: User Dissatisfaction in the Presence of Unusable Bicycles. (Under third round of review in IIE Transactions)

In this Chapter, we examine the effect of unusable bicycles on the service level provided in a single station. Obviously, the effect of the presence of unusable bicycles may differ between stations, depending on their capacities and the demand processes for bicycles and lockers. A better understanding of this effect will assist in better planning their collection. Clearly, in order to maximize the service level, all unusable bicycles should be removed from the stations or be repaired as soon as possible. However, since the transportation and maintenance resources of the operator are limited, a method to estimate the expected effect of the unusable bicycles at each station can help in prioritizing these operations.

The contribution of this chapter is as follows: we introduce an Extended User Dissatisfaction Function (EUDF) that represents the expected weighted number of users that are unable to rent or return a bicycle during a given period as a bivariate function of the initial number of usable and unusable bicycles at the station. This is an extension of the User Dissatisfaction Function (UDF) that was initially presented in Raviv and Kolka (2013), which assumed that bicycles are always usable. We prove some discrete convexity properties of the EUDF. In addition, we propose a method for calculating a convex polyhedral function that has nearly identical values as the EUDF in its range. This polyhedral approximation can be used to optimize the initial bicycle inventories in the system subject to various constraints using linear programming, see for example the Appendix D.

This Chapter is structured as follows. In Section 6.1 a formulation of the EUDF for a single station is presented. Properties of the EUDF, including a convexity analysis are provided in Section 6.2. In Section 6.3, a method to approximate the EUDF by a convex polyhedral function is presented. Results of a numerical experiment that examines the accuracy of the approximation are reported in Section 6.4. Concluding remarks are given in Section 6.5.

6.1. Extended user dissatisfaction function

In this section we present an extension of the UDF that was introduced by Raviv and Kolka (2013). Assuming that a station is not visited by repositioning vehicles throughout a given period (say, a day), the UDF represents the user dissatisfaction (a measure for the service level) as a discrete function of the initial number of bicycles in the station. Specifically, the user dissatisfaction is expressed as a weighted sum of the expected shortages of bicycles and the expected shortages of lockers along the given period. In Raviv and Kolka (2013) it is proven that the UDF is a convex function of the initial inventory of bicycles.

We begin by describing some notation and modeling assumptions that were presented in Raviv and Kolka (2013), which are also a part of the extended model.
Subsequently, we present some revised notation and additional assumptions needed for the extended model.

We model a single station during a finite period \([0, T]\). Initially, at time 0, there is a certain number of bicycles in the station at time 0. During this period, users who wish to rent or return a bicycle arrive to the station according to an arbitrary stochastic process. If the demand for a bicycle/available locker can be satisfied, the bicycle is rented/returned and the station’s inventory level is updated accordingly. On the other hand, if the demand cannot be satisfied, we assume that the user immediately abandons the station (she either roams to a nearby station or abandons the system). We note that the model does not take into account mutual influences of neighboring stations on the demand process.

There are two sources for user dissatisfaction, namely shortage of bicycles and shortage of lockers. For each type of shortage the system is penalized by an amount that represents the dissatisfaction caused due to this shortage. We denote by \(p\) the penalty for each user who faces shortage of bicycles and by \(h\) the penalty for each user who faces shortage of lockers. The total number of lockers in the station is denoted by \(C\). We refer to this value as the capacity of the station.

In the EUDF an additional dimension is introduced. The initial inventory of bicycles is divided into two groups, namely, usable and unusable (broken) bicycles, denoted by \(I_0\) and \(B_0\), respectively. This extension allows us to examine the effect of changes in the initial inventory level of each group on the service level, given their joint station capacity. In particular, the effect of the presence of unusable bicycles can be studied. However, the analysis of the EUDF becomes more difficult, as will be described in Section 6.2.

We assume that during the given period, the inventory level is not externally altered, that is, until time \(T\), no repositioning or repairing activities are performed in the station. In particular, this implies that the number of unusable bicycles in the station cannot decrease during the given period, since it is assumed that unusable bicycles would not be rented by the users. However, some bicycles may become unusable during the ride, that is, some bicycles may be returned unusable to the station. Therefore, the number of unusable bicycles in the station may increase during the given period. Lastly, we assume that there is no change in the condition of the bicycles while they are parked in the station.

Let \(E_R\) denote the time epochs in which the demands for bicycles or lockers occur under demand realization \(R\). We denote by \(I_j^R(I_0, B_0)\) the inventory level of usable bicycles right after the \(j^{th}\) demand occurrence under realization \(R\), given the inventory of usable and unusable bicycles \((I_0, B_0)\) at time 0. For the sake of brevity, we omit the conditioning on the initial inventory in subsequent notation. Similarly, the inventory level of unusable bicycles right after the \(j^{th}\) demand occurrence under realization \(R\) is denoted by \(B_j^R(I_0, B_0)\). The demand for bicycles or lockers at the \(j^{th}\) occurrence is denoted by the pair \((d_j^{R,1}, d_j^{R,2})\) \(\in\) \{\((1,0), (-1,0), (0,-1)\}\), where \((1,0)\) represents a demand for a usable bicycle, \((-1,0)\) represents a demand for a locker in order to return a usable bicycle and \((0,-1)\) represents a demand for a locker in order to return an unusable
Next, we present a recursive function, denoting the number of usable bicycles in the station after the occurrence of the $j^{th}$ demand, given the inventory levels after the $(j-1)^{st}$ demand occurrence:

$$I^R_j(I_0, B_0) = \begin{cases} 
0 & I^R_{j-1}(I_0, B_0) - d^R_j < 0 \\
I^R_{j-1}(I_0, B_0) - d^R_j & I^R_{j-1}(I_0, B_0) - d^R_j > C - B^R_{j-1}(I_0, B_0) \\
I^R_{j-1}(I_0, B_0) & \text{otherwise}
\end{cases}$$

And the number of unusable bicycles in the station is given by the following:

$$B^R_j(I_0, B_0) = \begin{cases} 
C - I^R_{j-1}(I_0, B_0) & B^R_{j-1}(I_0, B_0) - d^R_j > C - I^R_{j-1}(I_0, B_0) \\
B^R_{j-1}(I_0, B_0) - d^R_j & \text{otherwise}
\end{cases}$$

We refer to $C - B^R_j(I_0, B_0)$ as the effective capacity of the station. Since unusable bicycles cannot be rented, they block the lockers in which they are parked.

Let $\Delta^R_j(I_0, B_0)$ and $\Theta^R_j(I_0, B_0)$ be indicator functions that indicate whether a user faces shortage of a bicycle or a locker as the $j^{th}$ demand occurs. Let $(x)^+ = \max\{0, x\}$, then the bicycle shortage indicator is given by $\Delta^R_j(I_0, B_0) = (-I^R_{j-1}(I_0, B_0) + d^R_j)^+$ and the locker shortage indicator is given by $\Theta^R_j(I_0, B_0) = (I^R_{j-1}(I_0, B_0) + B^R_{j-1}(I_0, B_0) - d^R_j - d^R_{j-1} - C)^+$.

We denote by $F^R(I_0, B_0)$ the total dissatisfaction of users under demand realization $R$. The total dissatisfaction is obtained by summing all the shortages for bicycles and lockers and multiplying each shortage by the related penalty:

$$F^R(I_0, B_0) = \sum_{j=1}^{\mid\mathcal{E}^R\mid} \left( p \cdot \Delta^R_j(I_0, B_0) + h \cdot \Theta^R_j(I_0, B_0) \right)$$

Then, we denote by $F(I_0, B_0)$ the expected penalty (over all realizations) due to shortages of bicycles and lockers during the given period as a discrete function of the initial inventory of usable ($I_0$) and unusable ($B_0$) bicycles. We refer to this function as the EUDF and it is given by the following equation:

$$F(I_0, B_0) \equiv \mathbb{E}_R\{F^R(I_0, B_0)\} = \mathbb{E}_R\left\{ \sum_{j=1}^{\mid\mathcal{E}^R\mid} \left( p \cdot \Delta^R_j(I_0, B_0) + h \cdot \Theta^R_j(I_0, B_0) \right) \right\} \quad (6.1)$$

Note that the UDF is a special case of this model in which $B_0 = 0$ and there is no demand for lockers in order to return unusable bicycles.
6.2. Analysis of the EUDF

In this section we analyze the EUDF (6.1) and study its convexity, which is helpful for optimization purposes. In Section 6.2.1 we prove several properties of the EUDF, which are later used in its convexity analysis, presented in Section 6.2.2.

6.2.1. Properties of the EUDF

We begin our analysis of the EUDF by proving that it is non-decreasing in the initial inventory of unusable bicycles. Intuitively, this is true because an addition of unusable bicycles decreases the effective capacity of the station. We next prove this observation formally.

We denote by $\Omega$ a sequence of demand occurrences which do not include a returning of an unusable bicycle. Note that $\Omega$ can represent an entire demand realization or a subset of it. Thus, $F^\Omega(I_0, B_0)$ denotes the user dissatisfaction under this sequence of demand occurrences. After analyzing such sequences, we will extend our analysis to any demand realization.

**Lemma 6.1:** For any sequence of demand occurrences $\Omega$, the following inequality holds: $F^\Omega(I_0, B_0 + 1) \geq F^\Omega(I_0, B_0)$.

Proof: Consider two initial settings of a station: $(I_0, B_0 + 1)$ and $(I_0, B_0)$, i.e., when the number of usable bicycles is identical, but the number of unusable bicycles differs by one. We claim that the number of usable bicycles under both settings may differ by at most 1 at any time, namely, either $I^\Omega_j(I_0, B_0 + 1) = I^\Omega_j(I_0, B_0)$ or $I^\Omega_j(I_0, B_0 + 1) = I^\Omega_j(I_0, B_0) - 1$. This will be demonstrated using Table 6.1. In Table 6.1 we describe four different shortage events that may occur in either of these settings. In the second column we present the relations between usable inventory levels after the $(j - 1)^{st}$ demand occurrence under settings $(I_0, B_0)$ and $(I_0, B_0 + 1)$. In the third column we describe the type of shortage, namely, bicycle or locker. In the fourth and fifth columns we denote under which setting this shortage occurs. Finally, in the sixth column we present the relations between the inventory levels under the two settings that result from the shortage event. Note that at time 0, the inventory levels of usable bicycles are identical under both settings so that the first shortage event may be either 2 or 4. The sixth column in Table 6.1 demonstrates that the relation between the usable inventory levels under both settings may be either $I^\Omega_j(I_0, B_0 + 1) = I^\Omega_j(I_0, B_0)$ or $I^\Omega_j(I_0, B_0 + 1) = I^\Omega_j(I_0, B_0) - 1$. In the latter case, the subsequent shortage event may be either 1 or 3. Resulting again in the same possible inventory relations.
Table 6.1: Shortage events in two settings with initial inventories \((I_0, B_0 + 1)\) and \((I_0, B_0)\)

<table>
<thead>
<tr>
<th>Shortage event</th>
<th>Usable bicycles before the shortage occurs</th>
<th>Shortage type</th>
<th>((I_0, B_0 + 1))</th>
<th>((I_0, B_0))</th>
<th>Usable bicycles after the shortage occurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(l_{1,2}^0(I_0, B_0 + 1) = l_{1,2}^0(I_0, B_0) - 1)</td>
<td>Bicycle</td>
<td>(\Delta_1^0(I_0, B_0 + 1) = 1)</td>
<td>(\Delta_1^0(I_0, B_0) = 0)</td>
<td>(l_{1,2}^0(I_0, B_0 + 1) = l_{1,2}^0(I_0, B_0))</td>
</tr>
<tr>
<td>2</td>
<td>(l_{1,0}^0(I_0, B_0 + 1) = l_{1,0}^0(I_0, B_0))</td>
<td>Bicycle</td>
<td>(\Delta_1^0(I_0, B_0 + 1) = 1)</td>
<td>(\Delta_1^0(I_0, B_0) = 1)</td>
<td>(l_{1,0}^0(I_0, B_0 + 1) = l_{1,0}^0(I_0, B_0))</td>
</tr>
<tr>
<td>3</td>
<td>(l_{1,1}^0(I_0, B_0 + 1) = l_{1,1}^0(I_0, B_0))</td>
<td>Locker</td>
<td>(\Theta_1^0(I_0, B_0 + 1) = 1)</td>
<td>(\Theta_1^0(I_0, B_0) = 1)</td>
<td>(l_{1,1}^0(I_0, B_0 + 1) = l_{1,1}^0(I_0, B_0))</td>
</tr>
</tbody>
</table>

It is noticeable from the fourth and fifth columns of Table 6.1 that whenever a shortage occurs under setting \((I_0, B_0)\) it also occurs under setting \((I_0, B_0 + 1)\), but not the opposite. That is, for any occurrence of shortage, we have \(\Delta_j^0(I_0, B_0 + 1) \geq \Delta_j^0(I_0, B_0)\) and \(\Theta_j^0(I_0, B_0 + 1) \geq \Theta_j^0(I_0, B_0)\). In addition, for any demand occurrence where no shortage occurs, all indicators equal zero, and the inequalities hold trivially. By summing these inequalities for all demand occurrences and multiplying by the related penalties we obtain for the set \(\Omega\): \(F^\Omega(I_0, B_0 + 1) \geq F^\Omega(I_0, B_0)\)

**Theorem 6.1:** The EUDF \(F(I_0, B_0)\) is non-decreasing in the initial inventory of unusable bicycles \(B_0\).

Proof: Consider the shortage occurrences given two initial settings of a station: \((I_0, B_0 + 1)\) and \((I_0, B_0)\). We will show that \(F^R(I_0, B_0 + 1) \geq F^R(I_0, B_0)\), for any demand realization \(R\) and thus claim that it holds for the expectation. For a given demand realization \(R\), we divide the set of demand occurrences \(E^R\) to sequences of demand occurrences such that in each sequence there are no return attempts of unusable bicycles and the sequences are separated by return attempts of unusable bicycles. Note that for the first sequence the conditions are as in Lemma 6.1, i.e. the inequality holds. Following this sequence there are three different possibilities: (i) it is possible to return the unusable bicycle under both settings. (ii) it is not possible to return the unusable bicycle under both settings. (iii) it is possible to return the unusable bicycle under setting \((I_0, B_0)\) but not under \((I_0, B_0 + 1)\). When (i) or (ii) occurs, the difference in the total number of shortages up to this point, between the two settings remains unchanged and the same analysis may be repeated for the next sequence, since there is still a difference of one unusable bicycle between the two. After (iii) occurs the number of usable and unusable bicycles are identical under both settings, therefore, from this point and on the station faces exactly the same shortages under both initial settings. Now, since this is true for any demand realization it is also true for the expectation, thus we obtain: \(F(I_0, B_0 + 1) \geq F(I_0, B_0)\).

**Remark:** Since the EUDF \(F(I_0, B_0)\) is non-decreasing in \(B_0\), the function is minimized at \(B_0 = 0\), as expected. However, due to time and capacity constraints, the operator may not be able to remove all unusable bicycles (or even visit all stations in which there are unusable bicycles) therefore it is important to analyze the EUDF for all possible values of \((I_0, B_0)\).
Next, we prove the following three inequalities, which are needed for the convexity proof of the EUDF that will be presented in Section 6.2.2.

- $F(I_0, B_0 + 2) - F(I_0, B_0 + 1) - F(I_0, B_0 + 1) + F(I_0, B_0) \geq 0$
- $F(I_0 + 2, B_0) - F(I_0 + 1, B_0) - F(I_0 + 1, B_0) + F(I_0, B_0) \geq 0$
- $F(I_0 + 1, B_0 + 1) - F(I_0 + 1, B_0) - F(I_0, B_0 + 1) + F(I_0, B_0) \geq 0$

Observe that the first two inequalities mean that the EUDF is convex in each of the variables $(I_0, B_0)$ independently. The proofs for these inequalities are given under the following assumption:

**Assumption 6.1:** No unusable bicycles are returned to the station during the given period.

While this assumption may seem restrictive, note that the probability that a returned bicycle is unusable is low (see, for example, the discussion about the maintenance reports of NYC Bikeshare in Section 6.4). Hence, a major share of the effect of unusable bicycles is already captured by the unusable bicycles that are already parked in the station, i.e., in the initial state of the station. We note that without Assumption 6.1 it is possible to “cook” an example in which the EUDF is non-convex. However, note that the approximation method of the EUDF presented in Section 6.3 does not rely on Assumption 6.1. Moreover, in section 6.4, we evaluate the EUDF using real life demand data, including returns of unusable bicycles and confirm that the convexity conditions hold or, at worst, are violated with a negligible margin.

**Lemma 6.2:** Under Assumption 6.1, the EUDF $F(I_0, B_0)$ is convex in the initial inventory of unusable bicycles $B_0$, i.e.: $F(I_0, B_0 + 2) - F(I_0, B_0 + 1) \geq F(I_0, B_0 + 1) - F(I_0, B_0)$.

The proof of this Lemma is based on an approach similar to the one used in the proof of Lemma 6.1. For brevity of the main text, we present the complete proof in Appendix C.

**Lemma 6.3:** under Assumption 6.1, the EUDF $F(I_0, B_0)$ is convex in the initial inventory of usable bicycles $I_0$, i.e.: $F(I_0 + 2, B_0) - F(I_0 + 1, B_0) \geq F(I_0 + 1, B_0) - F(I_0, B_0)$.

We remark that the convexity proof provided in Raviv and Kolka (2013) can be used here since the effective capacity under all three settings is equal and remains constant during the entire given period. Here we prove this result through an alternative approach which is later used in the proof of Lemma 6.4.

Proof: Note that for each side of the inequality the station’s initial setting varies only by the initial inventory of usable bicycles. Under Assumption 6.1, the number of unusable bicycles remains the same in all these settings. Therefore, in a pair of settings, once a shortage occurs in one of the settings, either for a bicycle or for a locker, the number of usable bicycles equalizes and from that point on the number of shortages are equal under both settings. For example, for settings $(I_0 + 1, B_0)$ and $(I_0, B_0)$ if the first
shortage is for a bicycle, then the demand can be satisfied by setting \((I_0 + 1, B_0)\) but not by setting \((I_0, B_0)\) so that right after the shortage occurs, under both settings the station is empty. Similarly, if the first shortage is for a locker, then the demand can be satisfied under setting \((I_0, B_0)\) but not under setting \((I_0 + 1, B_0)\) so that right after the shortage occurs, under both settings the station is full. That is, for a given realization, the shortage difference between the two settings may be either -1,0 or 1. In Table 6.2 we compare the two sides of the inequality by exhibiting all possible combinations of first shortage occurrences for a given demand realization \(R\). As can be seen, for all possible combinations we obtain \(F^R(I_0 + 2, B_0) - F^R(I_0 + 1, B_0) \geq F^R(I_0 + 1, B_0) - F^R(I_0, B_0)\). Consequently, by summing over all demand realizations we obtain: \(F(I_0 + 2, B_0) - F(I_0 + 1, B_0) \geq F(I_0 + 1, B_0) - F(I_0, B_0)\). 

**Lemma 6.4:** under Assumption 6.1, for the EUDF \(F(I_0, B_0)\) the following inequality is maintained:

\[
F(I_0 + 1, B_0 + 1) - F(I_0, B_0 + 1) \geq F(I_0 + 1, B_0) - F(I_0, B_0)
\]

Proof: Observe again that in each side of the inequality the settings differ by one usable bicycle, therefore we can again compare the first shortage events (as in the proof of Lemma 6.3). In Table 6.3 we present the possible combinations of first shortage occurrences for the two sides of the inequality. Note that the last combination presented in Table 6.2 is not possible in this case and therefore does not appear in Table 6.3. It is observable from Table 6.3 that for all possible combinations we obtain \(F^R(I_0 + 1, B_0 + 1) - F^R(I_0, B_0 + 1) \geq F^R(I_0 + 1, B_0) - F^R(I_0, B_0)\). Consequently, by summing over all demand realizations we obtain: \(F(I_0 + 1, B_0 + 1) - F(I_0, B_0 + 1) \geq F(I_0 + 1, B_0) - F(I_0, B_0)\). 

**Table 6.2: Possible combinations of first shortage occurrences**

<table>
<thead>
<tr>
<th>First shortage occurrence</th>
<th>Difference</th>
<th>First shortage occurrence</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycle</td>
<td>(-p)</td>
<td>Bicycle</td>
<td>(-p)</td>
</tr>
<tr>
<td>Locker</td>
<td>(h)</td>
<td>Bicycle</td>
<td>(-p)</td>
</tr>
<tr>
<td>Locker</td>
<td>(h)</td>
<td>Locker</td>
<td>(h)</td>
</tr>
<tr>
<td>None</td>
<td>0</td>
<td>None</td>
<td>0</td>
</tr>
<tr>
<td>None</td>
<td>0</td>
<td>Bicycle</td>
<td>(-p)</td>
</tr>
</tbody>
</table>

**Table 6.3: Possible combinations of first shortage occurrences**

<table>
<thead>
<tr>
<th>First shortage occurrence</th>
<th>Difference</th>
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<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycle</td>
<td>(-p)</td>
<td>Bicycle</td>
<td>(-p)</td>
</tr>
<tr>
<td>Locker</td>
<td>(h)</td>
<td>Bicycle</td>
<td>(-p)</td>
</tr>
<tr>
<td>Locker</td>
<td>(h)</td>
<td>Locker</td>
<td>(h)</td>
</tr>
<tr>
<td>Locker</td>
<td>(h)</td>
<td>None</td>
<td>0</td>
</tr>
<tr>
<td>None</td>
<td>0</td>
<td>None</td>
<td>0</td>
</tr>
</tbody>
</table>
6.2.2. Convexity analysis of the EUDF

Recall that the EUDF is a bivariate discrete function. While the concept and definition of discrete convexity of univariate functions is quite similar to continuous convexity, this is not the case for multivariate discrete functions. In fact, several different definitions of convexity are given in the literature for multivariate discrete functions. In Murota and Shioura (2001), Murota (2009) and Moriguchi and Murota (2011), several classes of multivariate discrete convex functions are defined and the relationship among these classes is presented. We next outline some of these definitions and then prove that under Assumption 6.1, the EUDF is contained in these classes.

Definition 6.1: Convex extensibility (Murota 2009)
A function \( f : \mathbb{Z}^n \to \mathbb{R} \) is said to be \textit{convex-extensible} if there exists a convex function \( \tilde{f} : \mathbb{R}^n \to \mathbb{R} \) such that \( \tilde{f}(x) = f(x) \) for all \( x \in \mathbb{Z}^n \).

Definition 6.2: \( M^\natural \)-convex (based on Moriguchi and Murota 2011)
Denote the \( i^{th} \) unit vector by \( e_i \) and \( e_0 = 0 \), denote the domain of \( f \) by \( \text{dom} f = \{ x \in \mathbb{Z}^n | f(x) < +\infty \} \) and denote the positive and negative supports of a vector \( x \) by:

\[
\text{supp}^+(x) = \{ i \in \{1, \ldots, n\} | x_i > 0 \}
\]

\[
\text{supp}^-(x) = \{ i \in \{1, \ldots, n\} | x_i < 0 \}
\]

A function \( f : \mathbb{Z}^n \to \mathbb{R} \) is \( M^\natural \)-\textit{convex} if it satisfies the following exchange property:

\( (M^\natural \text{-EXC}) \forall x, y \in \text{dom} f, \forall i \in \text{supp}^+(x - y), \exists j \in (\text{supp}^-(x - y) \cup \{0\}) \) such that

\[
f(x) + f(y) \geq f(x - e_i + e_j) + f(y + e_i - e_j).
\]

See Murota and Shioura (2001) for a further discussion on \( M^\natural \)-convexity.

Definition 6.3: Discrete Hessian matrix (Moriguchi and Murota 2011)
The discrete Hessian \( H(x) = \left( H_{ij}(x) \right) \) of \( f : \mathbb{Z}^n \to \mathbb{R} \) at \( x \in \mathbb{Z}^n \) is defined by

\[
H_{ij}(x) = f(x + e_i + e_j) - f(x + e_i) - f(x + e_j) + f(x)
\]

Definition 6.4: (Theorem 3.1 in Moriguchi and Murota 2011)
A function \( f : \mathbb{Z}^n \to \mathbb{R} \) is \( M^\natural \)-\textit{convex} if and only if the discrete Hessian matrix \( H(x) \) in Definition 6.3 satisfies the following conditions for each \( x \in \mathbb{Z}^n \):

(i) \( H_{ij}(x) \geq \min \left( H_{ik}(x), H_{jk}(x) \right) \) if \( \{i, j\} \cap \{k\} = \emptyset \)

(ii) \( H_{ij}(x) \geq 0 \) for any \( \{i, j\} \).

Note that \( H_{ii}(x) \geq 0 \) means that \( f \) is convex in the variable \( i \).

Theorem 6.2: (Follows from Theorem 3.9 and Theorem 3.3 in Murota and Shioura 2001)
An \( M^\natural \)-convex function is convex-extensible.
Next, given the above definitions, we present and prove the main theorem of this study:

**Theorem 6.3:** Under Assumption 6.1, the EUDF $F(I_0, B_0)$ is $M^\square$-convex.

Proof: Since the EUDF is a bivariate function, condition (i) of Definition 6.4 is not relevant for our analysis, and condition (ii) of Definition 6.4 reduces to the three inequalities that were presented and proved in Section 6.2.1. Given the proofs of Lemmas 6.2-6.4, the EUDF satisfies the conditions given in Definition 6.4. Therefore the discrete Hessian of the EUDF is positive semidefinite and the EUDF is $M^\square$-convex.

Note that since the EUDF is $M^\square$-convex we conclude by Theorem 6.2 that it is also convex-extendible. Therefore, there exists a continuous convex function that has identical values at all integer points in the range of the EUDF. In the next section we present a method to approximate the EUDF by a convex polyhedral function that has the same values at integer points. Under Assumption 6.1, the EUDF is convex-extendible and therefore the approximation will provide an exact description of the function. More importantly, the results of the numerical experiment that will be presented in Section 6.4, demonstrate that even if Assumption 6.1 is relaxed, the approximation is very accurate.

### 6.3. A convex polyhedral function approximation

The approximation procedure of the EUDF is divided into two steps. In the first step we approximate the values of the EUDF for each possible combination of integer initial inventory levels $(I_0, B_0)$. In the second step an LP model is used to fit a convex polyhedral function to the values calculated in the first stage. That is, the epigraph of the EUDF is defined, approximately, as an intersection of half spaces.

Recall that the EUDF is the expectation of all possible demand realizations. One approach for estimating the expectations is by using Monte Carlo simulation. However, this process may require long calculation times and can be very noisy. Moreover since this calculation needs to be carried out for each possible initial setting and for every station in the bike-sharing system, this approach seems impractical. Instead, we adopt an approximation approach which is similar to the one presented in Raviv and Kolka (2013). This approach is based on a representation of the states of the station along the given period as a continuous time Markov chain.

Toward that, we assume that the arrival processes of renters and returners to the station are time heterogeneous Poisson processes, with arrival rates $\mu_t$ and $\lambda_t$, respectively. When a user returns a bicycle to the station there is a probability $\phi$ that the bicycle is unusable. That is, the returning rate of usable bicycles at time period $t$ is $(1 - \phi)\lambda_t$ and the returning rate of unusable bicycles at time period $t$ is $\phi\lambda_t$. Since the arrival processes of renters and returners reflect the arrivals of many independent users, we believe that this Markovian model is an adequate description of reality.
Recall that during the given period, no repositioning activities are being executed. While the inventory level of usable bicycles may increase or decrease along the day, the inventory level of unusable bicycles may only increase. Therefore, for any $\phi > 0$, in steady-state, the station will be full with unusable bicycles. However, we are interested in analyzing the dynamics of the station rather than its steady-state. Moreover, a station that is regulated in a sufficient manner is not likely to reach its steady-state. A description of the continuous-time Markov chain that represents the dynamic of the station is given in Figure 6.1.

![Figure 6.1 - Continuous-time Markov chain that represents the dynamics of the usable and broken bicycles inventory levels](image)

Let $\pi_{(I_0,B_0),(I,B)}(t)$ denote the probability that the station is in state $(I,B)$ at time $t$ given that in time 0 it was in state $(I_0,B_0)$. Now, it is possible to state the EUDF in terms of the transition probabilities as follows:

$$F(I_0, B_0) = \int_0^T \left( \sum_{k=B_0}^{C} \pi_{(I_0,B_0),(0,k)}(t) \right) \mu_t p \ dt + \left( \sum_{k=B_0}^{C} \pi_{(I_0,B_0),(C-k,k)}(t) \right) \lambda_t h \ dt \quad (6.2)$$

The first term in the integral represents the user dissatisfaction due to bicycle shortages. It is calculated by the probability that at time $t$ the station is empty, multiplied by the renting rate $\mu_t$ and the penalty for bicycle shortage $p$. The second term represents the user dissatisfaction due to locker shortages. It is calculated by the probability that at time $t$ the station is full, multiplied by the returning rate $\lambda_t$ and the penalty for locker shortage $h$. The evaluation of (6.2) is numerically obtained by discretizing the integral to short intervals of length $d$ and calculating the following sum:
\[ F(I_0, B_0) = d \sum_{i=1}^{T/d} \left( \sum_{k=B_0}^{C} \pi(I_0, B_0), (0,k) \right) ((i - 0.5)d) \mu_i P \]

\[ + \left( \sum_{k=B_0}^{C} \pi(I_0, B_0), (C-k,k) \right) ((i - 0.5)d) \lambda_i h \]

It is assumed that \( T/d \) is an integer. The value of \( \pi(I_0, B_0), (I,B) \) is numerically evaluated for each of the \( T/d \) points in time by the method presented in Raviv and Kolka (2013), we refer the reader to section 4 in their paper.

Next we discuss the fitting of a convex polyhedral function to the approximated values of the EUDF. As the EUDF is convex-extensible (under Assumption 6.1), there exists a continuous convex function that has identical values as the EUDF in all integer points. We denote this function by \( f(I_0, B_0) \). Though this function is unknown, we can use the fact that it is convex. First, let us state the following proposition:

**Proposition 6.1**: Supporting plane (Definition 1.2.3 in Ben-Tal and Nemirovski 2013)

Let \( M \) be a convex closed set in \( \mathbb{R}^n \), and let \( x \) be a point from the relative boundary of \( M \). A hyperplane

\[ \Pi = \{ y | a^T y = a^T x \} \ [a \neq 0] \]

is called supporting to \( M \) at \( x \), if it properly separates \( M \) and \( \{ x \} \), i.e., if

\[ a^T x \geq \sup_{y \in M} a^T y \quad \& \quad a^T x > \inf_{y \in M} a^T y \]

**Proposition 6.1**: Supporting a convex function (adapted from Proposition 2.6.2 in Ben-Tal and Nemirovski 2013)

For any point \( \bar{x} \) in the domain of a convex function \( f: \mathbb{R}^n \rightarrow \mathbb{R} \) there exists an affine function \( f_{\bar{x}}(x) = a^T x + b \), such that \( f_{\bar{x}}(\bar{x}) = f(\bar{x}) \) and \( f_{\bar{x}}(x) \leq f(x) \) for all \( x \in \mathbb{R}^n \).

Let \( \theta \) be the range of the EUDF, namely \( \theta = \{ (I_0, B_0) \in \mathbb{Z}^2 | I_0 \geq 0, B_0 \geq 0, I_0 + B_0 \leq C \} \). Since \( f(I_0, B_0) \) is convex and given Proposition 6.1, for each point \( (\bar{I}_0, \bar{B}_0) \in \theta \) there exists a plane that satisfies:

\[ \alpha(I_0, B_0) I + \beta(I_0, B_0) B + \gamma(I_0, B_0) = f(\bar{I}_0, \bar{B}_0) \] (6.3)

and

\[ \alpha(I_0, \bar{B}_0) I_0 + \beta(I_0, \bar{B}_0) B_0 + \gamma(I_0, \bar{B}_0) \leq f(I_0, B_0) \ \forall (I_0, B_0) \in \theta \] (6.4)

By generating a supporting plane for each point \( (I_0, B_0) \in \theta \), we obtain the following convex polyhedral function:

\[ \tilde{f}(I_0, B_0) = \max_{(I_0, B_0) \in \theta} \left( \alpha(I_0, \bar{B}_0) \cdot I_0 + \beta(I_0, \bar{B}_0) \cdot B_0 + \gamma(I_0, \bar{B}_0) \right) \]

Note that that for each point \((I_0, B_0) \in \theta \) we have \( \tilde{f}(I_0, B_0) = f(I_0, B_0) = F(I_0, B_0) \).

However, the calculation of a plane that satisfies (6.3) and (6.4) may, in some cases, be impossible due to the following reasons: (i) when Assumption 6.1 is relaxed, the
EUDF is not necessarily convex-extensible. (ii) in the calculation of the approximated EUDF values, numerical errors may occur. To overcome these issues, we construct a plane for point \((\tilde{I}_0, \tilde{B}_0)\) such that (6.4) is satisfied but some error is allowed in (6.3). Namely, the constructed plane may pass under \(F(\tilde{I}_0, \tilde{B}_0)\). Our goal is to construct a plane that passes as close as possible to \(F(\tilde{I}_0, \tilde{B}_0)\). We use the following LP formulation to achieve this goal:

Decision variables:

\[
\alpha, \beta, \gamma \quad \text{Coefficients of the fitted plane}
\]

\[
s \quad \text{Gap at the point } (\tilde{I}_0, \tilde{B}_0)
\]

Model:

\[
\text{Minimize} \quad s
\]

s.t.

\[
\alpha \cdot I_0 + \beta \cdot B_0 + \gamma \leq F(I_0, B_0) \quad \forall (I_0, B_0) \in \theta \backslash \{(\tilde{I}_0, \tilde{B}_0)\} \quad (6.6)
\]

\[
\alpha \cdot \tilde{I}_0 + \beta \cdot \tilde{B}_0 + \gamma = F(\tilde{I}_0, \tilde{B}_0) - s \quad (6.7)
\]

\[
\alpha, \beta, \gamma \quad \text{free} \quad (6.8)
\]

\[
s \geq 0 \quad (6.9)
\]

The objective function (6.5) minimizes the gap between the fitted plane and the EUDF at point \((\tilde{I}_0, \tilde{B}_0)\). Constraint (6.6) requires that the fitted plane pass under the EUDF at all other integer points. Constraint (6.7) defines the gap between the plane and the EUDF at point \((\tilde{I}_0, \tilde{B}_0)\). In Constraints (6.8)-(6.9) the definitions of the decision variables are given.

It is possible to redefine \(\tilde{f}(I_0, B_0)\) with the values of \(\alpha_{(I_0, B_0)}, \beta_{(I_0, B_0)}\) and \(\gamma_{(I_0, B_0)}\) obtained by the LP above for all \((I_0, B_0) \in \theta\). The maximal value of \(s\) over all the points \((I_0, B_0)\) is an upper bound on the gap between \(\tilde{f}(I_0, B_0)\) and \(F(I_0, B_0)\). In cases where the gap is zero for all constructed planes, we can say that the EUDF is convex-extensible.

### 6.4. Numerical results

In this section, we evaluate the accuracy of the polyhedral approximation of the EUDF and derive some insights regarding the effect of unusable bicycles on user dissatisfaction. As a case study, we use trip data of 232 stations in the Washington D.C. bike-sharing system, Capital Bikeshare, during the second quarter of 2013. The data can be downloaded from the system’s website: [http://www.capitalbikeshare.com/trip-history-data](http://www.capitalbikeshare.com/trip-history-data). Using this data we have estimated the renting and returning rates on weekdays in each station for each interval of 30 minutes for a period of 24 hours starting at midnight.

The approximation of the EUDF was coded in MathWorks Matlab. The plane fitting LP model was solved using IBM ILOG CPLEX Optimization Studio 12.6. The procedure was tested on an Intel Core i7 desktop. On average, the entire process of
calculating the EUDF took three seconds per station and the polyhedral function fitting took less than a second per station. This means that approximating the function for all stations as an input for a repositioning and collection optimization can be executed in acceptable time. Moreover, since the calculation for each station is done independently, the calculation procedure is amendable for parallelization. In addition, the renting/returning rates are typically not estimated on a daily basis and therefore the polyhedral functions will not be updated very often.

For each station we have approximated the EUDF by fine discretization to intervals of one minute. This was done for varying values of \( \phi \), the probability of a bicycle to be returned unusable, in the range 0%-5% in increments of 1%. The penalties for shortages were set to \( p = h = 1 \), so that the value of the function represents the expected total number of users who face shortages of bicycles or lockers. For each station and each polyhedral function, we calculated the maximal absolute gap with respect to the approximated values of the EUDF and the maximal relative gap over all possible points, where:

\[
\text{Relative gap} = \frac{\text{absolute gap}}{\text{EUDF value}}
\]

In Table 6.4, we present the aggregated values for all 232 stations. In the first column the probability for a bicycle to be returned unusable is presented. The second column presents the number of stations in which there was no gap at all in fitting the convex polyhedral function. The third and fourth columns represent the maximal absolute and relative gaps over all stations.

<table>
<thead>
<tr>
<th>Probability that a bicycle is returned unusable</th>
<th>Number of stations with no gaps (out of 232)</th>
<th>Expected daily bicycle and locker shortages</th>
<th>Maximal absolute gap</th>
<th>Maximal relative gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>210</td>
<td>0.000151</td>
<td>0.000015</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>211</td>
<td>0.000153</td>
<td>0.000014</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>209</td>
<td>0.000155</td>
<td>0.000014</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>214</td>
<td>0.000157</td>
<td>0.000014</td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td>214</td>
<td>0.000160</td>
<td>0.000014</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>217</td>
<td>0.000161</td>
<td>0.000015</td>
<td></td>
</tr>
</tbody>
</table>

One can observe in Table 6.4 that the maximal relative gap is negligible. That is, the EUDF is approximated very accurately by a convex polyhedral function and this accuracy is insensitive to \( \phi \) within the examined range. Indeed, for any practical purpose, the convex polyhedral function can be considered as an exact description of the approximated EUDF. The fact that the gaps are so small, strengthens our belief that in real life scenarios the EUDF is convex-extensible even when Assumption 6.1 is relaxed.

Moreover, recall that in the case \( \phi = 0 \), the EUDF is \( M^\delta \)-convex (Theorem 6.3), and therefore a polyhedral convex function should be fitted with no gap. The fact that we observe similar gaps in this case indicates that they may originate from numerical
errors in the approximation of the EUDF values and not from the true structure of the function.

In Table 6.4, the range of probabilities we examined was 0%-5%. In order to estimate the probability that a bicycle is returned unusable in a real system, relevant information should be collected. To the best of our knowledge, the only bike-sharing operator that publishes maintenance reports is NYC Bikeshare. The total number of trips taken in this system in 2014 was 8,791,987 and the total number of bicycle repairs was 34,806. Therefore, a reasonable estimator of $\phi$ is about 0.004 (0.4%). The results provided in Table 6.4 demonstrate that for such a probability, the approximation of a polyhedral convex function is very accurate.

Next, we focus on a single station and examine the effect of properly estimating the number of unusable bicycles. As an example, we present in Table 6.5 the approximated EUDF values for a station with 10 lockers. The values are calculated for $\phi = 1\%$ and $h = p = 1$. As can be observed, a change of one unit of usable bicycles in the initial setting, may lead to an increase or a decrease in the users’ dissatisfaction by less than one unit of shortage. However, a change in the initial number of unusable bicycles may lead to a greater change in the users’ dissatisfaction. This can be explained by the fact that increasing the number of unusable bicycles by one is equivalent to decreasing the effective capacity of the station by one, which may increase the bicycle or locker shortages by more than one. Indeed, wrong information about the number of unusable bicycles may lead to discrepancies in user dissatisfaction estimation. For example, in Table 6.5, if in the initial setting there are no unusable bicycles, there is a small difference in user dissatisfaction if the initial number of usable bicycles is between 2 to 6 [11.32-11.73]. Nevertheless, if two of these bicycles are actually unusable, the user dissatisfaction increases by at least 3 units [14.28-15.36]. This emphasizes the need for having correct information regarding unusable bicycles and for the proper planning of collecting them. In fact, collecting unusable bicycles may have a greater effect on the service level at a station as compared to addition/reduction of usable bicycles and therefore should be prioritized.

**Table 6.5: User dissatisfaction as a function of the initial usable and unusable bicycles in a station with 10 lockers**

<table>
<thead>
<tr>
<th>Initial number of unusable bicycles</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
</table>
6.5. Discussion
In this chapter, we have extended the user dissatisfaction function to account for the amount of unusable bicycles in addition to the number of usable bicycles. The presence of unusable bicycles highly affects the quality of service given to the users of bike sharing systems. Maintenance aspects in bike sharing are first studied in this paper and in Chapter 5 (Kaspi et al. 2015b).

We have demonstrated that a convex polyhedral function can be accurately fitted to the EUDF. In particular, we have proved that the EUDF is $M^n$-convex (and thus convex-extensible) for $\phi = 0$. The results of our numerical experiment suggest that in real life setting the EUDF is convex-extensible also for $\phi > 0$. As a consequence, the EUDF can be used in linear optimization models for planning of the operational activities. In addition, the extended user dissatisfaction model can assist in strategic planning, e.g., deciding on the size of the bicycle fleet, capacity of the stations, manpower requirement for operations and maintenance activities, etc.

The numerical results demonstrate that the presence of unusable bicycles may highly increase user dissatisfaction. Thus, even though only a small fraction of the bicycles is returned unusable the effect of these bicycles is significant. Therefore, this matter should receive more attention in the planning process. Particularly, system operators should invest resources in detection and collection of unusable bicycles. Accurate information regarding bicycle usability should be obtained and made available to the operators and the users of the system. The former can use it to optimize the maintenance and repositioning activities and the latter to better plan their itineraries.
7. Conclusions

In this dissertation, we have studied two aspects of bike-sharing systems that can enhance their operation: implementation of parking reservation policies and planning of maintenance operations.

In Part I we have proposed for the first time the implementation of parking reservation policies in bike-sharing systems. In Chapter 3, we demonstrated that the excess time spent by users under the CPR policy is lower as compared to having no passive regulation, the current practice in many systems (NR policy). This result was obtained by both an analytical analysis of a Markovian model of a simplified system and by a simulation of real world systems.

In Chapter 4, we further examined the effectiveness of parking reservations in bike-sharing systems as a method to enhance the quality of service provided to the users. We formulated MILP models to bound the excess time under any passive regulation, and in particular under any parking reservation policy. Using these bounds, we have demonstrated that a significant share of the excess time that could be theoretically saved by any passive regulation, is already saved by the CPR policy. Furthermore, we have shown that other, more sophisticated parking reservation policies, are not likely to be able to save substantially (if at all) more excess time.

In addition, we have studied several partial parking reservation policies. While these policies do not outperform the CPR, they also have shown to produce lower excess time with respect to the basic NR policy. These partial policies should be considered in cases where system operators are reluctant to implement completely parking reservations.

The MILP models presented in Chapter 4, can be used to evaluate the effectiveness of various other bike-sharing related policies. The models reflect the fact that each journey may be assigned to one of several itineraries. This adds a lot of flexibility to the system and affects its dynamics in a way that should not be ignored by a strategic planner. Though we have focused on reducing the excess time of users, our model can be extended to accommodate other objectives of the commuters. That is, each potential itinerary can be assigned with a measure that reflects a combination of several objectives.

Reservation policies reduce the uncertainty related to the usage of bike-sharing systems. The guarantee that a parking space will be available upon the return of the bicycle at the destination can save the time and anxiety associated with the possibility of having to search for a parking space. In addition, we note that by placing or trying to place reservations, the users reveal information that is currently not available to the operators of bike-sharing systems. Such information may be useful both for operational and strategic decisions. For example, information received via reservations can assist the operators in predicting the near future state of the system. This may allow better short term planning of repositioning activities. Furthermore, information on journeys that cannot be realized is collected. Such information is vital when planning capacity expansion of stations in the system.

In Part II we have studied for the first time maintenance operations in bike-sharing systems. In bike sharing systems a small share of the bicycles/lockers become unusable
every day. The negative implication of the presence of unusable bicycles/lockers is the reduction of the station capacity and the presentation of misleading information to the users. In the current setting of bike-sharing systems, there is no reliable on-line information that indicates the usability of bicycles/lockers.

In Chapter 5, we presented a method to detect unusable resources (bicycles and lockers) in bike sharing systems. We formulated a model that estimates the probability that a specific bicycle is unusable as well as the number of unusable bicycles in a station, based on available trip transaction data. In addition, we presented some information based enhancements of the model and discussed an equivalent model for detecting locker failures.

Unusable bicycles/lockers may have different effect on the quality of service in different stations, depending on the capacity of the station and the demand patterns. Evaluating this effect may assist in prioritizing the stations that should be visited by maintenance and repositioning workers.

In Chapter 6, we extended the User Dissatisfaction Function (introduced by Raviv and Kolka, 2013) to distinguish between usable and unusable bicycles. Specifically, the EUDF measures the expected shortages of bicycles and lockers as a bivariate function of the initial inventory of usable and unusable bicycles. We have analyzed and proved the convexity of the resulting bivariate function and provided a method to accurately approximate it by a convex polyhedral function. Consequently, the EUDF can be used in linear optimization models for operational and strategic decision making in bike sharing systems.

The numerical results of this chapter demonstrate that the presence of unusable bicycles/lockers may highly increase user dissatisfaction. Thus, even though only a small fraction of the bicycles/lockers become unusable, the effect of these bicycles/lockers is significant. This in turn emphasizes the need for having accurate real-time information regarding bicycle/locker usability.

For conclusion, in this dissertation we presented novel approaches that can improve the management of bike-sharing systems and the quality of service given to their users. We look forward to seeing the effect of the implementation of our ideas in real world systems.
References


Barlow, R.E., Proschan, F. (1975) Statistical theory of reliability and life testing: probability models, FLORIDA STATE UNIV TALLAHASSEE.


Appendix A - Proof of Lemma 3.4.

Lemma 3.4: In a NR policy where $\lambda T = 0$, the expected excess time rate due to waiting is a positive constant independent of $\lambda$ and $T$.

Proof: In this case the expected travel time is negligible. The system state representation can be degenerated to account only for the number of vehicles in each station, denoted by $y = (y_1, y_2, \ldots, y_S)$. We denote the set of all possible states by $Y$. A rent transition between two possible states $y, y'$ is denoted by the following indicator function:

$$
\epsilon_{ij}(y, y') = \begin{cases} 
1, & \text{if } y' = (y_1, \ldots, y_i - 1, \ldots, y_j + 1, \ldots, y_S) \\
0, & \text{otherwise}
\end{cases}
$$

The transition rates between any two possible states $y, y'$ are given by:

$$
\sum_{i,j} v_{ij} \lambda \epsilon_{ij}(y, y')
$$

The resulting set of steady state equations are:

$$
\sum_{y' \in Y} \sum_{i,j} v_{ij} \lambda \pi(y) \epsilon_{ij}(y, y') = \sum_{y' \in Y} \sum_{i,j} v_{ij} \lambda \pi(y') \epsilon_{ij}(y', y) \quad \forall y
$$

An equivalent system of equations can be received by multiplying all equations by $\frac{1}{\lambda}$ (eliminating $\lambda$ from all equations). Therefore, the limiting probabilities are independent of $\lambda$ and $T$. Recall that the Markov chain has a finite state space. Assuming that for any station $i$ there exists at least one station $j$ such that $\lambda_{ji} > 0$, the chain is both irreducible and positive recurrent, see Norris (1997). Therefore, all limiting probabilities are positive. As a result, when $\lambda T = 0$ the expected excess time rate due to waiting in a queue, (3.3), is equal to some positive constant. ■
Appendix B - Testing the accuracy of the approximation method presented in Section 5.2

We present here a numerical study carried out in order to test the accuracy of the approximated PoU (Section 5.2) as compared to the exact calculation (Section 5.1). We have used trip history transactions from the Capital Bikeshare system in Washington DC, during the 2nd quarter of 2013. We have selected 20 stations with capacities smaller than 15, for which the exact calculation could be done in a reasonable time. The preprocessing of the trip history data and the simulation were executed in the same manner as described in Section 5.3.

In Table B.1 we present the simulation results. The first five columns present information about the realizations, as in Table 5.1. In the two rightmost columns we present the average and the maximum absolute difference, over 100 realizations, between the exact and the approximated calculations of the expected number of unusable bicycles. Observing the rightmost two columns in the table, one can notice that the exact and the approximated calculation of the PoU result with very similar estimations of the number of unusable bicycles.

<table>
<thead>
<tr>
<th>Station ID</th>
<th>Capacity</th>
<th>Number of Rents</th>
<th>Number of Returns</th>
<th>Unusable Bicycles</th>
<th>Average difference</th>
<th>Maximal difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>73</td>
<td>14</td>
<td>151.89</td>
<td>151.11</td>
<td>1.46</td>
<td>0.0079</td>
<td>0.1683</td>
</tr>
<tr>
<td>118</td>
<td>14</td>
<td>134.81</td>
<td>136.56</td>
<td>1.35</td>
<td>0.0059</td>
<td>0.1147</td>
</tr>
<tr>
<td>183</td>
<td>13</td>
<td>128.74</td>
<td>127.95</td>
<td>1.34</td>
<td>0.0057</td>
<td>0.0576</td>
</tr>
<tr>
<td>187</td>
<td>14</td>
<td>119.29</td>
<td>118.01</td>
<td>0.97</td>
<td>0.0066</td>
<td>0.1049</td>
</tr>
<tr>
<td>186</td>
<td>14</td>
<td>118.32</td>
<td>123.33</td>
<td>1.30</td>
<td>0.0085</td>
<td>0.0958</td>
</tr>
<tr>
<td>201</td>
<td>11</td>
<td>115.52</td>
<td>113.97</td>
<td>1.16</td>
<td>0.0120</td>
<td>0.1117</td>
</tr>
<tr>
<td>188</td>
<td>10</td>
<td>113.77</td>
<td>114.15</td>
<td>1.08</td>
<td>0.0081</td>
<td>0.0949</td>
</tr>
<tr>
<td>126</td>
<td>13</td>
<td>84.08</td>
<td>84.31</td>
<td>0.77</td>
<td>0.0056</td>
<td>0.0545</td>
</tr>
<tr>
<td>136</td>
<td>14</td>
<td>76.64</td>
<td>80.10</td>
<td>0.77</td>
<td>0.0098</td>
<td>0.1626</td>
</tr>
<tr>
<td>155</td>
<td>11</td>
<td>47.31</td>
<td>41.89</td>
<td>0.53</td>
<td>0.0113</td>
<td>0.1346</td>
</tr>
<tr>
<td>209</td>
<td>14</td>
<td>46.63</td>
<td>46.19</td>
<td>0.56</td>
<td>0.0077</td>
<td>0.1269</td>
</tr>
<tr>
<td>163</td>
<td>11</td>
<td>44.15</td>
<td>41.15</td>
<td>0.32</td>
<td>0.0092</td>
<td>0.0973</td>
</tr>
<tr>
<td>107</td>
<td>11</td>
<td>43.43</td>
<td>41.18</td>
<td>0.48</td>
<td>0.0108</td>
<td>0.1934</td>
</tr>
<tr>
<td>191</td>
<td>11</td>
<td>40.85</td>
<td>43.93</td>
<td>0.43</td>
<td>0.0079</td>
<td>0.0892</td>
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<tr>
<td>80</td>
<td>14</td>
<td>38.35</td>
<td>31.61</td>
<td>0.33</td>
<td>0.0091</td>
<td>0.1023</td>
</tr>
<tr>
<td>88</td>
<td>10</td>
<td>35.54</td>
<td>36.58</td>
<td>0.36</td>
<td>0.0166</td>
<td>0.1731</td>
</tr>
<tr>
<td>175</td>
<td>11</td>
<td>34.88</td>
<td>34.26</td>
<td>0.27</td>
<td>0.0065</td>
<td>0.1235</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>34.55</td>
<td>37.84</td>
<td>0.45</td>
<td>0.0148</td>
<td>0.1498</td>
</tr>
<tr>
<td>154</td>
<td>14</td>
<td>32.18</td>
<td>22.61</td>
<td>0.17</td>
<td>0.0030</td>
<td>0.0881</td>
</tr>
<tr>
<td>140</td>
<td>14</td>
<td>30.54</td>
<td>30.79</td>
<td>0.37</td>
<td>0.0140</td>
<td>0.1271</td>
</tr>
</tbody>
</table>
Appendix C - Proof of Lemma 6.2

Lemma 6.2: under Assumption 6.1, the EUDF $F(I_0, B_0)$ is convex in the initial inventory of unusable bicycles $B_0$, i.e.: $F(I_0, B_0 + 2) - F(I_0, B_0 + 1) \geq F(I_0, B_0 + 1) - F(I_0, B_0)$. 

Proof: Consider the shortage occurrences given three settings of a station: $(I_0, B_0 + 2)$, $(I_0, B_0 + 1)$ and $(I_0, B_0)$, i.e., when the number of usable bicycles is identical, but the number of unusable bicycles differs by one and two. We study the bicycle shortage indicator differences, $\Delta^R_J(I_0, B_0 + 2) - \Delta^R_I(I_0, B_0 + 1)$ and $\Delta^R_J(I_0, B_0 + 1) - \Delta^R_I(I_0, B_0)$ and demonstrate that each shortage occurrence in which the latter equals 1 is preceded by at least one shortage occurrence in which the former equals 1. This is also demonstrated for locker shortages. Therefore, by summing over all demand occurrences we obtain that the inequality holds for any demand realization that satisfies Assumption 6.1.

We distinguish between eight possible shortage events, as described in Table C.1 and present the shortage differences in Table C.2:

<table>
<thead>
<tr>
<th>Shortage event</th>
<th>Usable bicycles before the shortage occurs</th>
<th>Shortage type</th>
<th>Usable bicycles after the shortage occurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) - 1 = I_{n+1}^U(I_0, B_0) - 2$</td>
<td>$\Delta^U_{n+1}(I_0, B_0 + 2) = 1$</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) = I_{n+1}^U(I_0, B_0) - 1$</td>
</tr>
<tr>
<td>2</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) - 1 = I_{n+1}^U(I_0, B_0) - 1$</td>
<td>$\Delta^U_{n+1}(I_0, B_0 + 2) = 1$</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) = I_{n+1}^U(I_0, B_0)$</td>
</tr>
<tr>
<td>3</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) = I_{n+1}^U(I_0, B_0) - 1$</td>
<td>$\Delta^U_{n+1}(I_0, B_0 + 2) = 1$</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) = I_{n+1}^U(I_0, B_0)$</td>
</tr>
<tr>
<td>4</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) = I_{n+1}^U(I_0, B_0)$</td>
<td>$\Delta^U_{n+1}(I_0, B_0 + 2) = 1$</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) = I_{n+1}^U(I_0, B_0)$</td>
</tr>
<tr>
<td>5</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) - 1 = I_{n+1}^U(I_0, B_0) - 2$</td>
<td>$\Theta^U_{n+1}(I_0, B_0 + 2) = 1$</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) - 1 = I_{n+1}^U(I_0, B_0) - 2$</td>
</tr>
<tr>
<td>6</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) - 1 = I_{n+1}^U(I_0, B_0) - 1$</td>
<td>$\Theta^U_{n+1}(I_0, B_0 + 2) = 1$</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) - 1 = I_{n+1}^U(I_0, B_0) - 2$</td>
</tr>
<tr>
<td>7</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) = I_{n+1}^U(I_0, B_0)$</td>
<td>$\Theta^U_{n+1}(I_0, B_0 + 2) = 1$</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) - 1 = I_{n+1}^U(I_0, B_0) - 2$</td>
</tr>
<tr>
<td>8</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) = I_{n+1}^U(I_0, B_0)$</td>
<td>$\Theta^U_{n+1}(I_0, B_0 + 2) = 1$</td>
<td>$I_{n+1}^U(I_0, B_0 + 2) = I_{n+1}^U(I_0, B_0 + 1) - 1 = I_{n+1}^U(I_0, B_0) - 2$</td>
</tr>
</tbody>
</table>

Table C.2: Shortage indicator differences

<table>
<thead>
<tr>
<th>Shortage event</th>
<th>$\Delta^J(I_0, B_0 + 2) - \Delta^J(I_0, B_0 + 1)$</th>
<th>$\Delta^J(I_0, B_0 + 1) - \Delta^J(I_0, B_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shortage event</th>
<th>$\Theta^R(I_0, B_0 + 2) - \Theta^R(I_0, B_0 + 1)$</th>
<th>$\Theta^R(I_0, B_0 + 1) - \Theta^R(I_0, B_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
In Figure C.1 the possible transitions between shortage events are presented. Since at time $t = 0$ (before the first demand occurrence) we have $I_0^R(I_0, B_0 + 2) = I_0^R(I_0, B_0 + 1) = I_0^R(I_0, B_0) = I_0$, the first shortage event may be either 4 or 8. Notice that each occurrence of event 6 is preceded by at least one occurrence of event 8. Similarly each occurrence of event 3 is preceded by at least one occurrence of event 1. Therefore by summing over all the demand occurrences of a given demand realization, we obtain:

$$
\sum_{j=1}^{\left|E^R\right|} \left( \Delta_j^R(I_0, B_0 + 2) - \Delta_j^R(I_0, B_0 + 1) \right) \geq \sum_{j=1}^{\left|E^R\right|} \left( \Delta_j^R(I_0, B_0 + 1) - \Delta_j^R(I_0, B_0) \right)
$$

and:

$$
\sum_{j=1}^{\left|E^R\right|} \left( \Theta_j^R(I_0, B_0 + 2) - \Theta_j^R(I_0, B_0 + 1) \right) \geq \sum_{j=1}^{\left|E^R\right|} \left( \Theta_j^R(I_0, B_0 + 1) - \Theta_j^R(I_0, B_0) \right)
$$

By multiplying the above inequalities by the relevant shortage penalties and summing the two inequalities we obtain:

$$
F^R(I_0, B_0 + 2) - F^R(I_0, B_0 + 1) \geq F^R(I_0, B_0 + 1) - F^R(I_0, B_0).
$$

Since this inequality holds for each demand realization that satisfies Assumption 6.1, it also holds for the expectation. ■
Appendix D – Integrated optimization model for bicycle repositioning and collection of unusable bicycles

In the following model, we assume that the same fleet of trucks is responsible for both repositioning activities and for collection of unusable bicycles. The Arc-indexed formulation presented in Raviv et al. 2013 is extended here to include decisions regarding the collection of unusable bicycles. In particular, the EUDF, which was presented in Chapter 6, is used as part of the objective function to represent the quality of service given to the users.

In the setting of this model, we have a fleet of repositioning trucks that operates during the night, when it is reasonable to assume that the movement of bicycles is negligible. The current state of each station in terms of usable and unusable bicycles is given. The model decides upon the routing of each truck and the loading/unloading of usable and unusable bicycles. The goal is to minimize a weighted sum of the trucks’ routing costs and penalties associated with the quality of service in each station (EUDF values), subject to a time constraint on the total travel time of each truck.

We assume that each station may be visited at most once by each vehicle, although a certain station may be visited by several vehicles. This assumption reduces the feasible solution set and allows solving larger instances, but may result in inferior solutions, see discussion in Raviv et al. 2013.

We use the following notations:

Parameters:

- \( N \) Set of stations, indexed by \( i = 1, \ldots, |N| \)
- \( N_0 \) Set of nodes, including the stations and the depot (denoted by \( i = 0 \), \( i = 0, \ldots, |N| \))
- \( V \) Set of vehicles, \( v = 1, \ldots, |V| \)
- \( s_i^0 \) Number of usable bicycles at node \( i \) before the repositioning operations start
- \( b_i^0 \) Number of unusable bicycles at node \( i \) before the repositioning operations start
- \( c_i \) Number of lockers installed at station \( i \in N_0 \), referred to as station’s capacity
- \( k_v \) Capacity (number of bicycles) of vehicle \( v \in V \)
- \( f_i(s_i, b_i) \) A penalty function for station \( i \in N \), the function is defined over the integers \( s_i = 0, \ldots, c_i \) and \( b_i = 0, \ldots, c_i \)
- \( t_{ij} \) Traveling time from station \( i \) to station \( j \)
- \( \alpha \) Weight/scaling factor of the operating costs relative to the penalty costs
- \( T \) Repositioning time, i.e., time allotted to the repositioning operation
- \( L \) Time required to remove a bicycle from a station and load it onto the vehicle
- \( U \) Time required to unload a bicycle from the vehicle and hook it to a locker in the station
Decision variables

\[ x_{ijv} \quad \text{Binary variable which equals one if vehicle } v \text{ travels directly from node } i \text{ to node } j, \text{ and zero otherwise} \]

\[ y_{ijv} \quad \text{Number of usable bicycles carried on vehicle } v \text{ when it travels directly from node } i \text{ to node } j. \ y_{ijv} \text{ is zero if the vehicle does no travel directly from } i \text{ to } j \]

\[ y'_{ijv} \quad \text{Number of usable bicycles loaded onto vehicle } v \text{ at node } i \]

\[ y''_{ijv} \quad \text{Number of usable bicycles unloaded from vehicle } v \text{ at node } i \]

\[ z_{ijv} \quad \text{Number of unusable bicycles carried on vehicle } v \text{ when it travels directly from node } i \text{ to node } j. \ z_{ijv} \text{ is zero if the vehicle does no travel directly from } i \text{ to } j \]

\[ z'_{ijv} \quad \text{Number of unusable bicycles loaded onto vehicle } v \text{ at node } i \]

\[ z''_{ijv} \quad \text{Number of unusable bicycles unloaded from vehicle } v \text{ at the depot} \]

\[ \tau_{iv} \quad \text{The time vehicle } v \text{ completes loading/unloading at node } i \text{ (zero if not visiting node } i) \]

\[ s_i \quad \text{Number of usable bicycles at node } i \text{ at the end of the repositioning operation} \]

\[ b_i \quad \text{Number of unusable bicycles at node } i \text{ at the end of the repositioning operation} \]

The integrated model for repositioning and collection of unusable bicycles is formulated as follows:

Min \[ \sum_{i \in N} f_i(s_i, b_i) + \alpha \sum_{i \in N_0} \sum_{j \in N_0} \sum_{v \in V} t_{ij} x_{ijv} \] \hspace{1cm} (D.1)

s.t.

\[ s_i = s_i^0 - \sum_{v \in V} (y'_{iv} - y''_{iv}) \quad \forall i \in N \] \hspace{1cm} (D.2)

\[ b_i = b_i^0 - \sum_{v \in V} (z'_{iv} - z''_{iv}) \quad \forall i \in N \] \hspace{1cm} (D.3)

\[ y''_{iv} - y'_{iv} = \sum_{j \in N_0, j \neq i} y_{ijv} - \sum_{j \in N_0, j \neq i} y_{jiv} \quad \forall i \in N_0, \forall v \in V \] \hspace{1cm} (D.4)

\[ z''_{iv} = \sum_{j \in N_0, j \neq i} z_{ijv} - \sum_{j \in N_0, j \neq i} z_{jiv} \quad \forall i \in N, \forall v \in V \] \hspace{1cm} (D.5)

\[ z''_{0v} = \sum_{j \in N_0, j \neq i} z_{0jv} - \sum_{j \in N_0, j \neq i} z_{0jv} \quad \forall v \in V \] \hspace{1cm} (D.6)

\[ y_{ijv} + z_{ijv} \leq k_v x_{ijv} \quad \forall i, j \in N_0, i \neq j, \forall v \in V \] \hspace{1cm} (D.7)

\[ \sum_{j \in N_0, j \neq i} x_{ijv} = \sum_{j \in N_0, j \neq i} x_{jiv} \quad \forall i \in N_0, \forall v \in V \] \hspace{1cm} (D.8)

\[ \sum_{j \in N_0, j \neq i} x_{ijv} \leq 1 \quad \forall i \in N, \forall v \in V \] \hspace{1cm} (D.9)

\[ \sum_{v \in V} y'_{iv} \leq s_i^0 \quad \forall i \in N_0 \] \hspace{1cm} (D.10)

\[ \sum_{v \in V} z'_{iv} \leq b_i^0 \quad \forall i \in N_0 \] \hspace{1cm} (D.11)

\[ \sum_{v \in V} (y''_{iv} - z''_{iv}) \leq c_i - s_i^0 - b_i^0 \quad \forall i \in N_0 \] \hspace{1cm} (D.12)
The objective function (D.1) minimizes the total cost of the system, consisting of the sum of the penalties associated with the quality of service in each station and the sum of the trucks’ routing costs. Constraints (D.2)-(D.3) set the final inventory level of usable and unusable bicycles, respectively, at each station. Constraints (D.4)-(D.6) verify the conservation of inventory levels on the trucks, of usable and unusable bicycles. Note that an underlying assumption in constraints (D.5)-(D.6) is that unusable bicycles can only be loaded onto the vehicles at the stations and can only be unloaded at the depot. Constraints (D.7) ensure that the total quantity of bicycles carried on a truck does not exceed its capacity. In addition, it allows a positive quantity to be carried on a direct path between two stations, only if the truck travels directly between those stations. Constraints (D.8) stipulate that if a truck enters a node, it also leaves that node. By Constraints (D.9), each vehicle is allowed to visit each station at most once. Constraints (D.10)-(D.11) limit the number of usable and unusable bicycles loaded in a station, respectively, to the initial inventory at the depot. Constraints (D.12) limit the number of usable bicycle that are loaded onto the vehicles at the stations to the residual capacity of the station plus the number of lockers that become vacant due to loading of unusable bicycles. Constraints (D.13) stipulate that all the usable bicycles that are loaded onto the vehicles are also unloaded. Constraints (D.14) ensure that all unusable bicycles that are loaded onto the vehicles are unloaded at the depot. Constraints (D.15) limit the total operation time of the vehicles that includes traveling times and loading/unloading times. Constraints (D.16)-(D.18) sets for each vehicle the completion time of loading/unloading operations. Note that by constraints (D.16)-(D.18) for each vehicle, the visiting order of the stations is determined, and therefore sub-tours are eliminated. Constraints (A.19) stipulate that the completion time of a vehicle at a station may be positive only if the vehicle visits the station. (D.20)-(D.22) are binary and general integrality constraints and (D.23)-(D.25) are non-negativity constraints.

Next, we replace the functions \( f_i(s_i, b_i) \) with the polyhedral functions calculated in Chapter 6 to obtain a MILP formulation. Let \( \theta_i = \)
\{(s, b) \in \mathbb{Z}^2 | s \geq 0, b \geq 0, s + b \leq c_i\} \text{ be the set of all possible inventory levels of station } i \text{ and let } g_i \text{ be the penalty incurred at station } i. \text{ In the objective function, we replace } f_i(s_i, b_i) \text{ by } g_i \text{ and add the following set of constraints to the model:}

\[ g_i \geq \alpha^i_{s,b} \cdot s_i + \beta^i_{s,b} \cdot b_i + \gamma^i_{s,b} \quad \forall (s, b) \in \theta_i \quad \forall i \in N \]

where the coefficients \( \alpha^i_{s,b}, \beta^i_{s,b}, \gamma^i_{s,b} \) are precomputed using the method described in Chapter 6.
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– מודל אופטימיזציה של שימור אופנים ואיסוף אופנים תקולים
מעודכרא תקציר

מערכות שיתוף כלוב של אופניים הפכו למראה שכיח בערים גדולות רבות ברחבי העולם. נכון לעתה, קיימות כ-900 מערכות שיתוף אופניים פעילות (למעלה ממיליון אופניים משותפים) ואまだまだ越多 least 200 מערכות נוספים מתוכן. מערכות אלה מותאמות לצרכים שונים ומשמשות לשיפור תחבורה ב toJsonратות שונות.

הทรבות החינוכיות ל봇 הביאו את חליפת אחריות והוס厣תكتشف phốות על למערכות שיתוף אופניים.

בעבודה זאת אנו בוחרים שני יבטים שעשויים להביא לשיפור התפעול של מערכות שיתוף אופניים: (1) חפגת מדיניות הזמנת חניות וכן (2) תכנון פעולות התחזוקה.

במערכת הפועלת תחנת מדיניות הזמנת חניות כלשהי, על המשתמשים לבצע הזמנה של חניית אופניים בעמדות עגינה בעת השכרת האופניים, וחניות אלו נשמרות עבורם. אנו מציגים מספר סוגים של מדיניות הזמינות חניה, בין היתר, מדיניות הזמנת חניה המלאה, בה כל המשתמשים נדרשים לבצע הזמנת חניה בעת השכרה. במדיניות האחרות, משתמשים נדרשים לבצע הזמנת חניה רק במשרדים מסוימים, או לא נדרשים כלל לעשות זאת. באמצעות מודל מרקוב של מערכות מפושטות וסימולציה של מערכותリアル של ממלכות ישנו את מדיניות הזמינות שונות בין זה.

בנוסף, אנו מציגים חסמים המבוססים על ת↙עunan מתמטי, על הזמן העודפי הכולל שמשתמשים שוהים במערכת תחת כל תור של פעולות פסיביות ובפרט תחת כל מדיניות הזמנת חניה. התוצאות מדגימות את הבטיחות של מדיניות הזמנת חניה המלאה וברחבי mondo כישון לשירות החינוךзначתיות של מדיניות הזמינות חניה.

החופה, אם כי קלאף כל מדיניות משובחים אין במעמד תחנת חניה.

הופעת תחנת חניה היא מת 된ת מחפשה חלופית לשער מערכות שיתוף אופניים. במעрактиו אלה, זוהי קבוצת מדיניות מחקוקית מידי. כי, לא יכין MediaTek אם בagnar ממאמץ במדינת רמות השם שיתוף, ישנה עמדת אך שיתוף אופניים,כמו כן מנהל ההספנות ושילוב של אופניים. אנו מציגים מדיניות התחזוקה של חניית אופניים והotentקויות של צעדים שמעונים במערך, ומציגים את עומס התקרה בשימוש.ccבפוקסמכ תק 않을 מחשבות בשום באלכסון אופטיים.

הטרובס מדיניות מגוון تقديم ר偎ר לברחת ומערכים שונים של מערכות שיתוף אופניים על מסיומין וגרמיה בשתייה של לאנים למיתוס אחריות הפונקיציות של מערכות שיתוף אופניים במערך אופטיים, או מערכים שונים של מערכות שיתוף אופניים לעבר טריב של מערכות שיתוף אופניים. }

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מאת

مور כספי

חיבורים לשנת קבלה החוארים ‘דוקטור לפילוסופיה’
הוגש לסנגור של אוניברסיטת תל-אביב

עבודה זאתגעשתהbang מיכל צור ודר' טל רביב

משרר תשע"י
מערכיון לשיתוף אופניים: מדיניות הזמנת חניית פעולות החוזקה

מאת

מר כסי

חיבור לשם קבלת התואר" דקוטר לפילוסופיה"
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תש"ע ו